Model of the flame front evolution with inertial effects

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1 Introduction

The general combustion theory concept assumes that flame evolution is controlled by burning velocity [1]. According to this concept the instant flame velocity may be calculated from stationary problem including parameters that are determined by current flame position and environment. In the case of flat flame propagating in the channel the flame position may be determined by one spatial coordinated counted along the channel. In the case of channel with variable cross section and walls temperature gradient the problem of description of flame evolution has emerged. For practical application it is desirable to use simple model to elucidate the physical mechanism responsible for the flame dynamics in microchannels. The objectives of the present study is the formulation of a reduced model of flame evolution in micro channel that takes into account variation of channel cross section and temperature of the channel walls. The basic equations constituting of the thermo-diffusive model of flame propagation in the channel were reduced to the system of the two ordinary differential equations describing the flame front dynamic. In the present study we demonstrate that classical concept of burning velocity meets with difficulties in modeling, for example, flame ignition, extinction and nonlinear flame oscillations. Modeling of flame oscillations, for example, requires at least a consideration of the flame acceleration (the flame front "inertia") and the rate of flame temperature variation (the "inertness" of the flame temperature). The ignorance of these inertial effects may lead to omission of oscillatory modes of flame propagation that in 2D or 3D can manifest itself in the forms of traveling waves, spinning modes of combustion or as chaotic motions of the flame front cells. Since oscillatory instability are exhibited near the flammability limits caused by heat losses, the account of the inertial effect is especially important for modeling of flame in this case. Flame propagation problems discussed in this paper probably may extend the existing flamelet models to describe combustion in non-uniform and non-stationary gas flows.

2 Flame front evolutionary equations

For a plane flame moving along the axis x in the channel with temperature $\theta(x)$ and with the transversal diameter $d(x)$ (see Fig.1) the conventional constant-density, localized-reaction-zone model reads,

$$\frac{\partial T_{1,2}}{\partial t} = \frac{\partial^2 T_{1,2}}{\partial x^2} + V(x)\frac{\partial T_{1,2}}{\partial x} - \Omega(x)(T_{1,2} - \theta(x)) \tag{1}$$

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$$\frac{\partial C_1}{\partial t} = \frac{\partial^2 C_1}{\partial x^2} + V(x)\frac{\partial C_1}{\partial x}\,,\ C_2 = 0 \tag{2}$$

Here T is the scaled gas temperature in units of T_b, the adiabatic temperature of combustion products; $\theta(x)$ is the channel walls temperature in units of T_b; C, scaled concentration in units of C_0, its value in the fresh mixture; x , scaled coordinate in units of $l_{th} = D_{th}/U_b$, the thermal width of the flame; D_{th} , thermal diffusivity of the mixture; U_b, velocity of a planar adiabatic flame; $V(x)$ is gas velocity in units of U_b and $V(x)d(x)^2 = const$; $\sigma = T_0/T_b$, where T_0 is initial temperature; $\Omega(x) = 4Nu\, l_{th}^2 / d(x)^2$ is the heat exchange parameter, where $Nu = \alpha d(x)/\lambda_g$ is Nusselt number. Subscripts 1 and 2 correspond to the fresh mixture and the combustion products, respectively. Equations (1) and (2) are subjected to the following boundary conditions in the far field

$$C_1 \to 1,\ \text{as}\ \ x \to +\infty;\ T_{1,2}, \theta \to \sigma\ \ \text{as}\ \ x \to \pm\infty \tag{3}$$

and at the flame interface $x = x_f(t)$

$$T_{1,2} = T_f\,;\ C_1 = 0\,; \qquad \frac{\partial T_2}{\partial x} - \frac{\partial T_1}{\partial x} = (1-\sigma)\frac{\partial C_1}{\partial x}\,;\ \frac{\partial C_1}{\partial x} = exp\left[\frac{N}{2}\left(1 - \frac{1}{T_f}\right)\right] \tag{4}$$

Here, $N = T_a/T_b$, scaled activation energy, with T_a being the activation temperature. It is assumed that the wall temperature $\theta(x)$ is the time-constant and the temperature gradient is negligibly small value $\left|\partial\theta / \partial x\right| = \varepsilon << 1$ as compared with flame heat width. The channel diameter $d(x)$ changes smoothly and the derivatives $\left|\partial d(x)/\partial x\right| = O(\varepsilon)$ is also assumed to be a small value.

The evolution equations were obtained by employing slow evolution approximation. This

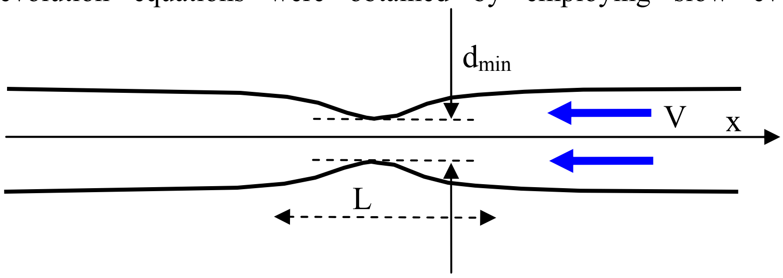

Fig.1
Scheme of microchannel of variable cross section

approximation assumes that characteristic length of variation of the channel diameter L or the wall temperature is larger than flame heat width $l_{th} = D_{th}/U_b$. In this case the time derivatives of temperature and concentration in (1)-(2) have smallness of $1/\tau_c$, where $\tau_c >> 1$ is the non-dimensional characteristic time of flame evolution and they may be considered as a small parameters. According to results [2] the asymptotic solution, for example for concentration, with boundary condition in the far field (3) can be written as:

$$C_1 = 1 - \widetilde{C}\,,\ \frac{\partial \widetilde{C}}{\partial x} = -V\widetilde{C} + \frac{1}{V}\frac{\partial \widetilde{C}}{\partial t} - \frac{1}{V^2}\frac{\partial^2 \widetilde{C}}{\partial t^2} + \ldots O\left(\tau_c^{-3}\right) \tag{5}$$

Omitting the secondary details we have written below the resulting equations describing flame front evolution that may be obtained at the same manner as in paper [2]:

$$\frac{d^2 x_f}{dt^2} = U - \frac{dx_f}{dt} - V \tag{6}$$

$$\frac{dT_f}{dt} = \frac{U}{2}\left[(1-\sigma)U - \left(U^2 + 4\Omega\right)^{1/2}(T_f - \sigma)\right] \tag{7}$$

Here $U = exp(N/2(1-1/T_f))$ and V,Ω and θ are the prescribed function of coordinate x determining at $x = x_f(t)$. Physically, the Eq.(6) without acceleration d^2x_f/dt^2 may be interpreted as condition of flame propagation relative to the fresh mixture with burning velocity U depending on flame temperature. Present modification of this equation by inclusion of flame front acceleration thereby taking into account the flame front "inertia" [2]. Another equation describes balance of heat fluxes at the flame front and it bounds T_f and x_f. In the case of constant temperature of the wall, so that $\Theta = \sigma$, and the stationary flame ($dT_f/dt = 0$), equations (7) and (8) transform into well known formulas [3] describing non-adiabatic flame *:*

$$D_f = U(T_f), \qquad \sqrt{D_f^2 + 4\Omega}\,(T_f - \sigma) = (1-\sigma)D_f \tag{9}$$

Here $D_f = V(x_f) + dx_f/dt$ is burning velocity. Two solutions of Eq.(9) with respect D_f exist if $\Omega < \Omega_c = 1/2e(1-\sigma)N$ and solution with larger velocity is stable and another is unstable. The critical diameter d_c is determined by formula $\Omega = 4Nul_{th}^2/d_c^2 = \Omega_c$.

3 Flame extinction and ignition in the model with inertial effects

The flame repetitive extinction ignition (FREI) in the straight tube with hot channel wall was modelled on the base of system of equation (7), (8). The FREI phenomenon was observed in experiments [4, 5] and it occurs in the tube with smaller diameter than critical diameter corresponding to the temperature of the tube cold side. This case was discussed in many previous papers (see for example [2], [5], [6]) and therefore we demonstrates here that our simple model can capture the FREI phenomena. The stationary temperature profile is approximated by formulae

$$\theta(x) = \sigma + (\Theta - \sigma)exp(-x/L) \text{ as } x>0; \quad \theta(x) = \Theta \text{ as } x<0 \tag{11}$$

Here L is the nondimensional characteristic length of the temperature drop in the tube wall. Fig. 2 show the results of numerical modeling of the FREI at the phase plane $(dx_f/dt, x_f)$. The upper part of isola in Fig 2 corresponds to normal flame propagating upstream $dx_f/dt > 0$ with velocity $D_f > D_{fc} = 1/\sqrt{e}$. Fig. 2 shows that flame front is almost passively transported by the gas flow when it moves downstream ($dx_f/dt > 0$). The flame front velocity with respect to the channel wall is close to the velocity of the gas flow. Fig. 3 shows a temporal dependence of flame temperature corresponding to the case is shown in Fig. 2. The temperature drops rapidly in the upstream turning point that is point of flame extinction. When the front moves downstream its temperature increases due to the heating by the hot channel wall. Then, in the vicinity of the downstream turning point, the temperature of the front increases still more due to chemical reaction. Therefore, the downstream turning point may be interpreted as point of flame ignition. Note that flame front in general interpretation is considered as surface with intensive heat liberation during the chemical reaction that occurs in narrow temperature interval around the adiabatic temperature $T_f \approx 1$. When the flame temperature falls outsides this temperature interval the heat liberated during reaction becomes negligible quantity and the flame front converts into the interface between burnt and unburned gas that is passively transported by the gas flow. Therefore flame front in new formulation is a surface between combustion products and unburned mixture with arbitrary temperature. We analyzed also flame passing through a bottle neck (see Fig.1) within a new model. According to general concept the flame can not pass through the tube narrowing if the diameter of the bottle neck is less than critical diameter $d_{min}<d_c$. The simulation of equations (6),(7) demonstrated possibility of flame passing through the bottle neck that is narrow than critical diameter. It occurs in the case of large velocity of the gas flow when the residence time of flame in the narrowing is small to cool reaction zone. This example shows principal novelties of new formulations taking into account inertial effects.

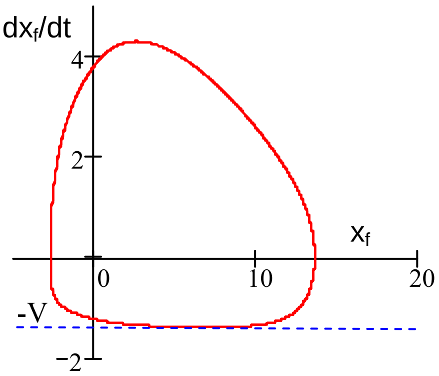

Figure. 2 Phase portrait is plotted on the base of Eqs.(6), (7) for the case σ=0.2, Θ=0.847, N=10, V=1.43, L=20, Ω=0.627.

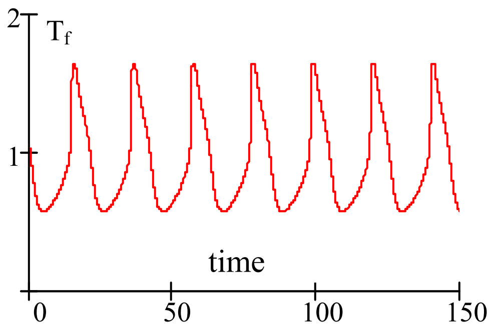

Figure. 3 Temporal dependence of flame temperature during FREI phenomena.

3 Conclusion

The system of 1D nonlinear equations for the flame coordinate and flame temperature was obtained that describe the non-stationary behavior of near-limit premixed flame propagating in a micro channel of variable area and non uniform temperature distribution in the channel wall. The nonlinear model outlined the flame stabilization, flame oscillations and periodical flame extinction and ignition processes and it may be considered as further development of classical concept on burning velocity.

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