

# Numerical study of continuous spin detonation with a supersonic flow velocity

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## 1 Introduction

A possible alternative for conventional burning of fuels in a turbulent flame is fuel burning in a Pulse Detonation Engine (PDE) [1] and in a Continuous Detonation Wave Engine (CDWE), which was first realized by Voitsekhovskii [2]. By the moment, regimes with a continuous spin detonation wave in rocket-type annular combustors and in flow-type combustors have been obtained and studied [3] for several fuel-oxygen and fuel-air mixtures with a subsonic velocity of oxidizer supply. A question arose whether the principle of continuous spin detonation could be extended to ramjet-type combustors with a supersonic velocity of the incoming flow. A positive answer was given in [4], where numerical simulations of the process of burning of a hydrogen-oxygen mixture in an annular cylindrical combustor demonstrated for the first time that it is possible to obtain a regime with a continuous spin detonation wave in a supersonic incoming flow up to the flow Mach number  $M_0 = 3$ . In the present paper, the formulation of the problem [4] is extended to the case of pre-compression of the incoming supersonic flow in the diffuser. The influence of the degree of partial deceleration of the flow on the existence of the continuous detonation regime and the value of the specific impulse is studied numerically.

## 2 Problem formulation

The problem of detonation burning of a  $2\text{H}_2 + \text{O}_2$  mixture in an annular cylindrical combustor is considered: a supersonic flow (with a flow Mach number  $M_0 > 1$ , pressure  $p_0$ , temperature  $T_0$ , and ratio of specific heats  $\gamma_0$  at the combustor entrance) passes through an annular diffuser (with an entrance cross-sectional area  $S_0$  and an exit cross-sectional area  $S_1$ ), is partly decelerated to the flow parameters  $p_1$ ,  $T_1$ , and  $M_1 \geq 1$ , and enters the annular cylindrical part of the channel of length  $L_1$  and width  $\delta$ . Under the assumption of isentropic compression of the gas in the constricting part of the supersonic diffuser and a given ratio of the areas  $S_0/S_1$ , the relation

$$\frac{M_1}{M_0} \left[ \frac{1 + 0.5(\gamma_0 - 1)M_0^2}{1 + 0.5(\gamma_0 - 1)M_1^2} \right]^{\frac{\gamma_0 + 1}{2(\gamma_0 - 1)}} = \frac{S_0}{S_1}$$

yields the Mach number  $M_1$  and the remaining parameters of the supersonic flow  $T_1 = T_0 [1 + 0.5(\gamma_0 - 1)M_0^2] / [1 + 0.5(\gamma_0 - 1)M_1^2]$ ,  $c_1 = c_0 (T_1/T_0)^{1/2}$ ,  $\rho_1 = (S_0/S_1) \rho_0 c_0 M_0 / (c_1 M_1)$ , and  $p_1 = p_0 (T_1/T_0) (\rho_1/\rho_0)$ . After that, the flow passes through the entrance junction of length  $L_2$  with linear expansion of the annular channel from  $\delta$  to  $\Delta$  and enters the annular cylindrical combustor of length  $L$  and cross-sectional area at the exit  $S_{\text{ex}}$ . If the distance between the tube walls is small, as compared with its radius, and there are minor changes in flow parameters in the radial direction, the problem can be simplified [4]. This

simplification makes the computations in a 2D rectangular domain with periodic boundary conditions along one coordinate direction converge. The size of the solution domain along the  $y$  coordinate (length of the period) is denoted by  $l$ , and its length along the  $x$  direction is denoted by  $L_1 + L$ . The flow occurs in the annular space of the combustor with the boundaries  $\Gamma_0$  (open entrance to the annular cylindrical part of the diffuser:  $x = -L_1$ ),  $\Gamma_1$  (entrance to the combustor:  $x = 0$ ), and  $\Gamma_2$  (open end of the combustor with exhaustion of combustion products:  $x = L$ ). The flow was described by a system of equations of unsteady gas dynamics in a quasi-two-dimensional approximation [4], which was supplemented by a two-stage model of chemical kinetics for reacting hydrogen-oxygen mixtures [5]. With properly imposed boundary and initial conditions, the problem was solved numerically by a finite-difference method with second-order approximation, based on the Godunov-Kolgan scheme. The numerical study was performed for a stoichiometric gaseous hydrogen-oxygen mixture ( $2\text{H}_2 + \text{O}_2$ ) with the following initial values of the constants: temperature  $T_0 = 300$  K, pressure  $p_0 = 1.013 \cdot 10^5$  Pa, molecular weight  $\mu_0 = 12$  kg/kmole,  $\rho_0 = p_0 \cdot \mu_0 / (RT_0)$ , and  $\gamma_0 = 1.4$ .

### 3 Results of computations

The first computations were performed for a cylindrical combustor ( $S_0 = S_1$ ;  $\Delta \cdot l = S_{ex}$ ;  $L_1 = 10$  cm,  $L_2 = 0.5$  cm,  $L = 5.2$  cm,  $l = 5$  cm, and  $\delta/\Delta = 0.5$ ) with a supersonic Mach number  $M_0 = 1.5$ , which corresponds to the initial specific flow rate of the mixture  $g_0 = (\delta/\Delta) \cdot \rho_0 \cdot c_0 \cdot M_0 = 197.2$  kg/(s·m<sup>2</sup>). The initiating detonation wave moving across the gas flow entering the combustor was found to generate an oblique compression wave, which enters the diffuser and propagates upstream over the supersonic flow. At the initial stage of the process, the pressure in the combustor performs irregular (seven or eight) oscillations with different amplitudes, followed later ( $t > 0.2$  ms) by almost periodic (with a period  $\Delta t \approx 20.6$   $\mu$ s) oscillations with an amplitude ratio  $p_{\max}/p_{\min} \approx 14.5$ . The velocity of the transverse detonation wave (TDW) averaged over the period is  $\langle D \rangle = l/\Delta t = 2.43 \pm 0.02$  km/s, and the velocity ratio is  $\langle D \rangle/D_0 = 0.85$ . Here  $D_0 = 2.84$  km/s is the ideal Chapman-Jouguet detonation velocity in the  $2\text{H}_2 + \text{O}_2$  mixture. The following specific feature was found in numerical simulations of the continuous spin detonation process in a supersonic incoming flow with  $M_0 = 1.5$ : the flow rate of the gas incoming into the combustor decreases. As a shock wave with the pressure behind its front  $p_{sw}/p_0 = 3.7$ , which is generated by continuous spin detonation, propagates upstream, it converts the initially supersonic flow with  $M_0 = 1.5$  in the diffuser to a subsonic flow with a specific flow rate  $\langle G \rangle \approx 0.7g_0$ .

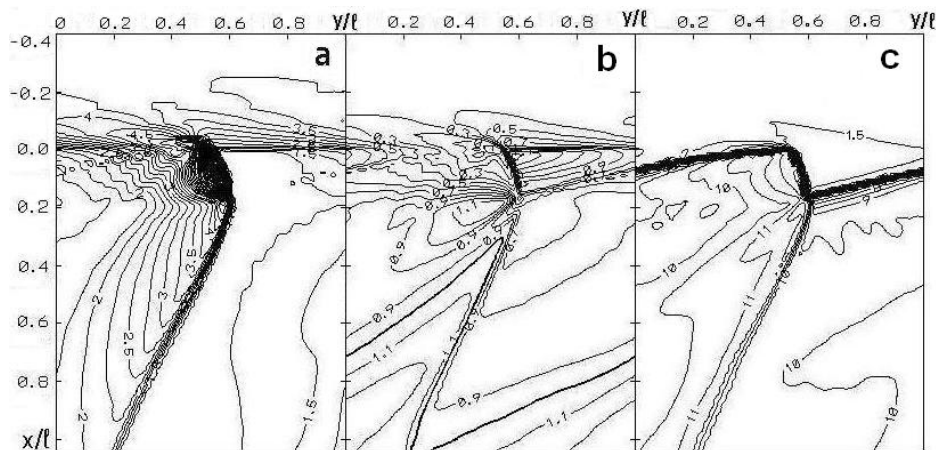


Figure 1. Calculated 2D structure of a continuous detonation wave for  $M_0 = 1.5$ ; and  $\delta/\Delta = 0.5$ : (a) isobars  $p/p_0$ ; (b) Mach contours  $M_x = u/c$ ; (c) isotherms  $T/T_0$ .

The calculated 2D structure of the flow is shown in Fig. 1. The upper part of the figure ( $x < 0$ ) refers to the gas-dynamic flow in the diffuser, and the lower part of the figure ( $x > 0$ ) shows the flow in the combustor channel. The TDW moves from left to right over a triangular low-temperature region containing the initial  $2\text{H}_2\text{-O}_2$  mixture escaping from the diffuser. Above the TDW, there is an oblique shock wave moving over the cold gas in the diffuser, and below the TDW, there is an oblique shock wave (tail) moving over the hot detonation products in the combustor. The height of the combustible mixture layer ahead of the TDW front is  $h = 0.77$  cm. The gas gradually expands behind the wave and becomes displaced by new portions of the gases if the pressure in the gas is lower than the pressure in the diffuser. Conditions for propagation of a new TDW in the next period are created. Varying the Mach number of the supersonic flow  $M_0$  revealed the existence of a continuous spin TDW in the range  $M_0 = 1\text{--}3$ . The calculated continuous spin detonation velocity  $D$ , pressure on the shock-wave front  $p_{sw}$ , mean pressure at the combustor entrance  $\langle p \rangle$ , and relative TDW size  $\eta = h/l$  are summarized in Table 1. It is seen that an increase in  $M_0$  leads to an increase in the pressure on the shock-wave front, pressure in the combustor, and TDW velocity.

Table 1: Characteristics of the continuous spin detonation wave

$M_0$	$g_0, \text{kg}/(\text{s}\cdot\text{m}^2)$	$\langle G \rangle/g_0$	$p_{sw}/p_0$	$\langle p \rangle/p_0$	$\eta = h/l$	$D, (\text{km}/\text{s})$
1	131.4	0.64	2.4	1.7	0.185	2.27
2	262.9	0.75	5.5	4.2	0.21	2.45
3	394.3	0.99	10.4	7.4	0.235	2.56

By contouring the combustor, the influence of the degree of pre-compression of the supersonic flow in the CDWE diffuser on combustion efficiency was determined. For this purpose, the area  $S_0$  was fixed, and the areas  $S_1 = \delta \cdot l$  and  $S_2 = \Delta \cdot l$  were reduced, the area ratio being retained constant:  $\delta/\Delta = 0.5$ . At a distance  $L_3 = 1$  cm from the combustor entrance, the cylindrical channel linearly expanded to the width  $\Delta_{ex}$ , which provided the mean pressure at the combustor exit  $\langle p_{ex} \rangle$  equal to the counterpressure ( $\langle p_{ex} \rangle = p_0$ ), i.e., the design regime of exhaustion from the nozzle was ensured on the average. At each time instant, the flow rate  $\langle G_{ex} \rangle$  and the specific impulse  $\langle J \rangle$  at the combustor exit ( $x = L$ ) were calculated by the formula

$$\langle G_{ex} \rangle = \frac{1}{l} \int_0^l \rho(L, y, t) u(L, y, t) dy, \quad \langle J \rangle = \frac{1}{l} \int_0^l (p(L, y, t) + \rho(L, y, t) u^2(L, y, t) - p_0) dy / \langle G_{ex} \rangle - c_0 M_0.$$

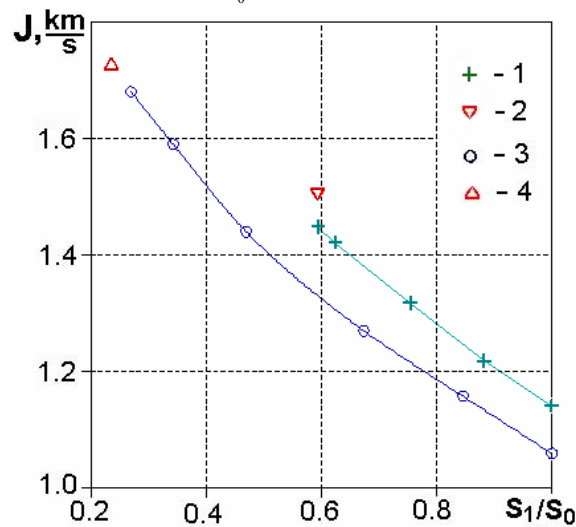


Figure 2. Specific impulse  $\langle J \rangle$  versus the degree of flow compression in the diffuser  $S_1/S_0$ . ( $M_0 = 2$ : 1- CDWE; 2 - ramjet;  $M_0 = 3$ : 3 - CDWE; 4 - ramjet).

Some calculated results for the specific impulse in the CDWE  $\langle J \rangle$  (km/s) for  $M_0 = 2$  and  $M_0 = 3$  are plotted in Fig. 2 versus the degree of diffuser constriction  $S_I/S_0$ . For a fixed Mach number of the incoming flow, the specific impulse of the CDWE is seen to increase monotonically with decreasing parameter  $S_I/S_0$  and to approach the specific impulse of an ideal ramjet. The ramjet impulse was calculated for isentropic compression of the flow in the diffuser to  $M = 1$ , its subsequent expansion to stagnation parameters, burning of the mixture at a constant stagnation pressure, and exhaustion of the products from the nozzle in the design regime. The numerical simulations show that the CDWE (in contrast to the ramjet) operates in the range  $M_0 = 1-3$  at an arbitrary (lower than the critical) degree of flow compression in the diffuser.

Nevertheless, attempts to compute continuous detonation regimes with TDWs in an annular combustor with  $M_0 > 3.1$  were not successful. In this case, though the shock wave propagating over the diffuser was formed at the initial stage as a result of TDW initiation in the combustor, its velocity became lower than the velocity of the supersonic incoming flow with time. Therefore, the shock wave was first entrained to the combustor entrance, then the TDW failed, and finally the hot combustion products were entrained by the supersonic flow away from the combustor. Thus, for the CDWE, there is the upper limit of the Mach number of the supersonic incoming flow:  $M_0 < 0.6 \cdot M_{CJ}$ . Here  $M_{CJ}$  is the Mach number of the Chapman-Jouguet detonation wave.

## 4 Summary

Thus, by an example of a hydrogen-oxygen mixture, the possibility of realization of continuous spin detonation in an annular combustor in the case of a supersonic ( $1 < M_0 \leq 3$ ) velocity of the flow at the diffuser entrance was demonstrated numerically; the TDW structure and the region of TDW existence depending on the flow Mach number were studied. With increasing degree of compression of the supersonic flow in the diffuser, the specific impulse was found to increase and approach the corresponding value for an ideal ramjet. The upper limit for the Mach number of the supersonic incoming flow was found:  $M_0 < 0.6 \cdot M_{CJ}$ .

## 5 Acknowledgement

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