Influence of External Noise on Dynamics of Expanding Flames

Roman V. Fursenko, Sergey S. Minaev,

Khristianovich Institute of Theoretical and Applied Mechanics

Siberian Branch of Russian Academy of Sciences, 630090, Novosibirsk, Russia

1 Introduction

A spherical flame expanding out from the ignition source is one of the most basic configurations of premixed combustion. It was observed in experiment [1] that the front of an outward propagating flame does not remain smooth given enough time. Instead, a self-similar structure of cells is formed on the flame surface and the flame front undergoes a noticeable acceleration. The cellularization of flame fronts was associated with the intrinsic long wave instability, also known as the Darrieus-Landau or the hydrodynamic instability [2].

To describe an outward propagating flame, Sivashinsky [3, 4] proposed a weakly nonlinear integro-differential equation of flame front that is motivated by the physics and captures a number of essential features typical for the flames that are driven by the hydrodynamic instability. Numerous works [4-6] dedicated to numerical simulations of expanding flames governed by the Sivashinsky equation clearly indicate the front cellularization and substantial flame acceleration. It was proposed that these phenomena resulted from explicit and/or implicit forcing, which is always presented both in experiments and calculations. However, numerical simulations of the Sivashinsky equation subjected to external white noise may provide only circumstantial support of this idea due to the extreme sensitivity of the equation to uncontrolled numerical noises that are always presented in the calculations. The attempt to exclude the effects of round-off errors was undertaken in [6] by using some filter. Yet these authors noted that even filtered numerical integrations of the Sivashinsky equation were not noise free and the remaining computational errors still affected the flame front dynamics.

The Sivashinsky equation allows for a whole class of exact pole solutions [7, 8], which have proved to be extremely useful for understanding of the wrinkled flame dynamics. The pole description gives an exact representation of the flame dynamics without noise. Nonsusceptibility of the pole solutions to the numerical noises and round-off errors presents the way to add totally controlled noise in the system so as to clarify its influence on the flame dynamics.

For the noiseless condition the pole dynamics always conserves the number of poles given by the initial condition. As a result, the exact solution of the Sivashinsky equation is incapable to describe the continuing self-similar cellularization of the expanding front and the flame self-acceleration that were observed in experiments and numerical simulations [9]. It is therefore tempting to conjecture that noise may play an important role in affecting the dynamical behavior of the flame fronts driven by the hydrodynamic instability.

Correspondence to: roman.fursenko@gmail.com

In terms of the pole decomposition technique the effect of noise may be manifested by the appearance of new poles in the system [10]. The theoretical estimation of the influence of additional pole on the flame dynamics has been presented in [11, 12]. It was shown that addition of a new pole may lead to the formation of a new secondary cusp on the outward propagating flame front. It reveals possibility of linking the self-similar structure and the pole solutions with controlled noises. In spite of this, the effects of external random forces on the exact solutions of the Sivashinsky equation were still not clearly understood.

The main goal of this work is investigation of the noise influence on the characteristics of the outwardly expanding flames in terms of the pole solutions. Calculations based on the exact solutions make it possible to exclude the effect of uncontrolled numerical noise and to examine the interplay between noises and the instability. The study demonstrates clearly that the presence of noises is a necessary condition for the flame acceleration.

From the physical point of view, the Sivashinsky equation with random force terms may describe the flames propagating through the space with localized nonuniformities or flow disturbances (such as dust or vortices). Therefore these issues are relevant also to the problems of combustion in the particles laden flows.

2 Results and Discussion

Investigations of outwardly propagating cylindrical flames reveal the sharp contradictions between the results of direct computation of the noiseless Sivashinsky equation [4, 5] and analytical results based on the exact pole solutions with constant number of poles. The numerous simulations shown substantial acceleration of the flame front [6] but the analytical results predicted the growing of the propagation velocity only over the short initial stage and its subsequent decay to the velocity of the undisturbed flame. Likewise, the fractal-like structure of the expanding flame front has been observed in numerical simulations but cannot be described in terms of pole solutions with constant number of poles.

A random point-wise set of perturbations uniformly distributed in time and in space is a suitable model for both the computational round-off errors and a variety of perturbations of physical origins [5]. The exact solution of the Sivashinsky equation with impulse-like periodical force term was obtained by numerically solving the ODEs system for pole positions which contains a variable number of poles. The N_g new poles appeared with periodicity T^{-1} at random spatial positions allowed modeling of the noise effects. We have assumed constant number of additional poles N_g per unit flame surface. This physically reasonable assumption was suggested by numerical simulations [13] which show that the number of microcusps on the flame surface grows with the flame size. Due to increasing of the mean flame radius the number of additional poles also increases proportionally to the flame radius. Physically this situation may be associated with sources of the flame perturbations uniformly distributed in the unburned region, for example, small particles. When a particle crosses the flame front, it generates a local perturbation that is described by a pole solution.

The time dependences of the flame front velocity evaluated for different values of ratio $P_N = N_g / 2\pi R_f$, which represents the number of additional poles per unit flame surface, are presented in Fig. 1. Power law approximations $(t - t_*)^{\alpha}$ for the expansion rate of the cylindrical flame are also depicted in Fig. 1. The growth exponent α is almost invariant in the wide range of parameters characterizing the noise intensity and its value is in a good agreement with the numerical results of direct computations of the Sivashinsky equation (marked curves in Fig. 1). Furthermore, the conclusion of strong correlation between the flame front velocity and the strength of the forcing demonstrated by numerical simulations of the Sivashinsky equation and by exact pole solutions coincides with preceding works [5, 11]. From the physical point of view it infers that upstream velocity perturbations, turbulence, or small particles (for example, for a flame propagating through dusted space) may affect the flame front expansion significantly. Hence, the flame expansion rate

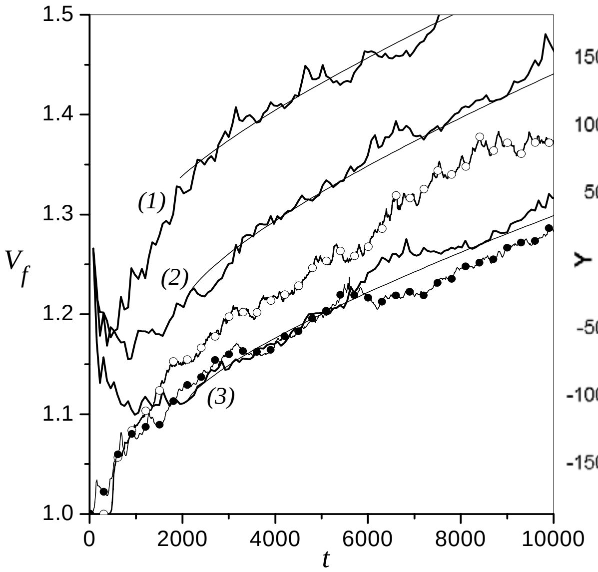

Figure 5. Time dependency of the outwardly propagating flame velocity calculated by exact pole solutions for E = 3 and (1): P_N = 0.005, T = 100; (2): P_N = 0.002, T = 100; (3): P_N = 0.002, T = 200. Marked lines are the results of direct computation of the Sivashinsky equation with zero (solid marks) and random (open marks) force

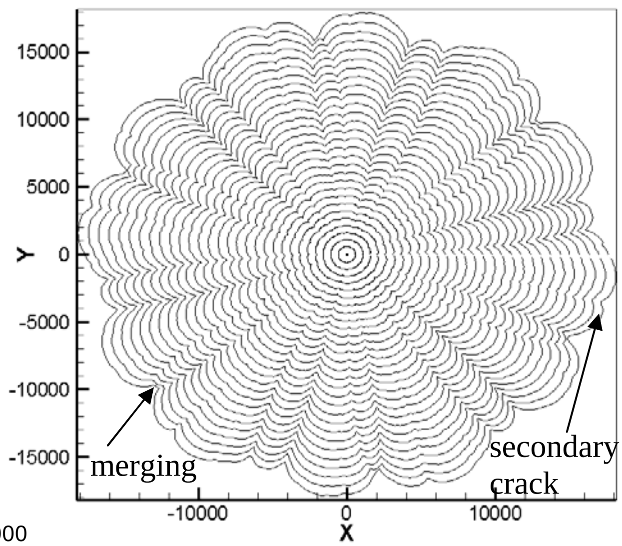

Figure 2. Evolution of the outwardly propagating cylindrical flame with E=3 and P_N = 0.002. T=100.

observed in experiments would depend on the testing conditions such as density of the dust or velocity heterogeneity. At the same time, the growth exponent of the power law approximation seems to be independent on noise intensity. It might be well to point out that comparison of the results obtained by exact pole solutions subjected to external noise and by direct computation of the Sivashinsky equation make possible to estimate the level of uncontrolled numerical noise peculiar to the used computational algorithm in terms of additional poles density.

The external noise in the absence of hydrodynamic instability as well as the development of initial perturbations driven by hydrodynamic instability without external noise do not lead to the flame self acceleration. In the both cases, the flame propagation velocity increases over the short time interval and subsequently decays to some constant velocity. It can thus be concluded that coupling of the noise effect and the hydrodynamic instability is the essential mechanism of the self-acceleration of the outward propagating flames. Moreover, the calculations testify that increasing of the number of additional poles (i.e. perturbations) N_g with growing of the flame radius is a necessary condition for the flame acceleration.

Typical shapes of the flame front obtained by calculations of the pole solutions taking into account the noise generation of new poles are illustrated in Fig. 2. A new small cusp formed due to noise may move toward a nearby big cusp and merge with it, as indicated by the arrow. In contrast to the planar geometry [11] it is not the only fate of a new cusp. The continual growing of the surface area of the outward propagating flame may result in tip splitting when the new secondary cusp remains near the tip between two existing cracks and does not merge with any one of them [12]. As demonstrated in Fig. 2, the pole solutions are capable to describe the cascading structure peculiar to the expanding flame fronts that have been observed in experiments [1] and numerical simulations [5]. That is the case only when new poles appear in the exact solution due to the noise effect and, moreover, if the number of additional poles N_g increases with the flame radius.

3 Concluding Remarks

The dynamics of hydrodynamically unstable flames has been investigated in terms of the exact solutions of the Sivashinsky equation with a random forcing term modeling an external noise. It is shown that modification of the pole solutions taking into account the appearance of new poles due to the noise captures the features typical for the outwardly propagating flames, which cannot be detected by the pole solutions with fixed number of poles. Therefore, the self-acceleration and cellularization of the expanding flame fronts can be described in terms of pole solutions simulating noises. It is demonstrated that the average velocity of the outwardly propagating flame grows as t^{α}. Although the rate of expansion depends on the forcing strength, the growth exponent α is the same in the wide range of parameters characterizing the noise intensity. These results obtained on the base of exact pole solutions quite in line with those of direct computation of the Sivashinsky equation.

Nonsusceptibility of the exact solutions to the round-off errors allows us to examine the separate influence of the external noise on the flame front dynamics. It was found that only joint action of the noise and the intrinsic hydrodynamic instability described by the set of pole solutions causes the flame cellularization and substantial self-acceleration. This conclusion provides a direct support to the idea that the flame acceleration results from explicit and/or implicit forcing which is always present in the numerical simulation and experiments.

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