Oscillatory Propagation of a Rich Premixed Spray Flame

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1 Introduction

The present work is a continuation of a previous study [1] in which it was found that the rich premixed laminar spray flame propagates in a steady manner and, depending on the operating conditions, the reaction fronts are eventually either plane or curved. Before a flame reaches its final steady state its velocity was found to oscillate around a certain positive mean value. However, values of the evaporation Damköhler number examined in [1] were restricted to the range $\Delta = 2 \div 50$ for which there is little direct interaction between the evaporation and the reaction front (see Fig. 1b). One can see that for $\Delta > 2$ the fuel completely evaporates ahead of the reaction zone. In this situation, it is natural, that the width of the evaporation zone does not impact significantly on the combustion.



Figure 1. Concentration of the liquid fuel fraction (bold line) and reaction rate (thin line) for different evaporation Damköhler numbers: $\Delta = 0.1(a)$ and $\Delta = 2(b)$.

In contrast, in the present research we focus on spray pre-mixtures with Δ significantly smaller than 2 for which droplets are traversed by the reaction zone and continue to evaporate in the oxygen-free region behind it (see Fig. 1a).

2. Description of Model

A conventional constant density reaction-diffusion model is adopted. The chemical kinetics is described using a single-step irreversible exothermic reaction. For the off-stoichiometric fuel–rich case this implies that a conservation equation need only be solved for the deficient reactant, the oxidant. The spray of evaporating droplets is modeled using the sectional approach [2] and under the assumption that the liquid fuel mass fraction is small (typically of the order of 10^{-2} , at most). For the sake of simplicity the mono-sectional case is considered here.

In appropriately chosen units the corresponding set of equations for the temperature, oxidant mass fraction and fuel liquid mass fraction reads

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$$=\nabla^2 T + (1 - \sigma)\Omega - \beta S_{\nu} H (T - T_{\nu}) \tag{1}$$

$$C_t = L e_F^{-1} \nabla^2 C - \Omega \tag{2}$$

$$(C_d)_t = -S_v H(T - T_v) = -\Delta C_d H(T - T_v)$$
(3)

$$\Omega = \frac{1}{2} (1 - \sigma)^2 N^2 L e^{-1} C \exp\left(N\left(1 - \frac{1}{T}\right)\right)$$
(4)

In these equations *T* is the non-dimensional temperature in units of $T_b = T_0 + QC_0/c_p$, the adiabatic temperature of combustion products in the pure gaseous case, *C* and C_d are the scaled concentrations of the oxidant mass fraction and fuel liquid mass fraction in units of C_0 - oxidizer concentration in the fresh mixture. In the fresh premixture: $C = C_0$; $C_d = C_{d0}$; *x*, *y* are the non-dimensional spatial coordinates in units of $l_{th} = D_{th}/U_b$ the thermal width of the flame, D_{th} is the thermal diffusivity of the mixture, U_b is the burning velocity of a planar adiabatic flame in the pure gaseous case, $C_{d0} = 0$, *t* is the non-dimensional time in units of l_{th}/U_b , $\sigma = T_0/T_b$ with T_0 the fresh mixture temperature, $N = T_a/T_b$ is the non-dimensional activation energy and T_a is the activation temperature, $T_v = T_v/T_b$ where T_v is the boiling temperature of the fuel, $Le = D_{th}/D$ is the Lewis number, *D* is the molecular diffusivity of the oxygen, c_p is the specific heat, $\Omega(C,T)$ is the appropriately normalized reaction rate to ensure that at large *N* the non-dimensional speed of a steadily propagating planar adiabatic flame is close to unity in the pure gaseous mixture, $\Delta = \left(D_{th}/U_b^2\right)\hat{C}$ is the vaporization Damköhler number

 T_{t}

and \hat{C} is the sectional vaporization coefficient and, finally, β is the nondimensional latent heat of vaporization of the droplets in the spray.

Equations (1)-(3) are to be solved in the two-dimensional region, 0 < x < d and $-\infty < y < \infty$ subject to the boundary conditions,

$$T_x = C_x = 0 \qquad at \qquad x = 0, d \tag{5}$$

The flame is assumed to propagate upwards. Hence, the appropriate boundary conditions at $y = \pm \infty$ are

 $T = \sigma$, $C = C_0$, $C_d = C_{d0}$ as $y \to +\infty$ $T_y = C_y = 0$ as $y \to -\infty$ (6) Initial conditions contain a small temperature perturbation,

$$T = \sigma, \quad C = C_0, \quad C_{d0} = C_{d0} \quad at \qquad y > 0$$

$$T = 1 + 0.01(x - d/2), \quad C = 0, \quad C_d = 0 \quad at \qquad y < 0 \tag{7}$$

The problem (1)-(7) is solved for N = 10, Le = 0.85; $\sigma = 0.127$, d = 100; $\Delta = 0.1 \div 0.5$, $T_v = 0.169$, $\beta = 0.041$, $C_{d0} = C_0 = 1$.

3 Results

The main results obtained from the numerical simulations are shown on Fig. 2 where the flame front amplitude, $\delta = \max_{0 \le x \le d} y(x) - \min_{0 \le x \le d} y(x)$, where y(x) is the flame front, is plotted as a function of time. One can see that there is a range of Δ for which the spray flame propagates in an oscillatory manner. For $\Delta = 0.2$ (not shown here) and 0.3 oscillations occur. The simulations with vaporization

Damköhler numbers outside the interval, namely $\Delta = 0.1, 0.4$ and 0.5, lead to stable propagation of a curved reaction front with results similar to those described in [1].

The impact of spray evaporation on rich flame propagation is via the heat consumed by the liquid drops for evaporation. It is very important where this phenomenon takes place. For a high enough

Damköhler number $\Delta \ge 0.4$ evaporation and heat losses occur in front of the reaction zone (Fig. 1b), whereas for small $\Delta \le 0.1$ the evaporation takes place after the flame front (Fig. 1a). Only for the narrow interval $0.2 \le \Delta \le 0.3$ the heat losses interact with the exothermic chemical reaction heat production and the interplay produces oscillations.



Figure 2. Flame front amplitude δ as function of time *t* for several values of Damköhler number $\Delta = 0.1, 0.3, 0.5$.

Figure 3 shows the flame front propagation during one period of oscillation. Here, for clarity of presentation, we define the flame front position as an isoline of deficient reactant (oxygen concentration) equal to 0.1. In addition, the flame front is stretched with the help of the following transformation

$$y'(x) = \overline{y} + 100(y(x) - \overline{y})$$
 where $\overline{y} = \int_{0}^{a} y(x) dx$.

It is clearly seen that there is no galloping propagation as is found for lean spray flames. Moreover the unsteady propagation arises only in two-dimensional modeling. The corresponding one-dimensional simulation ends up with steady propagation of the reaction wave for all studied Damköhler numbers.



Figure 3. Positions of the flame front (propagating from left to right) at several equidistant moments for a single period of oscillation ($\Delta = 0.3$).

During the period of oscillation (Fig. 3) the flame front consists of a different number of cells. At the beginning and the end there are two cells, whereas in the middle of the cycle there is a single cell. The periodic change of the number of the cells results in the oscillatory propagation. (Fig. 2).

Figure 4 shows a graph of the average velocity of the flame front for different values of the evaporation Damköhler number. This coincides with the results of a 1D simulation, despite the absence of flame curvature and unstable propagation. An explanation of this phenomenon is proffered in [1]. Fig. 4 clearly demonstrates that for small Δ the flame velocity is high. In this case evaporation occurs far after the reaction zone, and heat losses do not strongly impact on the reaction rate. If the evaporation Damköhler number is high enough then the evaporation consumes heat ahead of the chemical reaction. This results in drastic deceleration of the flame that is almost independent of Δ .



4 Conclusions

- 1. There is a narrow range of evaporation Damköhler numbers for which the interplay of heat production due to chemical reaction and heat losses due to evaporation of the fuel droplets results in oscillatory propagation of the spray flame.
- 2. The average flame velocity depends on the amount of evaporation of the liquid fuel fraction ahead of the reaction zone. In the limit of $\Delta \rightarrow 0$ all evaporation occurs after the flame zone and the flame average velocity goes to the velocity of the pure gaseous flame.

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