Simulation of Deciduous Tree Ignition by Ground Lightning Discharge in Approximation of Large Vessels

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1 Introduction

The electric current of a ground lightning discharge [1] proceeds in a tree trunk in the zones sated with moisture [2]. There are two basic types of vessel allocation in wood of deciduous trees. If vessels have basically identical diameter and one are distributed in regular intervals in the year ring, wood is named scatter vesseled [2]. Wood with vessels of unequal diameter, largest of which are concentrated in early wood, name ring vesseled [2]. There are also various mediate between these two extreme types. Within these large types of allocation separate vessels can be isolated from each other or met bunches of the different dimensions and forms. Located bunches vessels have borders parallel to surfaces r=const and φ =const along a line of their contact with each other [2]. Researches of conductivity at different kinds by means of radioactive phosphorus and dyes show, that at one kinds vessels are bound only within a ring layer whereas at others communication between separate layers of a ring [3] is observed. The purpose of research is development of deciduous tree ignition physical and mathematical model in approximation of large vessels and analysis of moisture influence on formation of a temperature field taking into account water evaporation.

2 Physical and mathematical statement of a problem

Separately standing deciduous tree is considered. Ground lightning discharge strikes in a tree trunk during the fixed moment of time. The electric current of a ground lightning discharge proceeds in a trunk. It is supposed, that the tree trunk is a conductor of resistor type electric current for which laws of the Ohm and Joule-Lans [4] are fair. Current parameters are accepted by the identical in various sections of a trunk. The basic assumptions: 1) water transport pathes are located in bunches (it can be united in large fascicles (vessels); 2) vessels is flatten along a contact line; 3) vessels are bound between separate rings. Set above the specified assumptions also compounds basis of large vessels approximation. Water evaporation is described by equation Knudsen-Langmuir [5]. Formed water steams instantly leave wood and as a result abjections of Joule heat there is its warming up. This assumption allows future modification of mathematical model to take into account mechanical demolition of tree trunk under water steam pressure. At reach of critical values of heat flows from large vessels of a core to a surface of ignition and its temperatures is started a deciduous tree ignition. The problem is solved by using polar coordinates in flat statement. The horizontal section of a tree...
Deciduous Tree Ignition

trunk is modelled. The scheme of decision area is presented on fig. 1 (Left), where 1 - core, 2 - burk, 3 - large vessels; \( R_s \) - external radius of a trunk, \( R_1 \) - border between the core and burk. \( \Gamma_i, \Gamma_j, \Gamma_{jk} \) - borders of zones which are presented on fig. 1 (Right).

![Figure 1. Geometry statement. Left: Decision area. Right: Borders.](image)

Mathematically process of a warming up of tree by ground lightning discharge till the ignition moment is described by system of the heat conductivity non-stationary differential equations:

\[
\frac{\partial T_1}{\partial t} = \frac{\lambda_{c_1}}{\rho c_1} \frac{\partial}{\partial r} \left( r \frac{\partial T_1}{\partial r} \right) + \frac{\lambda_{c_1}}{\rho c_1} \frac{\partial^2 T_1}{\partial \phi^2} - Q W \phi_5, \quad (1)
\]

\[
\frac{\partial T_2}{\partial t} = \frac{\lambda_{c_2}}{\rho c_2} \frac{\partial}{\partial r} \left( r \frac{\partial T_2}{\partial r} \right) + \frac{\lambda_{c_2}}{\rho c_2} \frac{\partial^2 T_2}{\partial \phi^2}, \quad (2)
\]

\[
\frac{\partial T_3}{\partial t} = \frac{\lambda_{c_3}}{\rho c_3} \frac{\partial}{\partial r} \left( r \frac{\partial T_3}{\partial r} \right) + \frac{\lambda_{c_3}}{\rho c_3} \frac{\partial^2 T_3}{\partial \phi^2} + JU - Q W \phi_5, \quad (3)
\]

\[
\rho_1 \frac{\partial \phi_1}{\partial t} = 0, \quad (4)
\]

\[
\rho_3 \frac{\partial \phi_3}{\partial t} = -W, \quad (5)
\]

\[
\sum_{i=4}^6 \phi_i = 1, \quad (6)
\]

\[
W = A \left( \frac{P^n - P}{2 \pi R T} \right). \quad (7)
\]

where \( T, \rho, c, \lambda \) - temperature, effective density, heat capacity and heat conductivity accordingly core (i=1), bark (i=2), large vessels (i=3) of trunk; \( J \) - force of a current; \( U \) - voltage; \( r, \phi \) - polar coordinates, \( t \) - time. \( Q \) - heat effect of water evaporation; \( W \) - mass speed of water evaporation, \( A \) - an accommodation coefficient, \( P^n \) - pressure of packed water steam, \( P \) - partial pressure of water steams in air, \( R \) - a universal gas constant, \( M \) - molecular weight of water. 4th sort boundary conditions (equality of heat flows) are exposed on borders of subareas. 3rd sort boundary conditions are exposed on border of decision area. High temperature production is considered in the field of large vessels, because water transport is implements in this zone. Vessels are a conductor of an electric current. Thermophysical properties of organic matter of wood [6]: \( \rho=650 \text{ kg/m}^3; c=1670 \text{ J/(kg} \cdot \text{K)}; \lambda=0.29 \text{ W/(m} \cdot \text{K)} \). Thermophysical properties of tree moisture [7]: \( \rho=1000 \text{ kg/m}^3; c=4180 \text{ J/(kg} \cdot \text{K)}; \lambda=0.588 \text{ W/(m} \cdot \text{K)} \). Thermophysical properties of water steam [7]: \( \rho=0.598 \text{ kg/m}^3; c=2130 \text{ J/(kg} \cdot \text{K)}; \lambda=0.024 \text{ W/(m} \cdot \text{K)} \).
3 Results of numerical modeling and discussion

The formulated mathematical model (1) - (6) with boundary and initial conditions is solved by locally-one-dimensional finite-difference method [8]. Sweep method in a combination to a method of simple iteration [8] was used for the decision of differential equations difference analogues.

The following scheme of investigated process is considered. Negative ground lightning discharge with duration 500 ms and peak current 23.5 kA and the 100 kV voltage strikes in birch tree trunk. Temperature distribution in horizontal section of a tree trunk to the various moments of time before ignition and at the moment of ignition of a tree trunk by a current of a ground lightning discharge (initial temperature 300 K) presented on fig. 2. Ignition conditions [6] (tab. 1) are used as ignition criteria.

<table>
<thead>
<tr>
<th>Time of ignition delay, s</th>
<th>Heat flow, kW/m²</th>
<th>Surface temperature, K</th>
</tr>
</thead>
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<tr>
<td>136</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>61.2</td>
<td>21</td>
<td>645</td>
</tr>
<tr>
<td>17.2</td>
<td>42</td>
<td>688</td>
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<tr>
<td>1.8</td>
<td>125</td>
<td>755</td>
</tr>
<tr>
<td>0.43</td>
<td>210</td>
<td>801</td>
</tr>
</tbody>
</table>

Table 1: Condition of deciduous wood ignition [7]

Figure 2. A temperature field on a horizontal section of tree during the various moments of time. Left: t=0.3 s. Right: t=0.5 s.

Analysis of temperature fields presented on fig. 2 shows, that as a result of action of a typical ground lightning discharge the tree trunk in the field of large vessels is warmed up to ignition temperature (more than 1000 K). Ignition conditions are reached for a typical ground lightning discharge by temperature criterion (801 K) and computed value of a heat flow (219 kW/m²). Raised field of temperatures (approximately on 100 K above, than in scatter vesseled wood) is formed in zone of large vessels. This results from the fact that much quantity of a water contains in large vessels. Formed pore space is filled with water steam (fig. 3). Large vessels zone heat effective volume decrease as a result and the field of the raised temperature in this area is formed.

It is necessary to notice, that intrinsic vessels at electric current transit also excrete Joule heat, sufficient for ignition of birch wood. However absence of an oxidant in this region can lead to that in real conditions on borders of intrinsic vessels the wood will not be ignited. Approximation of large vessels explains cases when on a trunk surface there are burn furrows after impact of a ground lightning discharge. Wood warmed up and a burn only in region of the large superficial vessel in which there passes an electric current. Parts of wood between the vessels are not warmed up to critical temperatures.
Figure 3. Dependences on time of volume fractions of organic matter, water and water steam in the large vessel.

4 Conclusion

The physical and mathematical model of deciduous tree ignition by ground lightning discharge is numerically solved. Approximation of large vessels is used. Possibility of deciduous tree ignition in the conditions of thunder-storm transit by the discharge of a cloud-to-ground class is shown. It is established, that presence of a considerable quantity of a water in large vessels at the initial stage slows down a warming up of birch wood. Effective thermophysical characteristics of large vessels changed after evaporation of all water (approximately during 0.4 s) and the raised temperature field is formed in comparison with scatter vesseled wood. The presented physical and mathematical models can become the additional module in systems of forest fire danger rating. Besides, received results have independent theoretical and fundamental values for the theory of forest fires. They allow to explain the physical nature of the investigated phenomenon of deciduous tree ignition by ground lightning discharge.

References