

The linear instability of detonations viewed as an advective-acoustic cycle

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1 How far do we understand the instability of detonations ?

The instability of detonations has been captured in its linear stage since the pioneering work of Erpenbeck [1] [2]. A perturbative analysis of the ZND model of detonations can account for the pulsating ("galloping") character of unstable detonations in one-dimensional flows, and also for the cellular pattern observed in multidimensional flows. Even in their simplest formulation, using a one-step Arrhenius chemistry, the ZND model of detonations depends on many parameters: the activation energy, the total heat release and the degree of overdrive, in addition to the adiabatic index γ of the ideal gas. The extensive numerical studies of this parameter space [3] [4] have provided us with a global view of the properties of unstable detonations for $\gamma = 1.2$. However complete, these studies are not sufficient to make the instability process physically intuitive. Some attempts to capture analytically the essence of unstable detonations were successfully led in the asymptotic limit of a Newtonian fluid ($\gamma \rightarrow 1$) for a large overdrive and large activation energy, both in one dimensional [5] and multidimensional flows [6]. Another analytical calculation was conducted in the limit of weak heat release [7], both for overdriven and Chapman-Jouguet detonations. Despite these analytical breakthroughs the physical understanding of the instability is still far from intuitive. The pulsating instability requires a high activation energy, whereas cellular detonations may exist even if the activation energy is small. In a recent review [8], the authors described the pulsating instability as "convective-acoustic, a perturbation at the shock being convected downstream (as an "entropy wave"), where the reaction causes a pressure disturbance propagated acoustically back upstream". The physical idea of a feedback between the reaction zone and the shock can be traced back to the early work of Schelkin (1959) [9]. The present study is a quantitative investigation of this feedback cycle, using the formalism of advective-acoustic instabilities recently developed in the astrophysical contexts of accretion onto a supersonic black hole [10] and core-collapse supernovae [11].

2 Comparison to known advective-acoustic cycles in different flows

Advective-acoustic cycles have been introduced in astrophysics in order to explain the instability, observed in the numerical simulations, of the accretion flow onto a black hole moving at supersonic velocity in a uniform gas [10]. The advective-acoustic cycle is based on the coupling between perturbations of pressure and those of entropy/vorticity. These two families of perturbations would be independent in a uniform flow, but become linearly coupled if the flow contains velocity and/or sound speed gradients. Acoustic perturbations reaching the shock produce entropy and vorticity perturbations which are

advected (i.e. "convected") with the flow to a region where the flow is both accelerated and heated. These flow gradients are responsible for an acoustic feedback, thus closing the advective-acoustic cycle. Although these cycles are new in astrophysics, similar cycles were studied more than 20 years ago in the context of combustion instabilities in ramjets [12] (the "rumble" instability) and rockets [13] ("aeroacoustic instabilities"). More recently, the advective-acoustic instability received particular attention in the context of core-collapse supernovae [11], where they can contribute to an asymmetric explosion mechanism, either neutrino-driven [14] or acoustic [15], with important consequences on the linear momentum of the residual neutron star [16], and its spin [17].

This series of examples illustrates the fact that unstable advective-acoustic cycles may exist for accelerated transonic flows as well as decelerated ones, either adiabatically heated, isothermal or even non adiabatically cooled. The essential common point between these flows is the existence of flow gradients which are responsible for the acoustic feedback from advected perturbations.

The flow describing a stationary, unperturbed ZND detonation is nearly adiabatic in the induction zone immediately behind the shock, where the heat released by exothermic chemical reactions is small. The flow is accelerated in the combustion zone up to a subsonic velocity if the detonation is overdriven, or up to the sonic velocity in the Chapman-Jouguet regime. This flow is thus accelerated and possibly transonic, and fundamentally not adiabatic: should we expect an unstable advective-acoustic cycle in this configuration? Acceleration with or without a sonic point is a common feature of unstable flows in a nozzle [12] or in shocked Bondi accretion [19]. The calculations of advective-acoustic instabilities in a flow where the gradients are produced by neutrino cooling [11] proved that adiabaticity is not a crucial ingredient for a significant acoustic feedback from advected perturbations. Similarly, the flow gradients induced by the chemical heat release are expected to produce a significant acoustic feedback when entropy and vorticity perturbations are advected through them. Whether this feedback is strong enough to produce an unstable advective-acoustic cycle is not obvious *a priori*, particularly in a parallel flow: the present study aims at a better understanding of this result.

3 The formalism of advected-acoustic cycles

The formalism of the advective-acoustic cycle, developed in the astrophysical context [18] [19], is meant to characterize the instability mechanism in the linear regime through the properties of two cycles taking place simultaneously, namely the advective-acoustic cycle and the purely acoustic cycle. The efficiency $\mathcal{Q}$ describes the amplification (or damping) of a pressure perturbation after an advective-acoustic cycle. Pressure perturbations reaching the shock generate not only entropy and vorticity, but also a reflected acoustic component, which can be partially refracted as it encounters the flow gradients. This leads to a purely acoustic cycle characterized by an efficiency $\mathcal{R}$, generally damped ($|\mathcal{R}| < 1$). The purely acoustic cycle can usually be neglected if the advective-acoustic cycle is strong enough $|\mathcal{Q}| \gg 1$. It becomes marginally important if $|\mathcal{Q}|$ is moderate, and can either act constructively or destructively depending on the relative phases of the two cycles which are characterized by different timescales $\tau_{\mathcal{Q}}$ and $\tau_{\mathcal{R}}$. The efficiency $\mathcal{Q}$ can be computed by separately calculating the efficiency of advective-acoustic coupling at the shock ($\mathcal{Q}_{\text{sh}}$), and in the flow ($\mathcal{Q}_{\nabla}$), such that $\mathcal{Q} \equiv \mathcal{Q}_{\text{sh}} \mathcal{Q}_{\nabla}$. Similarly, $\mathcal{R}$ is computed by separately calculating the efficiency of acoustic reflection at the shock ($\mathcal{R}_{\text{sh}}$), and in the flow ($\mathcal{R}_{\nabla}$), such that $\mathcal{R} \equiv \mathcal{R}_{\text{sh}} \mathcal{R}_{\nabla}$. In this approach, $\mathcal{Q}$ and $\mathcal{R}$ are two functions of the perturbation frequency ω_r , which is a real number. The resulting stability or instability of the flow can be described by a growth rate ω_i , determined from $\mathcal{Q}$, $\mathcal{R}$, $\tau_{\mathcal{Q}}$ and $\tau_{\mathcal{R}}$ according to a closure relation of the following type [19]

$$\mathcal{Q}e^{i\omega\tau_{\mathcal{Q}}} + \mathcal{R}e^{i\omega\tau_{\mathcal{R}}} = 1. \quad (1)$$

The eigenfrequency $\omega \equiv (\omega_r, \omega_i)$ deduced from this equation can be compared to the complex eigenfrequency $\tilde{\omega}$ obtained from a direct integration of the boundary value problem: this allows us to check whether the advective-acoustic formalism captured or not the mechanism responsible for the instability. Even without calculating accurately the timescales $\tau_{\mathcal{Q}}$ and $\tau_{\mathcal{R}}$, which can be difficult given the spatially

extended character of the acoustic feedback from flow gradients, the global efficiencies $\mathcal{Q}(\omega_r)$, $\mathcal{R}(\omega_r)$ can be compared to the values $\tilde{\mathcal{Q}}(\tilde{\omega})$, $\tilde{\mathcal{R}}(\tilde{\omega})$ directly estimated from the discrete spectrum of complex eigenfrequencies. This technique was conclusively used in the context of shocked Bondi accretion [18] and core-collapse supernovae [11]. Such a formalism can help us understand if the instability is purely acoustic or advective-acoustic. It can identify whether the unstable cycle is mainly fed by the efficiency of the coupling at the shock ($|\mathcal{Q}_{\text{sh}}| \gg 1$), or by the efficiency of the feedback ($|\mathcal{Q}_{\nabla}| \gg 1$).

4 Advective-acoustic cycles between the detonation shock and the region of heat release

4.1 Size of the region of heat release and frequency cut-off

The coupling of advected and acoustic perturbations requires low enough oscillation frequencies so that the wavelength of advected perturbations is long compared to the size of the flow gradients responsible for the coupling. In other words, the time spent by an advected perturbation in the region of advective-acoustic coupling (the region of heat release) should be at most comparable to the period of the perturbation. High frequency perturbations are indeed very weakly coupled because the flow is uniform at their scale. From this basic argument, a step-like flame is expected to lead to unstable advective-acoustic cycles up to very high frequencies [20], whereas a smoother burning profile should stabilize high frequency perturbations. This argument can help us understand recent results [21] concerning the size of the region of heat release.

4.2 Pulsating and cellular instabilities, entropy and vorticity

In a one-dimensional flow, the only advected perturbations are entropy ones. The advective-acoustic cycle restricted to one dimension is thus an entropic-acoustic cycle. In an adiabatic flow, the efficiency of the entropic-acoustic coupling depends crucially on the increase of enthalpy through the region of heat release [22]: the translation of this property in a non adiabatic flow can be determined using the advective-acoustic formalism.

A stationary shock perturbed by an oblique acoustic wave produces both entropy and vorticity perturbations. The acoustic feedback produced by oblique entropy waves can exceed the one dimensional case. Moreover, the presence vorticity is an additional source of acoustic feedback which is sensitive to velocity gradients rather than enthalpy ones. This may explain why a detonation which is very stable with respect to the pulsating instability (e.g. with a low activation energy) can still be unstable with respect to the cellular instability.

5 Conclusions

The formalism of advective-acoustic cycles can be used to improve our understanding of both the pulsating and the cellular instabilities of detonations in the linear regime. This approach goes beyond the numerical determination of eigenfrequencies, and aims at being physically more intuitive than existing asymptotic analytical calculations. This physical understanding is useful in order to anticipate the behavior of detonations involving chemical reactions more elaborate than the simple Arrhenius law.

References

- [1] Erpenbeck JJ (1962). Stability of steady-state equilibrium detonations. Phys. Fluids 5: 604
- [2] Erpenbeck JJ (1964). Stability of idealized one-reaction detonations. Phys. Fluids 7: 684

- [3] Lee HI, Stewart DS (1990). Calculation of linear detonation stability: one-dimensional instability of plane detonation. *J. Fluid Mech.* 216: 103
- [4] Short M, Stewart DS (1998). Cellular detonation stability. Part 1. A normal-mode linear analysis. *J. Fluid. Mech.* 368: 229
- [5] Clavin P, He L (1996). Stability and nonlinear dynamics of one-dimensional overdriven detonations in gases. *J. Fluid Mech.* 306: 353
- [6] Clavin P, He L, Williams FA (1997). Multidimensional stability analysis of overdriven gaseous detonations. *Phys. Fluids* 9: 3764
- [7] Short M, Stewart DS (1999). The multi-dimensional stability of weak-heat release detonations. *J. Fluid. Mech.* 382: 109
- [8] Buckmaster J et al. (2005). Combustion theory and modeling. *Proceedings of the Combustion Institute* 30: 1
- [9] Schelkin KI (1959). Two cases of unstable combustion. *Sov. Phys. JETP* 9: 416.
- [10] Foglizzo T, Galletti P., Ruffert M. (2005). A fresh look at the unstable simulations of Bondi-Hoyle-Lyttleton accretion. *A&A* 435: 397
- [11] Foglizzo T et al. (2007). Instability of a stalled accretion shock: evidence for the advective-acoustic cycle. *ApJ* 654: 1006
- [12] Abouseif GE, Keklak JA, Toong TY (1984). Ramjet Rumble : The low-frequency instability mechanism in coaxial dump combustors. *Combustion Science and Technology* 36: 83
- [13] Mettenleiter M, Haile E, Candel SM (2000). Adaptive control of aeroacoustic instabilities. *Journal of Sound and Vibration* 230: 761
- [14] Buras R et al. (2006). Two-dimensional hydrodynamic core-collapse supernova simulations with spectral neutrino transport II. Models for different progenitor stars. *A&A* 457: 281
- [15] Burrows A et al. (2006). A new mechanism for core-collapse supernova explosions. *ApJ* 640: 878
- [16] Scheck L, Kifonidis K, Janka HT, Müller E (2006). Multidimensional supernova simulations with approximative neutrino transport I. Neutron star kicks and the anisotropy of neutrino-driven explosions in two spatial dimensions. *A&A* 457: 963
- [17] Blondin J, Mezzacappa A (2007). Pulsar spins from an instability in the accretion shock of supernovae. *Nature* 445: 58
- [18] Foglizzo T (2001). Entropic-acoustic instability of shocked Bondi accretion I. What does perturbed Bondi accretion sound like? *A&A* 368: 324
- [19] Foglizzo T (2002). Non-radial instabilities of isothermal Bondi accretion with a shock: Vortical-acoustic cycle vs. post-shock acceleration. *A&A* 392: 353
- [20] Zaidel RM (1961). The stability of detonation waves in gaseous detonations. *Doklady AN SSSR* 136: 1142
- [21] Short M, Sharpe GJ (2003). Pulsating instability of detonations with a two-step chain-branching reaction model: theory and numerics. *Combust. Theory Modelling* 7: 401
- [22] Foglizzo T, Tagger M. (2000). Entropic-acoustic instability in shocked accretion flows. *A&A* 363: 174