Stability of cylindrical flame in radial micro channel with a wall temperature gradient

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1 Introduction

Understanding of near -limit premixed flame stability is important for development of new combustion technologies such as lean burn or micro-scale combustion [1–4]. The present interest in micro scale combustion devices arises due to major advantages associated with these small-scale systems. Some of these advantages include higher heat and mass transfer coefficients and ~20–50 times higher energy densities of hydrocarbon fuels compared to electrochemical batteries [1, 2, 4]. Lower operating temperatures are expected in small-scale systems that allows reduction of thermal NOx formation [4]. At the same time, such systems could act as efficient heaters for steam reformers in integrated micromechanical systems to produce hydrogen for fuel cell and other related applications [4]. It is difficult to obtain stable flame at small scales (smaller than quenching distance) due to increased heat loss by a large surface-to-volume ratio. To overcome this difficulty, heat recirculation is employed to enhance the flame stability limits [4-6]. Thermal wall interaction plays an important role in controlling heat recirculation and affects the combustion stability limits in these devices [7-9]. Therefore knowledge on combustion in small channel with non-uniform temperature distribution in the wall is important for future implementation of combustion technology at small scales. Recent experimental studies on premixed gas combustion in micro channels with temperature gradient revealed interesting phenomena – *flames with repetitive extinction and ignition* (FREI) [10]. Recent experiments [11] in radial micro channel displayed existence of multiple stationary and nonstationary flame configurations in the case when spacing between discs was smaller than critical diameter corresponding to the reference temperature. In the experiments [11] two quartz plates are maintained parallel to each other and methane-air mixture is injected at the center through mixture delivery tube. In some cases the flame was observed in the form of rotating spiral configuration and traveling waves [11]. This study is an attempt to understand mechanism of 2D flame structures formation by using linear stability analysis of stationary solutions describing circular flame. In experiments [11] , at large enough flow rates a stable circular flame existed at the hot part of radial micro channel. With decreasing flow rate the breathing and oscillating structures were observed. Further decreasing of the flow rate sometimes led to appearance of rotating spiral flame like that is shown in Fig.1. In the case of small spacing between discs and small values of the flow rates the traveling waves were observed (Fig.2). These structures sometimes could take pelton like form [11]. If the breathing flame and oscillating structures could be referring to the same type of phenomena as FREI then belonging of rotating spiral and traveling waves to any mechanism was not well clear. In the current paper a two-dimensional (2D) linear stability analysis was applied as a first stage in description of complex nonstationary patterns like spiral flames observed in experiment. In a sense the present model may be considered as extension of 1D stability analysis of the flame front in the channel with temperature gradient [13].

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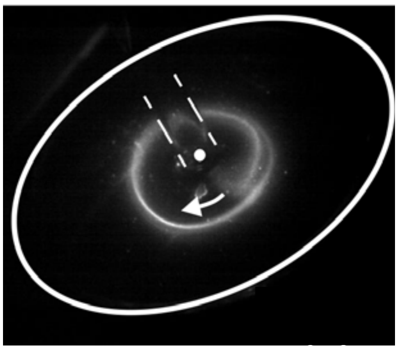


Figure 1. Rotating spiral flame. The ellipse marks the edge of the quartz disk. White arrow shows direction of rotation around center (point).

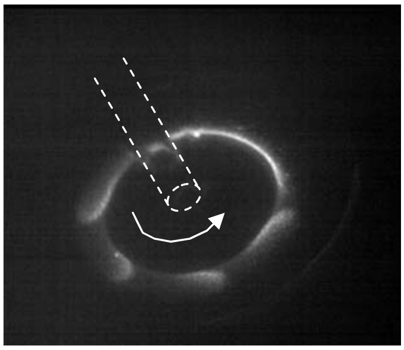


Figure 2. Travelling wave moving in direction depicted by arrow. Dashed lines mark contour of supply tube.

2 Mathematical model

For cylindrical flame the conventional constant-density and constant thermal properties model reads,

$$\frac{\partial T}{\partial t}+\frac{Q}{r}\frac{\partial T}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}-\Omega\left(T-\theta(r)\right)+(1-\sigma)W \qquad (1)$$

$$\frac{\partial C}{\partial t}+\frac{Q}{r}\frac{\partial C}{\partial r}=\frac{1}{Le\, r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{1}{Le\, r^2}\frac{\partial^2 C}{\partial \varphi^2}-W \qquad (2)$$

The theoretical analysis have been conducted within frame of localized-reaction-zone assumption assuming delta-function representation of the term responsible for the chemical reaction: $W=\delta(r-r_f)\exp\left(N/2\left(1-1/T_f\right)\right)$. Here $N=E/RT_b$ is the normalized activation energy and $r=r_f(t,\varphi)$ is the flame interface in polar coordinates. In equations (1), (2) the non-dimensional variables t and r are respectively the time normalized by D_{th}/U_b^2 and the radius in units of D_{th}/U_b, D_{th} is the gas thermal diffusivity and U_b is the adiabatic flame speed. C is the deficient reactant in units of C_0, its value in the fresh mixture; the channel walls temperature θ and the gas temperature T are normalized by flame adiabatic temperature T_b. Q is the flow rate in units of D_{th}; $Le=D_{th}/D_{mol}$ is the Lewis number. Heat exchange parameter $\Omega=2Nu/Pe^2$, where $Nu=\alpha b/\lambda_g$ and $Pe=bU_b/D_{th}$ are the Nusselt and Peclet numbers for the plane channels with spacing b and α is heat-transfer coefficient. Equations (1), (2) are subjected to the following boundary conditions:

$$r\to r_0:\ \frac{\partial T}{\partial r}-\frac{Q}{r_0}T=-\frac{Q}{r_0}\sigma\ ;\ \frac{1}{Le}\frac{\partial C}{\partial r}-\frac{Q}{r_0}C=-\frac{Q}{r_0},\ \theta\to\sigma;\quad r\to+\infty:\ \theta\to\Theta, \qquad (3)$$

$\sigma=T_0/T_b$, $\Theta=T_w/T_b>\sigma$, where T_0 is the fresh mixture temperature and the T_w is the temperature at the periphery of the system. The temperature distribution in the wall is defined by formula,

$$\theta(r)=\sigma+\frac{\Theta-\sigma}{r_1-r_0}(r-r_0),\ r_0<r<r_1;\qquad \theta(r)=\Theta,\ r_1<r, \qquad (4)$$

which roughly approximates experimental profile. In the stationary, axisymmetric case the solutions of the problem (1)–(4) is straightforward and for example solution for temperature may be expressed in terms of modified Bessel functions. Substituting stationary solutions into boundary condition at the flame interface $r=r_{fs}$ one can obtain the system of algebraic equation with respect T_{fs} and r_{fs} variables.

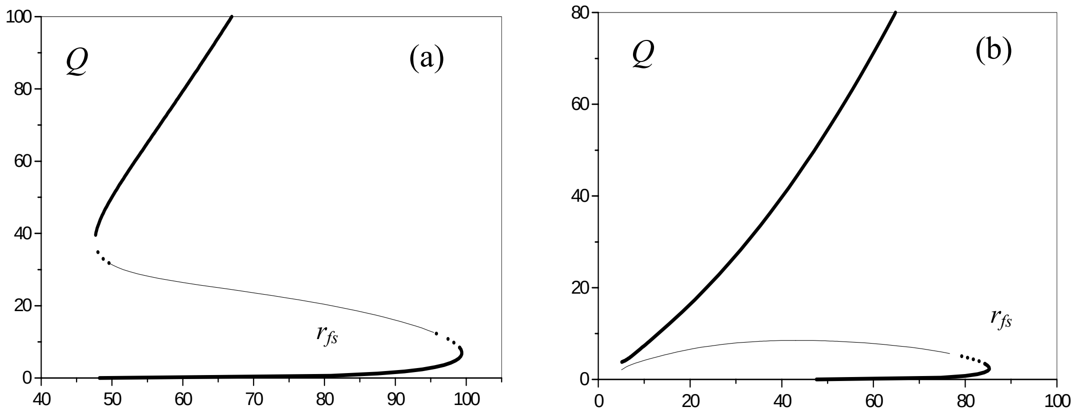
3 Stationary solutions and stability analysis

Solutions of the algebraic equations with respect T_{fs} and r_{fs} variables were found by numerical calculations and the results of these calculations are shown in Fig.3a and Fig.3b. The two typical configurations are came in the cases when the diameter of the channel gap is respectively less Fig.3a and larger Fig.3b of critical diameters. The linear stability of the stationary solution can be determined by examining the growth rate of infinitesimal perturbations that is assumed to be proportional to *exp(ωt+imφ)*. The linear analysis resulted in the dispersion relation for *ω*:

$$(\beta-\alpha)\left(\frac{dT_{2s}}{dr}(r_{fs})+\frac{2T_{fs}^2}{N}\left(\gamma+\frac{1-QLe}{r_{fs}}\right)\right)-\frac{(1-\sigma)Q}{r_{fs}}\left(\alpha-\gamma+\frac{Q(Le-1)}{r_{fs}}\right)=0 \qquad (5)$$

Here the α, β and γ are defined by formulas $\alpha(\omega,m)=d\ln(\tilde{T}_1)/dr$, $\beta(\omega,m)=d\ln(\tilde{T}_2)/dr$, $\gamma(\omega,m)=d\ln(\tilde{C}_1)/dr$ and subscripts 1 and 2 correspond to the region of fresh mixture and the combustion products, respectively.

Equation (5) is solved numerically to identify unstable branches (Re(ω) > 0). First of all, the growth rate of zeros harmonic (m=0) had been calculated as function of the flow rate. This type of perturbation is responsible for expansion or shrinking of the flame without changing its cylindrical shape. The calculation shows that upper branches marked by thick lines in Figs. 3a, 3b are stable solution with negative real part of the growth rate Re(ω) < 0. This fast stable mode exists when the gas flow rates have large values (Q > 40 in Fig.3a) and it may be associated with normal non-adiabatic flame. As expected the low flow rate (Q < 10 in Fig.3a) regime is also stable that is similar to weak flame stabilized in the tube with walls temperature gradient [13]. In the intervals of



Figures 3a, 3b
Dependency $Q(r_{fs})$ (non-dimensional) evaluated for, Nu=4, σ =0.176, Θ=0.588, Pe=9.1(3a) and Pe=18.2(3b), Pe_c=17.8, r_1=4.55, r_2=113.6 . Bold, dashed and dotted lines denote stable, unstable and pulsating branches of the $Q(r_{fs})$ curves respectively.

the flow rates adjoining to the stable regions the growth rates have positive real and non zeros imaginary parts. According to the linear analysis the development of such perturbations can leads to periodically oscillations of the flame radius that may be associated with breathing flame observed in experiments. The linear stability analysis with respect to spatial perturbations has been conducted and three typical dependencies of growth rate ω on harmonic number m were distinguished as a function of velocity and channel width. For type I instability, the real part of growth rate is maximum for m = 0 along with an imaginary part of growth rate. The imaginary part is also nonzero in case II and real part exists only for $m \neq 0$. In these cases, either structures traveling in angular direction or breathing flame with localized distortion of its cylindrical shape (standing waves) may be formed under nonlinear stabilization of instability. The zero harmonic m = 0 dominates in the third type of instability (III) and the imaginary part is absent. The development of such instabilities can lead to either strong nonlinear oscillations like FREI phenomena or flame quenching. The transformation of the cylindrical flame into other geometric configuration like a spiral rotating flame is also non-exceptional variant.

4 Conclusion

In the present study, stability of circular flame in radial microchannels with a positive temperature gradient and negative velocity gradient is analyzed. Stable flame propagation modes at high and low mixture velocities are predicted. At moderate velocities, unstable flame propagation modes are found. Results of these investigations are qualitatively in agreement with the experimental fact that nonstationary spatial structure were observed only in some interval of gas velocities and outside of this interval flame has stable circular shape. Though the modeling of, for example, rotating spiral configurations lies beyond of linear theory, nevertheless the existence of pulsating instability revealed by linear analysis demonstrates that small perturbations can potentially converts into rotating structures at the nonlinear stage of flame evolution.

This work was partially supported by the RFBR (grant No. 05-02-16408).

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