Self-organization in Film Flow with Combustion Wave

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1 Introduction

Studies of fundamental regularities governing film flow regimes are of interest for wide range of practical problems appearing in projecting and optimization of technological plants in energetic, chemical industry and other branches of industry. The present work is devoted to theoretical study and numerical modeling of processes in film flow of fluid on inclined surface with local heat source. Experimental researches carried out at the Institute of Thermophysics SB RAS [1] show that the effect of thermocapillarity under certain conditions can significantly influence the character of film flow. Forming of "roller" of fluid is observed in the experiments in the area with high gradient of film surface temperature. If the temperature (or surface tension) gradient exceeds certain critical level then the periodical 3-D flow structure appears. The main quantity of fluid is gathered in periodical streams (or "fingers"). Between the streams the thickness of film decreases significantly [2].

Qualitatively the same structure was observed in the Institute of Chemical Kinetics and Combustion SB RAS [3, 4] in the experiments with combustion wave propagating over fuel liquid film. In this case the heat source moves with the velocity of flame front. Heat is transferred from the reaction zone to the thin metallic substrate with high thermal conductivity. Cold liquid ahead the flame front is heated by the hot substrate. This results in temperature (and surface tension) gradient at the free surface. The thermocapillary force locally induces the flows which leads to liquid roller forming. So the flow structure becomes two-dimensional. If the heat flux is high then this structure losses stability. It is substituted with periodical "fingers", i.e. 3-D structure.

The authors' previous theoretical results describe 2-D regime of locally heated film flow [5-7]. That results allow us to state the following hypothesis: 2-D flow structure becomes unstable and 3-D perturbations grow as the local arrest of liquid is achieved due to thermocapillary effect (in the frame of reference moving with the heat source) [8, 9]. The results of linear stability analysis and numerical modelling are presented below.

2 2-D steady-state problem

The action of a local heat source with small power results in 2-D regime of flow down an inclined plane with the angle θ to horizon in gravitation field $\mathbf{g}$. The liquid is far from boiling. The effects of heat, mass and momentum transfer at the free surface are neglected. The heat source may be of different nature (for example – heat release in combustion wave), it moves with the velocity c=const and cause a thermocapillary effect. The shape of free surface in steady-state case $y=h(x)$ depends on distribution of surface tension $\sigma(x)$.

The flow of viscous (ν=const) incompressible (ρ=const) liquid is described by equation of continuity, Navier-Stokes equations with boundary conditions at free surface and wall and also by condition of constant flow rate. 2-D steady-state problem is analyzed (in moving frame of reference) supposing the change of film thickness to be small relatively to characteristic space scale of non-uniformity of the solution: $|dh/dx| \ll 1$. The following equation for $h(x)$ can be derived if $\sigma(x)$ is known function:

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$$\left(\frac{h^3}{h_\infty^3}-1\right)\sin\theta+\frac{h^3}{h_\infty^3}\frac{d}{dx}\left\{\frac{\sigma}{\rho|\mathbf{g}|}\frac{d^2h}{dx^2}-h\cos\theta\right\}+\frac{3h^2\, d\sigma/dx}{2\rho|\mathbf{g}|h_\infty^3}=\frac{3\nu c}{|\mathbf{g}|h_\infty^2}\left(\frac{h}{h_\infty}-1\right), \tag{1}$$

where h_∞ is the film thickness far from the heat source.

Using the known dependence $\sigma(x)$ (from infrared thermography) one can calculate the solution, see Fig. 1.

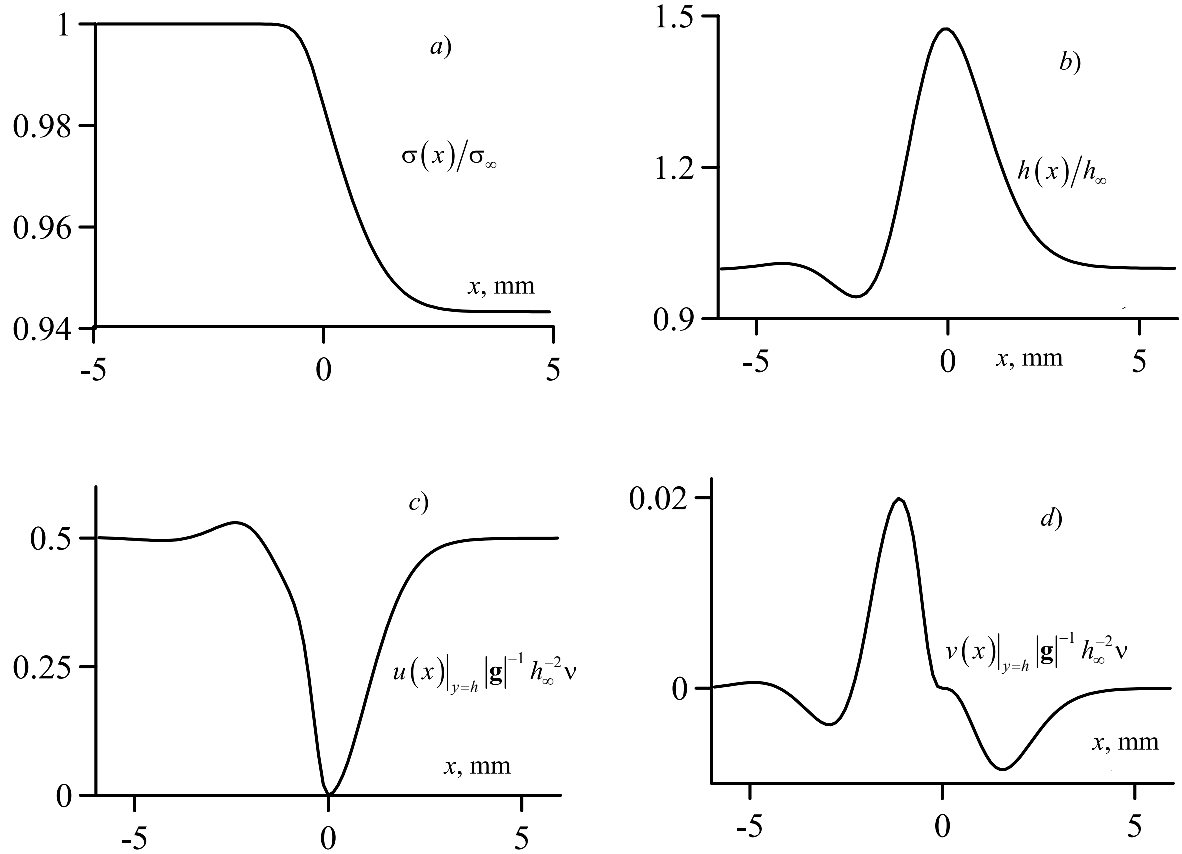

Figure 1. Distributions of main parameters in 2-D regime of film flow (critical regime): *a*) surface tension, *b*) film thickness, *c*) *x*-component of velocity on the free surface and *d*) *y*-component of velocity on the free surface. 25% C_2H_5OH + 75% H_2O, $\theta=\pi/2$, h_∞=0.126 mm, T_∞=303 K, ρ=956 kg/m^3, $\nu=1.8\cdot10^{-6}$ m^2/s, σ_∞=0.034 kg/s^2, $d\sigma/dT=-1.1\cdot10^{-4}$ kg/(s^{2}K), c=0 m/s.

According to equation (1), as c=0 the critical value of the gradient of σ exists. If the value of the gradient of σ is supercritical then a zone of reverse flow would be appeared at the film surface. This probably means the limit of 2-D steady-state solution stability and bounds validity of 2-D model.

When $|c| \gg h^2\nu^{-1}|\mathbf{g}|\sin\theta$ one could write the quadratic equation for $h(x)$, neglecting the dependence of pressure on σ. This equation possesses real roots only before the critical condition of stability. Thus condition of the local arrest of liquid at the surface means the limit of 2-D steady-state regime existence.

3 Analysis of stability of 2-D solution and modelling

The obtained information about the solution of the 2-D stationary problem allows us to carry out the linear analysis of the flow stability in the case of critical condition is fulfilled.

The system of linearized equations of heat conduction, continuity, Navier-Stokes with conditions at the perturbed free surface is sought in the vicinity of point x=0 where u=0 on the free surface. Near the point of the flow arrest the well-known Orr-Sommerfeld equation can be derived for y-component of velocity. If z-periodical small non-stationary perturbations of basic 2-D solution don't depend on x then this equation can be reduced to the linear ordinary differential equation of the fourth order with constant coefficients. As a result it is possible to find the expressions for the amplitudes of perturbations of all parameters and derive the characteristic equation:

$$\Omega^2\left(\Omega^3 - 2\kappa^2 B_1 + 4\kappa^2\Omega^2\right) + \left(\Omega^3 + 2\kappa^2 B_1\right) B_2 \kappa^2\, h/h_\infty + 4\kappa^4 \Omega B_1 = 0, \qquad (2)$$

where Reynolds number $\mathrm{Re} = \dfrac{h_\infty^3 |\mathbf{g}|\sin\theta}{\nu^2}$, $B_1 = \left(\dfrac{d\sigma}{dx}\dfrac{\mathrm{Re}}{\rho h_\infty |\mathbf{g}|\sin\theta}\right)^2$, $B_2 = \left(\dfrac{\kappa^2\sigma}{\rho h_\infty^2 |\mathbf{g}|\sin\theta} + \mathrm{ctg}\theta\right)\mathrm{Re}$, Ω and κ are dimensionless increment and wave number of perturbation, . In the limiting case σ=const equation (2) brings the well-known result describing the propagation of capillary-gravitational waves on the surface of viscous liquid layer [10]. In the long-wavelengh part of spectrum the solution of equation (2) can be approximately written as:

$$\Omega \approx \Omega_0 - 2\kappa^2 - 2\kappa^2 B_2\, h/\left(3h_\infty\Omega_0\right), \qquad \Omega_0^3 = 2\kappa^2 B_1\,. \qquad (3)$$

According to (3) the most unstable perturbation has the wavelength $\Lambda_* = 2\pi\left[250\left(\sigma h_{\max}/3\right)^3 \rho\nu^2\right]^{1/8} \left|d\sigma/dx\right|_{\max}^{-1/2}$. This formula predicts the scale, which corresponds quantitatively to measurements of mean period of 3-D flow structure. For 25% C_2H_5OH + 75% H_2O, $\theta=\pi/2$, h_∞=0.126 mm, T_∞=303 K the observed period is 7 mm [1, 2]. Theoretical result is $\Lambda_* = 7.7$ mm. As Re=const the obtained dependence predicts: $\Lambda_* \sim \left(\sin\theta\right)^{-11/24}$. This is very close to the experimental data: the period is proportional to $\left(\sin\theta\right)^{-1/2}$ [1, 2]. So the presented results of linear analysis of stability can describe the main features of film flow with local heat source.

The obtained solution of linear problem is used for numerical modelling of periodical film structure. The following evolution equation taking into account the non-linear terms of kinematic nature was solved by spectral method with periodic boundary conditions (see Fig. 2):

$$\partial h/\partial t = v' - u'\,\partial h/\partial x - u\,\partial h'/\partial x - w'\,\partial h'/\partial z\,,$$

here h', u', v', w' are the perturbations of film thickness and velocity components.

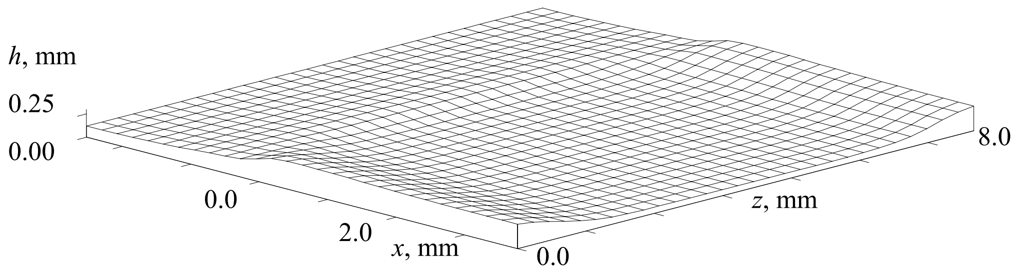

Figure 2. Steady-state periodical film structure.

References

[1] Kabov OA, Marchuk IV, Chupin VM (1996). Thermal imaging study of the liquid film flowing on vertical surface with local heat source. Russian Journal of Engineering Thermophysics. 6: 105-138

[2] Kabov OA (1998). Formation of regular structures in a falling liquid film upon local heating. Thermophysics and Aerodynamics. 5: 547-551

[3] Korzhavin AA, Bunev VA, Gordienko DM, Babkin VS (1998). Behaviour of flames propagating over liquid films on metallic substrates. Combustion, Explosion and Shock Waves. 34: 15-18

[4] Korzhavin AA, Bunev VA, Namyatov IG, Babkin VS (2000). Flame propagation over fuel liquid film on metallic substrates. Explosion and Shock Waves. 36: 25-30

[5] Sharypov OV, Medvedko KA (2001). Concept of the limit of existence of 2-D steady-state structure of fuel liquid film under flame propagation. Proc. 18th International Colloquium on the Dynamics of Explosions and Reactive Systems, Seattle, USA, July 29 – August 3, 2001

[6] Medvedko KA, Sharypov OV, Fomin AV (2002). Limits of existence of a two-dimensional steady-state structure of a liquid film in combustion-wave propagation. Combustion, Explosion and Shock Waves. 38: 19-23

[7] Sharypov OV, Medvedko KA (2002). Instability of a two-dimensional film flow with inhomogeneous temperature field on the free surface. Technical Physics Letters. 28: 541-543

[8] Sharypov OV, Medvedko KA (2004). Instability of 2D flow structure of fuel film. Abstracts. International conference on combustion and detonation. Zel'dovich memorial, Moscow, Russia, August 30 – September 3, 2004

[9] Sharypov OV, Medvedko KA (2005). On the film flow stability with combustion. Proc. 13th Symposium on Combustion and Explosion, Chernogolovka, Russia, February 7-11, 2005 (in Russian)

[10] Landau LD, Lifshits EM (1986). Hydrodynamics. Nauka, Moscow (in Russian)