Mathematical Modelling of Two Problems of Wave Dynamics in Heterogeneous Media

A.V. Fedorov, I.A. Fedorchenko, and I.V. Leont'ev
Institute of Theoretical and Applied Mechanics SB RAS, Novosibirsk, Russia
Corresponding author A.V. Fedorov: fedorov@itam.nsc.ru

Introduction

We investigate two problems. The first one is the problem of dust layer interactions with shock waves. It is of considerable applied and theoretical interest. The practical aspect is related to the phenomenon of initial phase of a "layer detonation" process. The theoretical aspect is a possibility to get new information about a mechanism of heterogeneous medium and air mixing phenomenon at contact surface. Fedorov (2004) presented a review of numerous works made from the early 60s and devoted to the physical and mathematical investigations of the phenomenon. The solution of this problem was shown to be far from the completion.

The second problem is the simulation of Riemann problem for a layered heterogeneous system. The intense interest in this problem is determined by the problem of estimating accidental explosion consequences. This problem was investigated both theoretically and experimentally in the paper of Gelfand et al (1987). In their theoretical description the volume concentration of particles had, however, a value characteristic of rarefied gas-particle mixtures (of the order 10^{-3}), which differed significantly from actual values (0.28 for acrylic plastic and 0.72 for sand).

The main goal of the paper is to present and verify the mathematical models to describe these problems.

Problem 1. Dust layer interactions with shock waves. Mathematical model and problem statement

The problem statement is as follows. The planar SW travels over the dust layer in a still gas. A particle layer with thickness *h* is situated on the rigid wall. Behind the SW the flow is forming in air which parameters are derived from planar shock relations.

The problem has been investigated within the framework of two mathematical models. The first one is the 2D Euler equations for the mixture with additional equation for the volume concentration of solid phase m_2 and special equation of state for the mixture. The higher density of the layer is caused by the presence of a dispersed phase. The description of CIP-scheme implemented in the work are presented in Xiao et al. (1994). The second approach is based on two-dimensional Favre-averaged Navier-Stokes equations closed by the k-ω turbulence model of Wilcox. The dust layer in this case has been simulated by a lower temperature gas.

Results and discussion

The calculations have shown that the flow scheme is similar in both of these cases. In the near-wall dense layer the front of the leading shock wave becomes curved. The curved wave falls onto the rigid wall and reflects from it. It has been shown that either regular and irregular regimes of reflection of the curved SW from the surface can be observed. The reflected wave arrives at the contact surface between clean and dusted gases and is reflecting from it as an expansion wave, which again comes onto the wall. Then the process is repeated.

Three possible mechanisms of dust lifting behind the travelling SW have been found. The first one is associated with the formation of an area with a rather high positive vertical velocity of the mixture behind the curved SW, which may lead to upward ejection of large particles. The second mechanism is conditioned by an unsteady vortex structure into which the dusty gas jet is "swirled". The third possible mechanism of particle lifting is related to the Kelvin—Helmholtz instability of the shear layer, which develops in the stratified layer under the action of internal waves and external perturbations.

The verification of the model has also been carried out with the help of experimental data (Matsui 1992, Gelfand et al. 1989).

In experiments by Matsui (1992) a shock wave is travelling through the shock tube of circular section and 5.2 m length. The tube wall has been covered by soot which has been formed previously as a result of an acetylene decompounding under the action of a detonation wave. Then shock wave is organized and spread along this soot layer.

Initial parameters for the problem were the layer thickness h, density of particles material ρ_{22}, particle volume concentration m_2 and shock wave Mach number M_s. The following initial conditions has been accepted: $M_S = 1.62$, $\rho_{22} = 1900\, kg/m^3$, $h = 4\ mm$, m_2 was varied from 0.001 to 0.002.

Two pressure probes were situated on the top and bottom tube walls to measure the pressure values for different time moments. In fig. 1 pressure behaviors at a bottom of the tube for two m2 values are shown. Here the bold solid line presents the simulation results for $m_2 = 0.001$ and thin solid line shows results for $m_2 = 0.002$. The experimental data is presented by dotted line.

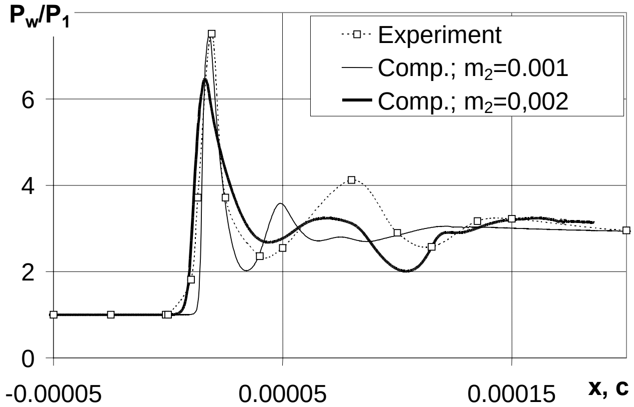

Fig. 1 Comparison of computed pressure behavior with experimental data Matsui (1992)

A good agreement between experimental and calculated values in the first peak of the wall pressure distribution has been observed. This peak is concerned with the shock wave falling on a rigid wall. After this peak a rarefaction wave appears as a result of the shock wave/contact surface interaction. Moreover, the numerical solution demonstrates exactly the steady pressure value far downstream. In the case of a low dust volume concentration, the waves period inside the layer decreases while the amplitude of waves increases.

The comparison of the calculations with the experimental data by Gelfand et al. (1989) has also been done. In this case the normal shock wave falls on the fine particle layer situated on a rigid wall. The material of particles is polystyrene, $\rho_{22} = 1060\, kg/m^3$, h=20 mm, m_2=0.29. Pressure ratio between driving and driven sections is equal 2.

Comparison between numerical and experimental pressure distributions on the tube end is presented in fig. 2. In this case only qualitative agreement between numerical solution and experiments takes place since the experimental m_2 value is higher than that in the computations.

In Gelfand et al. (1989) the experimental investigation of the two-dimensional problem of shock wave/dust layer interaction has also been conducted. The pressure distribution along the wall has been measured. The sand particles were used in experiments which density was $\rho_{22} = 2450\, kg/m^3$ and the layer thickness was 25 mm. Initial pressure in the driving section was 0.1 Mpa, pressure ratio on the diaphragm was equal to two. Different to 1-D problem, the first peak in pressure distribution on the layer surface appears earlier than on the wall. The delay time between shock falling on the contact surface and the wall takes place depending on layer thickness. Fig. 3 presents the numerical and experimental surface pressure distributions showing the qualitative agreement.

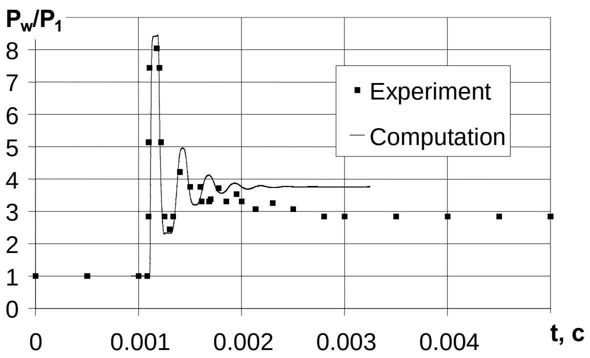

Fig. 2 1-D problem pressure behavior at the end of tube comparing to experiment Gelfand et al. (1989)

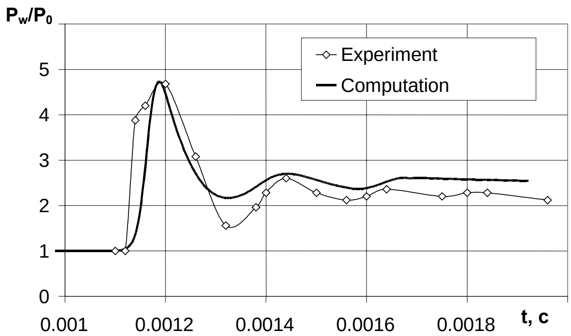

Fig. 3 Comparison of experimental data Gelfand et al. (1989) with the results of 2-D computation pressure distribution

The calculations of the shock wave interacting with the dust layer whose initial shape differs from a rectangle and the investigation of the influence of the shock shape on the interaction flow field picture have also been carried out. And with it computations with and without taking into account turbulence of the mixture have been made. Computed streamline velocity contours for these two cases are demonstrated in fig. 4.

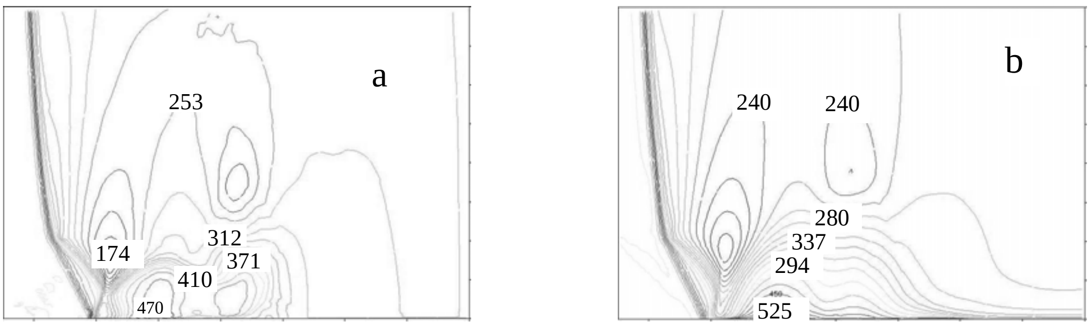

Fig. 4 Longitudinal velocity in the case (a) non-turbulent calculation; (b) turbulent approach

From fig. 4, the main qualitative difference between the turbulent and non-turbulent mixture flows can be seen that is the shape of the leading layer edge. When the turbulence is taken into account (fig. 4, b) a thin near-wall high-speed jet is formed which existence can be explained by high shear stresses due to turbulent viscosity.

Problem 2. Riemann problem for stratified heterogeneous system. Mathematical model and problem statement

Consider a vertical shock tube in the high-pressure chamber (HPC) of which there is a layer of fine particles on the diaphragm. The gas located in the HPC represents a mixture of helium and nitrogen. After the diaphragm rupture the outburst of particles into the low-pressure chamber (LPC) occurs. It is required to determine the wave structure of the flow of heterogeneous mixture as it propagates into the low-pressure chamber. The given statement models the phenomenon of a disastrous explosion of a volume filled with a two-phase medium of fill density. This problem is studied below within the framework of the model of two compressible isothermal media with equal pressures and velocities of components (Fedorov (1990)). The consideration of such a model enables an extension of the range of the values of concentration of solid particles in the layer, for which it is possible to perform validated computations. As we will show here, the computed data give credible results for the pressure in the front of the leading shock wave, etc., which agree with the data of experiment. The mathematical modeling is based on a system of conservation laws for mass and momentum for the mixture on the whole, which is augmented by a kinetic equation for the relative mass concentration of helium. To close it an equation of state (EOS) is used, which was obtained after equating the pressures of phases. It is here possible to determine the equilibrium concentration of particles depending on the mean density of the mixture. Upon substituting this expression into the EOS of one of mixture phases we obtain a convex EOS for the mixture on the whole.

Results and discussion.

Within the framework of the given model, the computations of the problem were carried out by the method of coarse particles, and the results obtained were compared with experiment Gelfand et al. (1987).

To verify the numerical method the test computations were performed in the problem of expansion of a pure gas volume. Figure 5 shows the computed pressure profiles in the LPC, here $\Delta P = P - P_0$, where P_0 is the LPC pressure. The symbols mark the experimental data. The agreement in the results may be considered to be satisfactory.

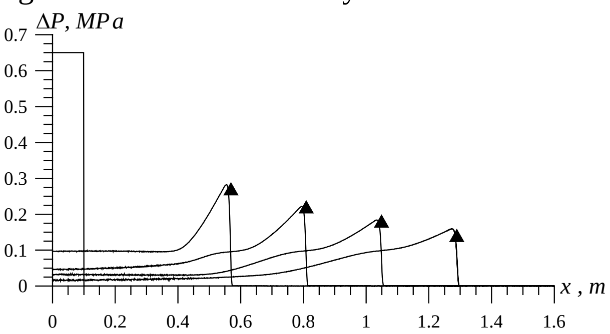

Fig. 5 Pressure distributions along the shock tube

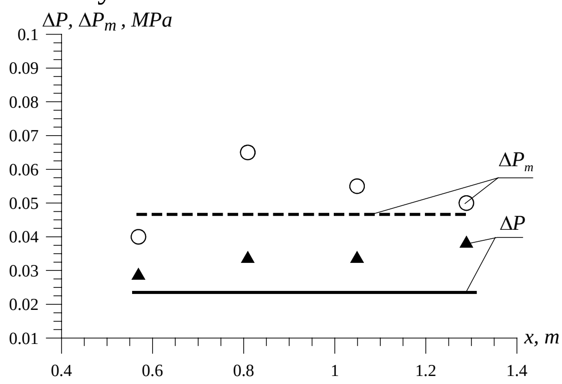

Fig. 6 Pressure dependencies on the shock front and pressure wave (sand)

The subsequent computations of the expansion of a stratified heterogeneous structure have shown that after the diaphragm rupture, a wave pattern is realized in the shock tube with the formation of a leading shock wave (SW) and the following compression wave (CW), which catches up the SW front. The compression wave arises because of the presence of a contact boundary in the HPC between the gas and the layer of the fill density lying on the diaphragm. Due to the satisfaction of the condition for the acoustic impedances (Gelfand et al. (1987)) on the left and on the right from the contact boundary the rarefaction wave propagating in the layer of particles after the diaphragm rupture is reflected from the contact boundary by a compression wave. As the process develops the CW catches up the leading SW and can enhance it.

The computations have also show the possibility of the CW turn-over at large times.

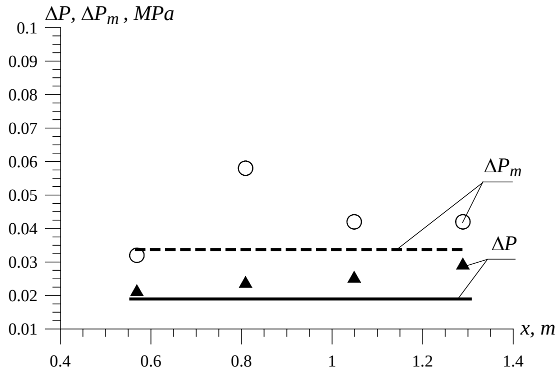

Fig. 7 Pressure dependencies on the shock front and pressure wave (plexiglass)

Some integral data characterizing the expansion of a sand layer 2.5 cm in thickness are depicted in Fig. 6. Here ΔP is the pressure at the leading SW front, it corresponds to triangular symbols, and ΔP_m is the pressure behind the CW, it corresponds to circular symbols. The computation was carried out for $m_{20} = 0.25$. The computed estimates for the pressure ΔP (solid line) and ΔP_m (dashed line) can be seen to be a bit lower than the corresponding experimental values, the agreement may nevertheless be considered satisfactory.

A similar comparison for the layer of particles of acrylic plastic 2.5 cm in thickness is presented in Fig. 7. The value $m_{20} = 0.28$ (experiment). As can be seen for the acrylic plastic particles the mean size of which is 0.01 mm the considered model gives a better agreement with experiment than for the sand with size 0.3 mm.

Acknowledgements

The work was supported by the Russian Foundation for Basic Research (Grant No. 03-01-00453).

References

A.V. Fedorov *Mixing in Wave Processes Propagating in Gas Mixtures (Review)* // Combustion, Explosion, and Shock Waves. 2004. Vol. 40, No. 1. P. 21–37.

B.E. Gelfand, Yu.V. Kazakov, S.P. Medvedev, A.N. Polenov, A.V. Fedorov, V.M. Fomin *Spread of stratified compressed gas-dust system* // Numerical methods of multiphase flows filtration. ITAM, SB AS USSR, Novosibirsk, 1987, P. 88–98. (in Russian)

Xiao F., Yabe T. *A method to trace sharp interface of two fluids in calculations involving shocks* // Shock Waves. 1994. Vol. 4. P. 101–108.

Matsui H. *Structure and Propagation Mechanism of the Soot Layer Detonation* // Proceedings of Research on the Processes of Combustions and Modelling of Fires, P. 57–62, Khabarovsk 1992.

B.E. Gelfand, S.P. Medvedev, A.A. Borisov, A.N. Polenov, S.M. Frolov, S.A. Tsyganov *Shock Loading of Stratified Dusty Systems* // Archivum combustionis, vol. 9, No 1/4, 1989. Pp. 153–165.

A.V. Fedorov *Mathematical description of flows of condensed mixtures under high pressure* // Physical fluid dynamics of reactive media. Novosibirsk, Publishing house Science, SB AS USSR, 1990. P. 119 – 128. (in Russian)