How Large the Rate of Expansion of a Spherical Flame Can Grow?

V. Karlin and J. Mai
University of Central Lancashire, Preston, UK

Corresponding author, V. Karlin: VKarlin@uclan.ac.uk

Introduction

In [1] extensive experimental data on premixed expanding spherical flames have been analysed and it was concluded that starting from a certain moment the averaged flame radius $<r>$ grows as $<r>\propto t^{3/2}$ rather than $<r>\propto t$. The phenomenon was linked to the cellularization of flames, which was well known from experiments too. Indeed, the appearance of cellular patterns increases the flame surface area, hence the fuel consumption, and hence the averaged flame expansion rate.

The cellularization of flame fronts was, in its turn, associated with intrinsic combustion instabilities. The effect of the hydrodynamic combustion instability on expanding spherical flames was studied in [2] using the linear perturbation theory combined with phenomenological assumptions. Later, the approach was further improved and freed from the phenomenological assumptions [3]. These linearized solutions confirmed the onset of the instability of the flame front but could not quantify its cellularization and acceleration because the latter phenomena are essentially nonlinear.

A simple, yet physically reasonable, nonlinear model of hydrodynamically unstable expanding spherical flames was suggested in [4, 5]. Numerical studies of this model confirming that there is a time instance t_*, such that the flame expansion rate behaves like $<r>\propto t^{3/2}$ for $t > t_*$.

Permanent growth of size of a spherical flame as it expands prompts studies of the effect of the size of a planar flame on its propagation speed as the first step towards the understanding of the acceleration mechanism of the expanding flames. The investigation of dynamics of planar flames revealed a definite correlation between the size of the flame and its spatially averaged propagation speed, see e.g. [6]. The effect was explained by proving high sensitivity of planar cellular flames to particular types of linear perturbations, see [6–9]. By continuing calculations reported in [6] for even larger planar flames, we obtained that their propagation speed no longer grows after a certain critical flame size is reached. In this paper we are interested in extending these findings for planar propagating flames to the expanding ones. In particular, we are studying the possibility of stabilization of the expansion rate for large enough time intervals, when the flame size grows sufficiently large.

Because of their physical nature simple nonlinear models of expanding flames [4, 5] are valid only locally. Therefore, the results obtained when applying them to the whole flame are instructive indeed, but still inconclusive and cannot be accepted as the adequate theoretical model of cellularization and acceleration. A physically consistent global model of flames of arbitrary smooth enough geometry was developed in [10]. Mathematically, the approach projects the governing equations to the flame surface reducing mathematical dimension of the problem by one. However, the resulting equation is still extremely costly from the computational point of view and only two-dimensional simulations have been carried out so far.

A compromise between universality and computability was suggested in [11], where consideration was limited to a narrow but still very practical case of flames which do not deviate from the spherical ones significantly. On technical side the model combines the operator of the linearized problem obtained in [2] for the expanding spherical flame in terms of spherical harmonics expansions and a Huygens type nonlinearity specific to the local nonlinear model [4, 5]. Physically, model [11] is consistent with [10] and is robust and plausible enough to simulate the cellularization of expanding spherical flames in three spatial dimensions. At the time of writing of this paper, the flame sizes we were able to reach in our computations do not exceed those reported in [11] significantly and are not large enough to match our two-dimensional calculations. However, an extrapolation of data obtained in computations we carried out so far, shows that numerical studies of the parallelised three-dimensional model [11] on the time scales required for comparison with the two-dimensional model [4, 5] are possible.

Mathematical Models and Computational Algorithms

Let us consider an expanding flame front and assume that its surface is close enough to a sphere and that every point on the flame surface is uniquely defined by its distance $r = r(\theta, \phi, t)$ from the origin for $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$, and $t > 0$. It is convenient to represent such a flame as a perturbation $\Phi(\theta, \phi, t)$ of a spherical surface of a reference radius $r_0(t)$, i.e. $r(\theta, \phi, t) = r_0(t) + \Phi(\theta, \phi, t)$. Then, the Fourier image of the governing equation of the flame front evolution in the nondimensional notations suggested in [4, 5] can be written as

$$\frac{d\widetilde{\Phi}_k}{dt} = \left\{ -\frac{1}{[r_0(t)]^2}|k|^2 + \frac{\gamma}{2r_0(t)}|k| \right\} \widetilde{\Phi}_k - \frac{1}{2[r_0(t)]^2} \sum_{l=-\infty}^{\infty} l(k-l)\widetilde{\Phi}_l \widetilde{\Phi}_{k-l} + \widetilde{f}_k(t). \qquad (1)$$

Here $|k| < \infty$, $t > 0$, $\widetilde{f}_k(t)$ are the Fourier components of the properly scaled upstream perturbations of the unburnt gas velocity field $f(\phi, t)$, and initial values of $\widetilde{\Phi}_k(0) = \widetilde{\Phi}_k^{(0)}$ are given.

System (1) is solved numerically by neglecting the harmonics of orders higher than a finite integer number $K > 0$. Then, the nonlinearity can be represented as a circular convolution and evaluated effectively with the FFT. Also, we found that the stability of the numerical integration scheme suggested in [4] can be improved significantly by calculating the contribution from the linear terms in (1) analytically. Thus, the linear terms, i.e. the source of physical instability, are tackled exactly and only the nonlinear ones, with the dumping effect, are approximated.

Equations of the three-dimensional model [11] can be written in terms of the spherical harmonics expansion coefficients $\widetilde{\Phi}_{n,m}(t)$ of $\Phi(\theta, \phi, t)$ as

$$\frac{d\widetilde{\Phi}_{n,m}(t)}{dt} = \omega(n,t)\widetilde{\Phi}_{n,m}(t) + \frac{1}{2[r_0(t)]^2}\widetilde{N}_{n,m}(t) + \widetilde{T}_{n,m}(t) + \widetilde{f}_{n,m}(t). \qquad (2)$$

Here $|n|, |m| < \infty$, $t > 0$, $\widetilde{f}_{n,m}(t)$ are the spherical harmonics coefficients of the properly scaled upstream perturbations of the unburnt gas velocity field, and initial values of $\widetilde{\Phi}_{n,m}(0) = \widetilde{\Phi}_{n,m}^{(0)}$ are given. The mathematical formulas for the linear response $\omega(n, t)$, the spherical harmonics coefficients $\widetilde{N}_{n,m}(t)$, of the nonlinear term, and $\widetilde{T}_{n,m}(t)$, of the artificial term, added in order to prevent translations of the flame front as the whole object in space, are available somewhere else.

A computational algorithm similar to [11], was used in this work. In addition, the stability of the numerical integration scheme was improved by evaluating the contribution from the linear terms analytically and the code was parallelized in order to speed up the computations and to use larger data sets.

Computational results

Typical shapes of the flame fronts governed by (1) over large time intervals are illustrated in Fig. 1. The graph of $[r(\phi, t) - <r>]/<r>$ for $t = 7.65 \times 10^4$ shows that the wrinkle amplitudes are up to 10% of the averaged flame radius. The explicit forcing was not applied in this example.

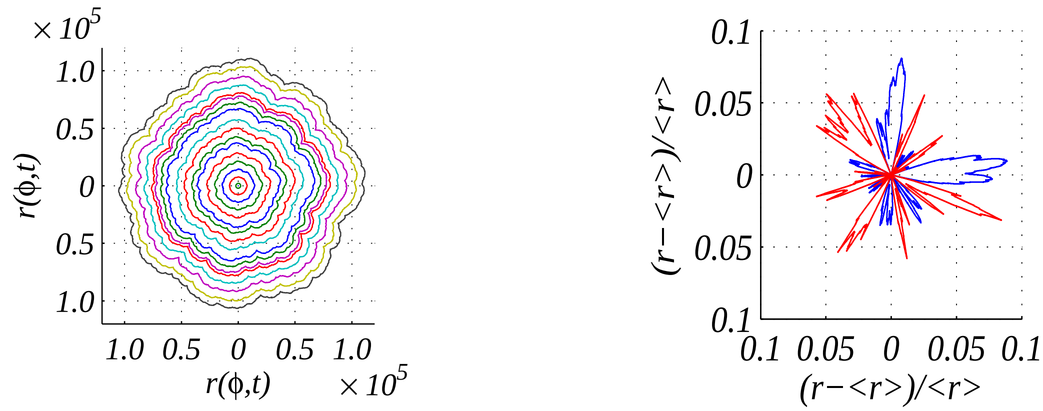

Figure 1: Evolution of a spherical flame governed by (1). Here values of $r(\phi, t)$ are in the left and $[r(\phi, t) - <r>]/<r>$ for $t = 7.65 \times 10^4$ are in the right. Positive values of the latter are in blue and negative ones are in red; $\gamma = 0.8$, $f(\phi, t) \equiv 0$.

Velocities of the planar and spherical flames are compared in Fig. 2. Power law approximations $(t - t_*)^\alpha$ for the expansion rate of the spherical flame are also depicted there. For the time interval $[2.2 \times 10^3, 7.65 \times 10^4]$ the optimal $\alpha \approx 0.34$. For earlier times $t \in [2.2 \times 10^3, 2.0 \times 10^4]$, the best approximation is with $\alpha \approx 0.47$, i.e. almost $1/2$. On the other hand, as time goes by, the expansion rate slows down and for $t \in [3.0 \times 10^4, 7.65 \times 10^4]$ we got $\alpha \approx 0.23$.

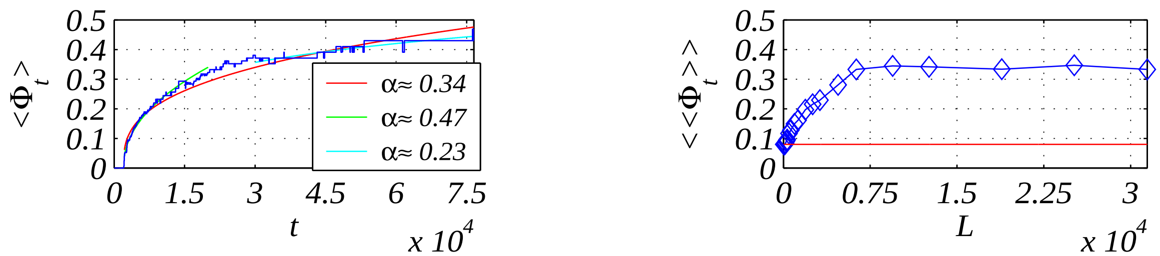

Figure 2: Space averaged velocity for spherical (left) and time-space averaged velocity for planar (right) flame fronts versus time t and flame size L respectively. Here $\gamma = 0.8$ and $f(\phi, t) \equiv 0$.

An example of evolution of a random three-dimensional perturbation of a spherical flame is illustrated in Fig. 3. Note, the space and time scales in (1) and (2) are different. Their ratios are $4\pi/\gamma$ and $\dfrac{4\pi\gamma(2-\gamma)}{(1-\gamma)[\sqrt{1+\gamma(2-\gamma)/(1-\gamma)}-1]}$ respectively, see [4] and [11]. .

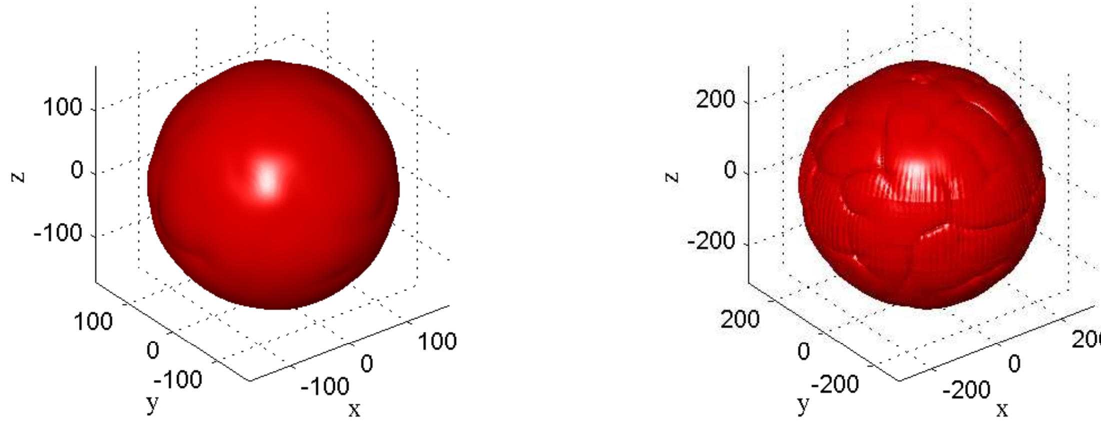

Figure 3: Evolution of a spherical flame; $t \approx 40$ (left) and $t \approx 72$ (right). Here $\gamma = 0.764$.

Conclusions

Long time interval simulations of a simplified model of the expanding spherical flames indicated that their expansion rate slows down as the flame size grows. The saturation of the planar flame propagation speed as their size grows was established too. Hence, a hypothesis of stabilization of the spherical flame expansion rate over finite time interval is suggested.

In order to verify the hypothesis, numerical simulations of a more sophisticated model of a three-dimensional spherical expanding flame were initiated. Using parallel techniques the evolution of such a flame has been successfully simulated to a stage when wrinkles appear and form a well developed cellular structure.

Acknowledgements

This work was supported by EPSRC grant GR/R66692.

References

[1] Y.A. Gostintsev, A.G. Istratov, and Y.V. Shulenin. *Phys. Combust. Expl.*, 24(5):63–70, 1988.
[2] A.G. Istratov and V.B. Librovich. *Acta Astronautica*, 14:453–467, 1969.
[3] J.K. Bechtold and M. Matalon. *Combust. Flame*, 67:77–90, 1987.
[4] L. Filyand, G.I. Sivashinsky, and M.L. Frankel. *Physica D*, 72:110–118, 1994.
[5] G. Joulin. *Phys. Rev. E*, 50(3):2030–2047, 1994.
[6] V. Karlin. *Proc. Combust. Inst.*, 29(2):1537–1542, 2002.
[7] G. Joulin. *J. Phys. France*, 50:1069–1082, 1989.
[8] V. Karlin. In A.B. Movchan, editor, *IUTAM Symposium on Asymptotics, Singularities and Homogenization in Problems of Mechanics*, pages 549–556. Kluwer, 2003.
[9] V. Karlin. *Math. Methods Modelling Appl. Sci.*, 14(8):1191–1210, 2004.
[10] M.L. Frankel. *Phys. Fluids A*, 2(10):1879–1883, 1990.
[11] Y. D'Angelo, G. Joulin, and G. Boury. *Combust. Theory Modelling*, 4:1–22, 2000.