On the Inclusion of Frictional Work in Non-Ideal Detonations

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Abstract

It is found that different authors have treated non-ideal detonations (detonations with friction) in different ways. Some authors include a source term in the conservation of energy equation, work done by friction, while others do not. This difference is addressed here and it is concluded that the work term should be included in the analysis. The conservation laws are integrated with and without the frictional work term to evaluate the magnitude of the term. It is found that the qualitative nature of the solution is not changed but the inclusion of the work term significantly increases the detonation velocity as compared with the case without a work term.

Introduction

Many researchers have studied non-ideal detonations (detonations with friction and heat losses) analytically: Zel'dovich [1], Zel'dovich *et al.* [2], Zhang and Lee [3], Brailovsky and Sivashinsky [4] and Dionne *et al.* [5]. While most of the above authors used a similar approach, integrating the steady one-dimensional conservation laws with appropriate source terms for friction and heat loss, there is a discrepancy in the way friction was included in the conservation laws. The following conservation laws (in the steady detonation frame of reference) are used by the authors in the case with friction and no heat loss:

$$\frac{d}{dx}(\rho u) = 0$$
 continuity – all authors

$$\frac{d}{dx}(P + \rho u^2) = -f$$
 momentum – all authors

$$\frac{d}{dx}[u(\rho e + P)] = 0$$
 energy – Brailovsky and Sivashinsky

$$\frac{d}{dx}[u(\rho e + P)] = -Df$$
 energy – all other authors

where ρ , u, P, f, e and D are density, flow velocity in the detonation reference frame, pressure, friction force, internal energy (including kinetic energy and chemical heat release), and detonation velocity in the lab reference frame, respectively. The discrepancy lies in the conservation of energy: Brailovsky and Sivashinsky, as opposed to all other authors, omit to include in the conservation of energy, work done by the friction force. It should be noted that Brailovsky and Sivashinsky did not simply forget to include the work term since the paper

includes a lengthy discussion of why the work term should not be included. They point out that friction is an internal force as opposed to an external force such as a body force. They also point out that in the Fanno flow model (one-dimensional flow of a compressible perfect gas in a duct with friction), no work term is included in the energy equation. Also, this study has lead to a whole body of work on detonability and flammability limits [6], deflagration to detonation transition (DDT) [7] and low-velocity detonations in porous media [8].

The objective of the present study is to resolve the above discrepancy, i.e. determine whether or not a frictional work term should be included in the conservation of energy in general and in the case of non-ideal detonations. Furthermore, the effect of this work term will be quantitatively assessed.

Effect of Work Done by Friction

To quantify the effect of the work done by friction, the conservation laws were integrated with and without the work term. The conservation laws are first rearranged into the ZND equations and subsequently integrated subject to the generalized Chapman – Jouguet criterion. This is



Figure 1. Detonation Mach number as a function of friction intensity (friction factor coefficient k_f for two activation energies ($E_a = 22$ and 27) with and without frictional work term.

done via an iterative procedure. It is relatively standard and straightforward to do this; for more details, see Dionne *et al.* [5] for example. A friction factor k_f was varied. Figure 1 shows the detonation Mach number as a function of friction intensity for two different activation energies ($E_a = 22$ and 27), with and without the work term.

The solutions show qualitatively the same behavior: decreasing Mach number for increasing friction intensity, larger effect of friction for higher activation energy and multi-valued solutions for high activation energy. However, it is clear that the work term is quantitatively significant. Note that the added work results in a higher detonation Mach number (smaller velocity deficit) as compared to the solutions without the work term. This is no surprise since the work done by friction is energy that is added to the flow. It may be counter intuitive to think that friction results in increased velocity but this is not the case. The net effect of friction is still to reduce the detonation velocity from the CJ velocity. This means that the source term in the momentum equation dominates the source term in the energy equation.

Fanno Flow Model

Fanno flow is the steady one-dimensional compressible flow of a perfect gas in a duct with friction. In the formulation of this model, the steady conservation laws supplemented with a friction source term in the conservation of momentum equation are integrated from initial conditions. However, no source term is included in the conservation of energy equation. The reason for this is generally not discussed. In Fanno flow, the walls of the duct are always at rest in the steady reference frame. One could argue that the work term vanishes since the velocity at the wall is zero. However, this is inconsistent with the one-dimensional formulation. Since the flow is one-dimensional, the velocity of the gas at the wall is not zero. This means that a work term, *fu*, should be included. In Fanno flow, consistency with the one-dimensional formulation is forgone in order to capture the physics of frictional flow in a duct, where in reality, the no-slip boundary condition forces the velocity of the fluid at the wall to be zero. Therefore, in a real physical system, no energy is added to the flow from the force at the wall.

Detonations with Friction

In the case of detonations, unlike in Fanno flow, the walls are moving at velocity D in the steady reference frame. Therefore, in a multi-dimensional formulation, energy is being supplied to the flow at a rate of Df. However, to be consistent with the one-dimensional approximation, the work term would be fu. Either way, energy is being added to the flow and a work term should be included. Again, it appears evident that the correct work term should be Df, even though it is inconsistent with the one-dimensional formulation.

Consider a steady detonation stabilized in a wind tunnel. Clearly this problem is different than the case of the detonation propagating at constant velocity in a duct at rest. In the first case, the walls are at rest in the steady reference frame and move at velocity D in the second case. To make the two problems alike, consider that the walls of the wind tunnel are conveyor belts moving at velocity D in the steady reference frame, see Fig. 2. It is obvious that an external source of power needs to be input into the system (e.g. electrical motor or pulley and weights, etc.) to keep the conveyor belts moving against the friction force f. It is also obvious that the



Figure 2. Detonation propagating in fixed tube in detonation reference frame (left) and detonation propagating in a wind tunnel with moving walls (right).

work done to turn the conveyor belt goes into the flow. Now, from the perspective of the detonation (or the flow), the two problems are identical, i.e., the flow cannot "know" the difference between a moving wall and a conveyor belt. Therefore, it is clear that since work is input into the first case, work must also be input in the second case. It is also clear that the magnitude of the rate of work term is Df since D is the velocity of the conveyor belt and the force acting on it is f.

Galilean Transformation

The formal way to obtain the appropriate form of the steady conservation laws in the detonation reference frame is to start with the unsteady conservation laws in the lab frame (with appropriate source terms) and to perform a Galilean transformation to a reference frame moving with velocity D. The unsteady Euler equations have the following general form:

$$\frac{\partial \rho}{\partial t'} + \frac{\partial (\rho u')}{\partial x'} = m$$
$$\frac{\partial (\rho u')}{\partial t'} + \frac{\partial}{\partial x'} (\rho u'^2 + P) = f$$
$$\frac{\partial (\rho e')}{\partial t'} + \frac{\partial}{\partial x'} [u'(\rho e' + P)] = q$$

where t', x', u' and e' are in the lab reference frame and m, f, q are sources of mass, momentum and energy respectively, and

$$e' = \frac{1}{\gamma - 1} \frac{P}{\rho} + \frac{1}{2} u'^2.$$

To obtain the Euler equations in the detonation reference frame, it is necessary to make the following transformations:

 $x = Dt - x', \qquad t = t', \qquad u = D - u'$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + D \frac{\partial}{\partial r}$ and $\frac{\partial}{\partial r'} = -\frac{\partial}{\partial r}$.

Carrying out the above substitution in the above equations yields:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = m$$
$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x} (\rho u^2 + P) = Dm - f$$
$$\frac{\partial (\rho e)}{\partial t} + \frac{\partial}{\partial x} [u(\rho e + P)] = \frac{1}{2} D^2 m - Df + q$$

Clearly the energy equation contains the term Df, work done by the force f on the flow.

Detonations in Unsteady Reference Frame

It is clear that in the lab reference frame, the friction force does no work since the wall is at rest (just as in Fanno flow). Therefore, one could perform an unsteady Euler calculation with friction (included in the momentum equation only) and let the detonation evolve to a constant velocity. This constant velocity can then be compared to that obtained from the steady state analysis with and without friction. The unsteady calculation should agree exactly with one of the two solutions (the correct one).

Unsteady calculations have been done by Dionne *et al.* [5]. It was shown that ZND detonations are in fact unstable and oscillate. However, as the activation energy is decreased, the oscillations decrease in amplitude and eventually, for low enough activation energy, the oscillations disappear. For such activation energies then, it is possible to find the terminal velocity (or Mach number) of the detonation and compare it to the steady analyses. Figure 3 is one such unsteady calculation performed with a friction coefficient of 0.9299. Detonation Mach number is plotted versus distance. The detonation was initiated with a strong blast wave, hence the high initial Mach number. The Mach number decays and asymptotes a constant value. Two horizontal lines on this figure indicate the results of the steady calculations: the upper one includes the frictional work term, while the lower one does not. It is clear from this figure that the unsteady calculation is consistent with the steady analysis only if the frictional work term is included in the energy equation.



Figure 3. Detonation Mach number vs. distance for a detonation initiated by a strong blast wave ($E_a = 10$, $\gamma = 1.2$, $k_f = 0.9299$). The horizontal dashed lines represent the steady state solutions with (above) and without (below) the frictional work term.

Ram Accelerator with Frictional Walls

Another application besides quasi-detonations where work done by friction must be considered is ram accelerators. Just as with detonations, in the steady (or quasi-steady) reference frame, the wall has a non-zero velocity. Therefore, work done by friction from the wall should be included in the energy equation. While it is true that in a conventional ram accelerator, friction on the tube wall is probably negligible, modifications to the ram accelerator, such as adding baffles to the tube wall, will significantly increase the drag on the tube wall [9]. In this case, friction must be included in the model for thrust on the projectile.

Assuming quasi-steady, one dimensional flow, and operation in the thermally choked regime (flow leaving the control volume is sonic), the one-dimensional steady conservation laws applied to a control volume moving at constant velocity, *D*, attached to the projectile are:



Figure 4. Schematic of operation of a ram accelerator with friction on walls.

$$\rho_o DA = \rho_1 u_1 A$$

$$P_oA + \rho_oD^2A + T + FL(perimeter) = P_1A + \rho_1u_1^2A$$

$$\rho_o DA\left[h_o + \frac{D^2}{2} + Q\right] + FL(perimeter)D = \rho_1 u_1 A\left[h_1 + \frac{u_1^2}{2}\right].$$

The momentum equation contains a thrust term, T, (force applied by the projectile on the control volume) and a friction term from the wall, FL(perimeter), (and/or obstacles or baffles fixed to the wall). L is the length of the control volume. The energy equation contains a source of heat term, Q, (chemical energy released by combustion) and a work term from the friction force. The thrust term from the momentum equation does not have an associated work term in the energy equation. This is because the velocity of the projectile is zero in the control volume reference frame and therefore, no work is done on the flow by this force.

The shear stress is taken to be proportional to the square of the relative velocity between the wall and the flow:

$$F = \frac{1}{2}k_f \rho_o \left| \frac{D - u_1}{2} \left[\frac{D - u_1}{2} \right] \right|.$$

Since this control volume approach does not tell us how the velocity varies inside the control volume, the flow velocity is taken as the average of the velocities of the flows entering and leaving the control volume. The friction coefficient k_f is estimated by distributing uniformly the drag of the baffles on the surface between baffles. This yields the following expression:

$$k_f = C_D \frac{A_p}{A_w}$$

where C_D is the drag coefficient of the baffle, A_p is the projected area of the baffle and A_w is the surface area of the tube wall between baffles. Taking the drag coefficient as 1, setting the spacing between baffles to 2r and approximating the projected area of the baffle as πr^2 gives a friction coefficient k_f of 0.25.

Once non-dimensionalized, the system of equations becomes independent of the cross-sectional area but depends on the aspect ratio of the control volume, L/d. As a rule of thumb, the length of the control volume is taken as twice the projectile length, which itself is taken as 4 radii. Therefore, the aspect ratio of the control volume is 8. The specific heat ratio is taken as 1.4 and the normalized heat release Q/c_pT_o is taken as 5.

The above system of equations can be solved simultaneously to produce normalized thrust as a function of projectile Mach number. This result is plotted in Fig. 5 with and without friction. Without friction, thrust is maximum when the exit flow velocity is equal to the wall velocity. Therefore, at this point the inclusion of friction has no effect since friction depends on relative velocity between the gas and the wall (which is zero both at the inlet and the exit of the control volume). For higher Mach numbers, the thrust is reduced by friction since friction acts in the



Figure 5. Normalized thrust on projectile in ram accelerator with and without friction.

negative direction. For lower Mach numbers, thrust is increased since friction acts in the positive direction.

Conclusion

It was shown that the magnitude of the frictional work term is of considerable importance for detonation with friction. This term represents work that is done by friction on the flow and results in energy input to the flow. The effect of this term on the ZND solution is to increase the detonation velocity (or Mach number). However, the net effect of friction (source terms in momentum and energy equations), is still to reduce the detonation velocity from the CJ velocity.

It is also concluded that the work term should be included in the conservation laws and the ZND equations. This was explained from a physical point of view as well as demonstrated with a Galilean transformation from the lab frame to the detonation frame of reference. Unsteady Euler calculations also showed that the detonation velocity is consistent with the steady analysis only if the frictional work term is included in the energy equation.

Even though this work term is formally inconsistent with a one-dimensional formulation, it must be included whenever the wall has a non-zero velocity in the reference frame of interest. In other words, if a steady analysis is performed, the work term must be included if the wall has a nonzero velocity in the steady reference frame. In Fanno flow, the wall velocity is zero in the steady reference frame and therefore the work term vanishes. However, in quasi-detonations or ram accelerators, the wall has a non-zero velocity in the steady reference frame and therefore the work term must be included.

The effect of friction on ram accelerator projectile thrust was also investigated. It was found that for high Mach numbers, friction decreases thrust while thrust is increased for lower Mach numbers. Thrust is unaffected at the point where the flow exiting the control volume is equal to the wall velocity (point of maximum thrust when there is no friction).

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