# Prediction of Flame Propagation in a Tube with Obstacles Using a Fictitious Domain Method

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## Introduction

Among various numerical techniques for partial differential equations, the finite difference method is one of the most attractive. These methods on structured meshes are usually faster than on unstructured meshes and in the case of regular domain it is very efficient in terms of computation cost. In this sense, a fictitious domain method (FDM) allows to work in a regular domain using a regular meshes independently of the geometry of the problem investigated. FDM techniques for partial differential equations have recently shown an interesting potential for solving complicated problems in Science and Engineering (Smagulov 1981, Glowinski 1994, Collino 1997). The main reason for the popularity of FDMs is that they allow to use structured meshes on a geometrically simple shape (typically a rectangle in 2D) together with an auxiliary (fictitious) domain containing the actual domain, therefore allowing the use of fast solvers. Another important point is that the stability condition of the resulting scheme is the same as the one of the finite difference scheme (Smagulov 1981,).
Up to now a FDM was used for solving complicated problems of physics with Dirichlet boundary conditions (Astrakhantsev 1978, Glowinski, Pan 1995, Glowinski, Kearsley 1995). But many problems of science (for example such as reacting flow problems or combustion problems) are Neumann problems and here the application of FDM has essential peculiarities.
In the context of the study a fictitious domain method is extended and implemented for combustion problems in a non-regular domain. The most prominent example of these problems is the propagation of a flame in a tube with obstacles. It is known that the combustion process of premixed gases in tubes or vessels is strongly affected by obstacles. For freely propagating flames (larger) obstacles can cause violent flame acceleration and enhance the transition from deflagration to detonation.

## Physical, chemical and mathematical models

We are considering the case when a 2D flame propagates in a rectangular tube with length $L$ and a cross section of $2h$ x $2h$. A plate obstacle with height $h/2$ and thickness $l$ is mounted perpendicular to the axis of the combustion vessel. The combustible gas is a stoichiometric methane-air mixture. The ideal gas law is assumed to be valid.
The study is based on the assumption of a low Mach number flow (sound wave propagation is neglected). The 2D unsteady governing conservation equations of energy, species mass, mass and momentum, transport coefficients, thermodynamic data, kinetic coefficients, numerical techniques and solution algorithm are given elsewhere (Kaltayev 2001, 2002). A

single step, global irreversible reaction of the methane-air laminar flame is used with Arrhenius type reaction rate (Westbrook 1981).

The fictitious transport coefficients are:

$$\lambda^{\varepsilon}(x)=\begin{cases}\lambda, if\ \ x\in\Omega\\ \varepsilon, if\ \ x\in\Omega_f\end{cases},\qquad D^{\varepsilon}(x)=\begin{cases}D, if\ \ x\in\Omega\\ \varepsilon, if\ \ x\in\Omega_f\end{cases},\qquad \xi^{\varepsilon}(x)=\begin{cases}\xi, if\ \ x\in\Omega\\ \varepsilon^{-1}, if\ \ x\in\Omega_f\end{cases},$$

where $\Omega$ is the physical flow domain, $\Omega_f$ is the fictitious domain (obstacle domain), $\varepsilon$ is small positive parameter.

On the boundaries we use the non-slip condition for the velocity and the system is considered heat- and mass isolated. At initial time the gas mixture is at rest at a temperature of *300 K* and an average pressure of $P(0)=P_0=10^5$ *Pa*. Other parameters are: $\varepsilon=10^{-6}$, *L=38 mm*, *2h=10 mm*, *l=0.5 mm.* Due to the symmetry of the problem the calculations are performed in the upper part of rectangular vessel using the uniform mesh with *100* x *760* (cases 1a, 1d, Fig. 1) or *200* x *1520* grid-points (cases 1b, 1c, Fig. 1).

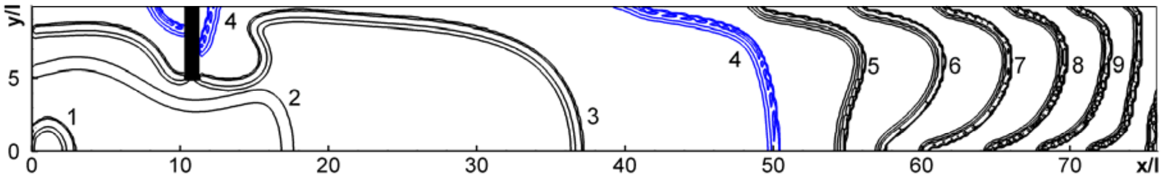


Fig. 1a

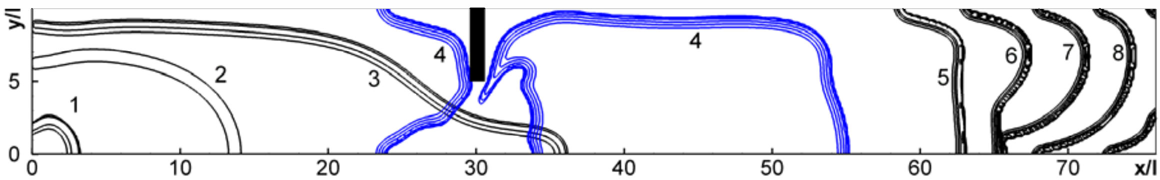


Fig. 1b

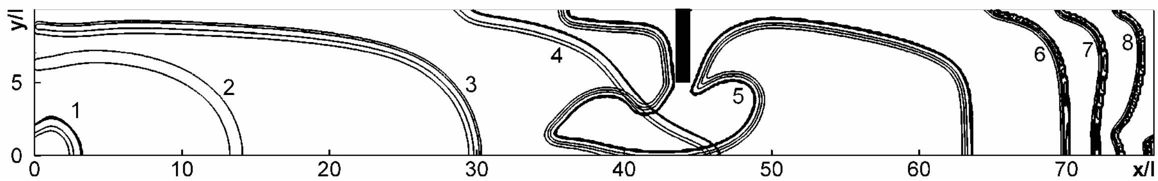


Fig. 1c

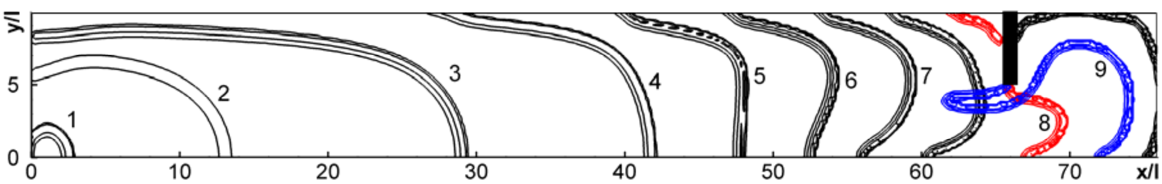


Fig. 1d

Fig. 1. Flame position for four different positions of the orifice : *1 – 1 ms; 2 – 5 ms; 3 – 9 ms; 4 – 13 ms; 5 – 17 ms; 6 – 21 ms; 7 – 25 ms; 8 – 29 ms; 9 – 33 ms; 10 – 37 ms.* Position of orifice: a – $x_{ob}/l=11$; b – $x_{ob}/l=30.4$; c – $x_{ob}/l=44$; d – $x_{ob}/l=66$, $l=0.5mm$.

**Results of flame and pressure simulation**

Similar to the experiments of Starke (Starke 1989) we consider the case when the flame travelling in a tube with an orifice as an obstacle at various distances *x* from the point of ignition. Figure 1 shows four snapshots of the travelling flame. Like in experiments: a real tulip formation occurs only when the orifice is placed close to endflanges (Fig. 1a and Fig. 1d); a flame front refraction takes place in case 1b (flame position between 6 and 8, Fig. 1b) and 1c (flame position between 7 and 9, Fig. 1c); a fastest mixture burning out occurs in case 1c then 1b (Fig. 2); a strongest pressure jump and flame acceleration takes place in case 1c (Fig. 2). The baroclinick vorticity generating and increasing at the refracting flame is observed (Fig.3).

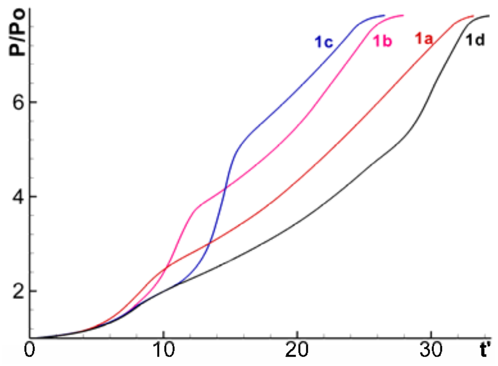


Fig. 2. Average pressure records of 1a, 1b, 1c and 1d cases, *t=t'*1.25sec*.

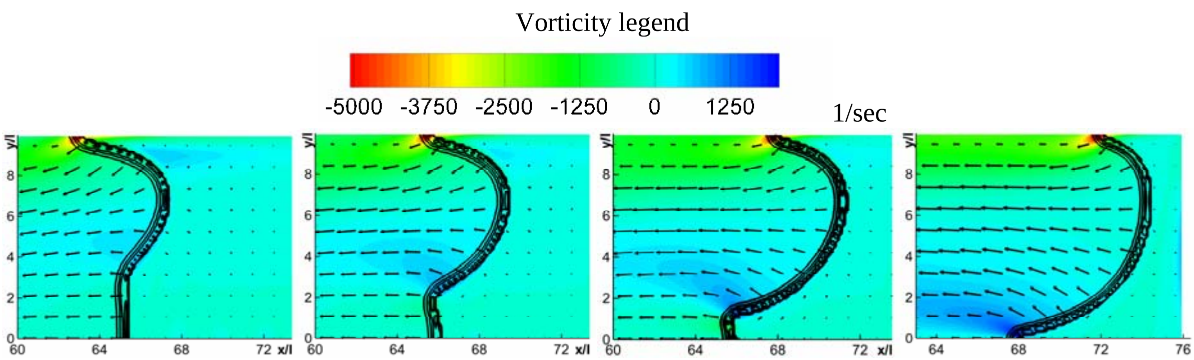


Fig. 3. Velocity (arrows) and vorticity (colour field) fields near the refracting flame front in case of 1b (position 6-8).

Simulations of other cases will be presented, too:

1. A plate obstacle with height *h* mounted at various distances x from the point of ignition in the centre of combustion chamber;
2. A cylinder obstacle with semi-circular cross section and diameter *h* mounted at various distances x from the point of ignition at the wall of combustion chamber;
3. A cylinder obstacle with circular cross section and diameter *h* mounted at various distances x from the point of ignition in the centre of combustion chamber;

4. An obstacle with new moon like cross section and chord *h* mounted at various distances x from the point of ignition in the centre of combustion chamber.

## Conclusion

Using the FDM techniques a detailed simulation of flame propagation in a tube with obstacles is implemented. The computer code of flame propagate at various parameters is carried out. The results obtained are compared with experimental data available in the literature (Starke 1989) and analyzed. Evolution of the average pressure of combustion chamber, velocity, vorticity, temperature, and dynamic pressure fields are shown. Flame travelling in a tube with various types of obstacles is animated.

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