The Effect of Discrete Energy Sources on Detonation Propagation

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Introduction

A century after it was postulated, the Chapman-Jouguet criterion has been exhaustively validated by analytical and experimental findings. Yet, exactly why the CJ criterion is so successful in predicting the average propagation velocity of unsteady and multidimensional detonation waves is intriguing, as the CJ condition was formulated for steady, one-dimensional detonations. This point is illustrated by gaseous detonations that propagate with highly irregular structure and detonations in multiphase and heterogeneous condensed phase systems. Despite the enormous complexity of their structure, most of these detonation waves propagate with an average velocity within one percent of that predicted by the CJ criterion. While it might be argued that, since a sufficiently large steady control volume can encompass the wave, the one-dimensional relations must still dictate the average propagation of a detonation, this answer is not entirely satisfying.

In order to address this issue by reducing it to an extreme case, it is of interest to consider a reactive wave propagating in an idealized heterogeneous system of discrete sources imbedded in an inert medium. The propagation mechanism becomes one of “sympathetic detonation,” where the decaying blast from one source initiates the next. This could be, for example, a suspension of primary explosive or bubbles of a sensitive gaseous explosive (acetylene-oxygen) in inert gas. Whether a wave of sympathetic detonation propagates, on average, at the equivalent CJ detonation velocity of the homogenized mixture is an outstanding question.

The effect of discrete heat sources on flame propagation was considered by Goroshin et al. (1998), who treated the case of point-sources of heat that are activated at a specified temperature as heat diffuses outward from previously-activated sources. Since the governing equation was the diffusion (heat) equation, the solution could be constructed analytically, using superposition of the solution for a single source (Green’s functions). For the present problem, the task is considerably more difficult, since the mechanism of propagation will by via nonlinear shock waves (blast waves) generated by the discrete sources, which are not amenable to superposition. For a system governed by the Euler equations, it is likely that numerical solutions will be required. This paper, however, strives to address the problem analytically as far as feasible. This is done by considering simplified models in which the blast waves to not interact, or even simpler model systems (Burgers equation) in which analytic solutions are possible.

Zeroth Order Blast Wave Analysis

Begin by considering a homogenous energetic medium with energy source density E_o [J/kg] (e.g., a cloud of fine explosive powder suspended in inert gas). Now, imagine that the energy sources can be collected from the medium and collapsed onto thin sheets separated a distance L apart (see Fig. 1). The energy per unit area of each sheet is now given by $E_{planar} = \rho\, L\, E_o$ [J/m^2]. If one source is triggered instantaneously, it will generate a blast wave that, for sufficiently strong blast, is governed by the similarity solution (Jones 1961)

$$t = \frac{3}{2}\left(\frac{\eta E_{planar}}{B\rho_o}\right)^{-1/2} x^{3/2}$$

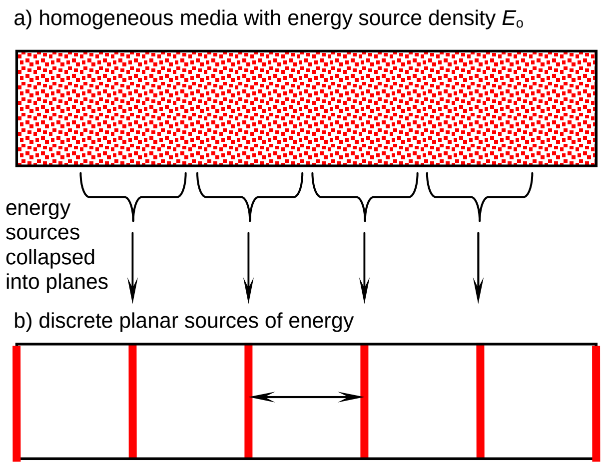

Figure 1 Schematic showing how system of discrete energy sources is derived from equivalent homogenous system.

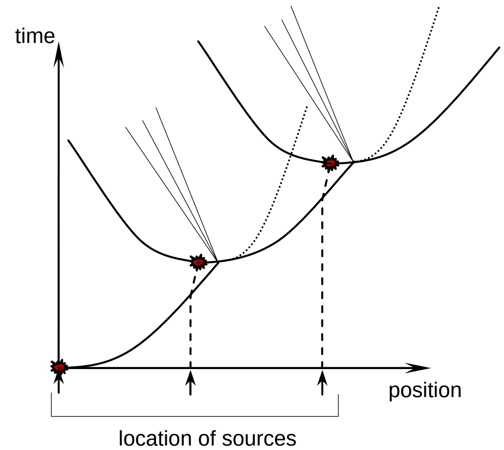

Figure 2 Schematic of wave processes in propagation of a detonation in a discrete-source system.

Here, B is a dimensionless parameter and η is an energy partition function (for symmetric blast waves propagating in both directions, $\eta = 0.5$).

If we assume that as the blast encounters the second source, it is triggered instantaneously, then the second blast is coincident with the first. If we further assume that there is no influence from one blast to the next, then the average propagation of this wave of "sympathetic detonation" will simply be the average time required for the first blast to span a distance L

$$(U_s)_{avg} = \frac{x_s}{t} = \frac{3}{2}\sqrt{\frac{\eta}{B}E_o}$$

Note that this velocity is independent of L: the more widely spaced the sources are, the more energy accumulated in each source, and the stronger (and faster) the blast is from that source. Thus, the average propagation speed depends only on the average energy density of the equivalent homogenous system. Since the interaction between sources has been neglected, this model can be considered a "zeroth order" analysis. We can compare this propagation velocity to the CJ detonation velocity of the original, homogenous system with source energy density E_o:

$$U_{CJ} = \sqrt{2(\gamma^2 - 1)E_o}$$

The velocities are compared in Table 1. Although the propagation velocities have the same functional dependence on E_o and are of the same order of magnitude, the sympathetic detonation in the discrete source system propagates about 30% slower than CJ detonation velocity for this simplified model. It is unclear if this velocity will increase as the ensemble of blast waves begins interacting. To construct a "first order" model, it would be necessary to treat the unsteady flowfield in Fig. 2. Rather than attempt this, we will instead consider a model conservation law system (the Burgers equation) that can be solved analytically.

Table 1. Comparison of CJ and Discrete Source Detonation

	$\gamma = 1.4$	$\gamma = 1.666$
Homogenous	$U_{CJ} = 1.38\sqrt{E_o}$	$U_{CJ} = 1.89\sqrt{E_o}$
Discrete (zeroth order model)	$U_{Avg} = 0.96\sqrt{E_o}$	$U_{Avg} = 1.29\sqrt{E_o}$

Exact Solutions Using Model Equation

Using the one-dimensional scalar Burgers equation as a model for detonation phenomena was proposed by Fickett (1979) and independently by Majda (1981). Here, we will follow the treatment of Fickett (1985a, 1985b), although the systems can be shown to be equivalent. The analog system is based on the inviscid Burgers equation

$$\rho_t + p_x = 0, p = \frac{1}{2}(\rho^2 + q\lambda)$$

Here, ρ is the conserved property and is analogous to density, p (the flux term) is the analog to pressure in the momentum equation, λ is a reaction progress variable (0 for unreacted, 1 for reacted), and q is the energy release. A reaction rate can specify the dependence of λ on ρ, however, for comparison purposes, in the homogeneous CJ detonation the reaction can be taken as instantaneous behind the leading shock. The shock or detonation propagation velocity is given by:

$$U_s = \frac{[p]}{[\rho]}$$

Assuming the strong shock limit ($\rho_o = 0$), the nonreacting shock and detonation velocities become

$$U_s = \frac{1}{2}\rho_s\,,\ U_{CJ} = \sqrt{q} = \rho_{CJ}$$

For the discrete system, we can specify an array of delta function sources along an array. The sources remain fixed as the shock passes (x corresponds to a Lagrangian coordinate, not a physical position in space). In the homogeneous system, the source term is a source with respect to time, not space. Thus, there is some question as to how to construct the equivalent source array in space. The approach taken here is to make each source a source of area equal to the increase in area that the homogenous denotation would experience traveling the same distance L. This source is assumed to have a constant delay τ_c. More realistic rate laws (Arrhenius) could be considered as well. When a source is activated, a delta function is inserted as a triangular source (width w, height $h = \frac{2A}{w}$, where the width is taken in the limit $w \to 0$) into the solution profile.

Except for the instants in which the sources are activated, the solution of the discrete system is governed by the nonreactive Burgers equation, $\rho_t + (\frac{\rho^2}{2})_x = 0$. As this system has only one characteristic (a right-running wave of velocity ρ), the solution of a shock propagating with a linear gradient in ρ upstream and downstream can be constructed analytically using the Method of Characteristics. Because the sources are inserted as triangular profiles, the solution consists of a "sawtooth" profile of shock waves and linear rarefaction waves for all time. Shock-shock and shock-contact discontinuity mergings are treated analytically, and the solution is advanced until the next source is "deposited" in the solution.

For comparison purposes, a case where sources of unit area ($A_{source} = 1$) were distributed along the x-axis at intervals two units apart ($L = 2$) is considered. The sources have fixed delay of $\tau_c = 5$ units. The corresponding CJ detonation in this case propagates a unit velocity ($U_{CJ} =$ 1). A profile of the solution is shown in Fig. 3, and the instantaneous and average velocity of the front is shown in Fig. 4. The average velocity was computed by simply dividing the distance the front had traveled divided by the time elapsed. Note that the velocity of the wave appears to be converging to the CJ detonation of the equivalent homogenous source detonation.

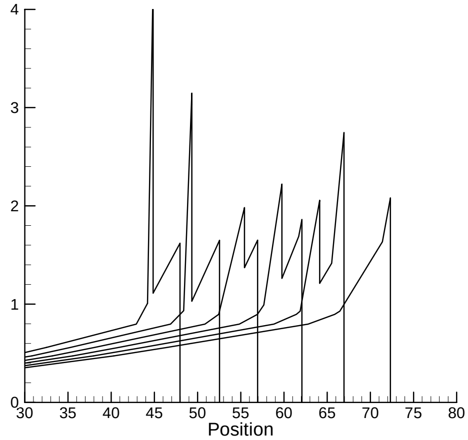

Figure 3 Solution profiles of system with sources (A_{source} = 1) spaced L = 2, with a constant τ_c = 5 delay.

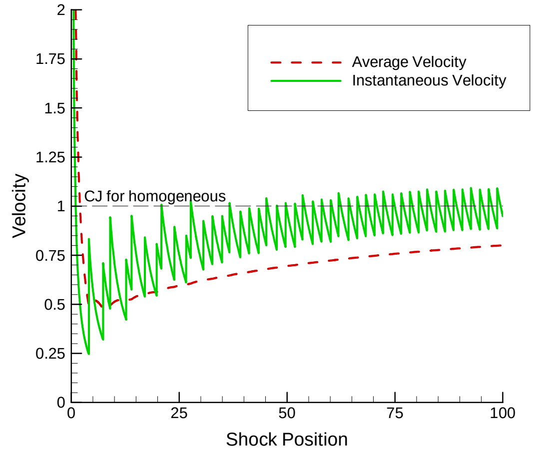

Figure 4 Velocity history of system with sources (A_{source} = 1) spaced L = 2, with a constant τ_c = 5 delay.

If we examine the *x-t* diagram in Fig. 5 (note: this *x-t* diagram is constructed in a reference frame that is moving with the average velocity U_{CJ}) of the case of a constant delay, we see that the trailing characteristics associated with prior sources appear to form an envelope that passes extremely close to the locus of new sources. This feature of the limiting characteristics may be the discrete-source analog to the steady sonic plane of the CJ detonation in this system.

Conclusions

In conclusion, it may be speculated that any distribution of energy sources that are initiated in coherence with a shock wave will propagate, on average, at the Chapman-Jouguet speed of the corresponding homogenized medium. This hypothesis, if proven, would provide an alternative and perhaps more satisfactory explanation for what experimentally observed detonations exhibit such excellent agreement with the CJ criterion, particular in highly irregular or heterogeneous detonations.

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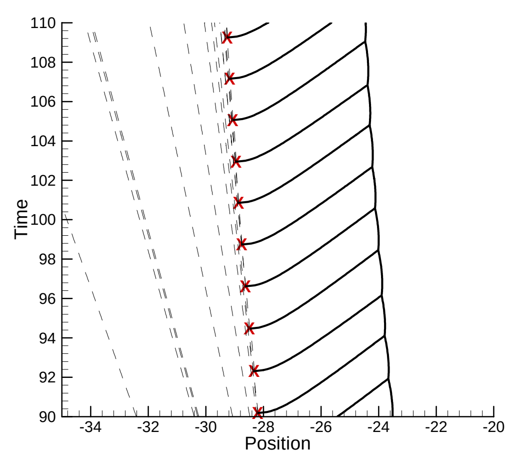

Figure 5 *x-t* diagram of wave processes in system of discrete sources. Solid lines indicate shocks, dashed lines contact surfaces, and "X" the time and location of source firings.