A Theoretical Evaluation of Turbulent Markstein Number for Expanding Spherical Flames

Andrei N. Lipatnikov and Jerzy Chomiak
Chalmers University of Technology, Gothenburg, Sweden

Corresponding author, A.N. Lipatnikov: andrei.lipatnikov@chalmers.se

Introduction

In the combustion literature, the Markstein number is commonly used to characterize the effects of weak external perturbations on a laminar flame speed, the magnitude of the Markstein number being very sensitive to the definition of the flame speed (Clavin 1985). In a recent paper (Lipatnikov & Chomiak 2004a), we have proposed to use a turbulent Markstein number Ma_t in order to parameterize effects of large-scale stretching of premixed turbulent flames by mean flow on the flame speed via analogy with laminar flames. This hypothesis has been supported by processing a number of published experimental data on the growth of the radius of expanding, statistically spherical, premixed, turbulent flames and the obtained results have shown that the considered effects are strong.

The goal of this work is to evaluate Ma_t regarding the burning velocity by invoking recent theoretical results obtained for self-similar developing flames (Lipatnikov & Chomiak 2002a, 2004b). As discussed in detail elsewhere (Lipatnikov & Chomiak 2002b), a typical premixed turbulent flame is a self-similar developing flame such that the spatial profiles of mean gas density, normal to the flame brush, are described by the same function at different instants t when using a time-dependent flame brush thickness $\delta_t(t)$ to normalize the spatial coordinate as follows $\xi = (x - x_f(t))/\delta_t(t)$, where $x_f(t)$ is the flame position.

Analysis

The following two theoretical results are invoked: First, for an expanding, statistically spherical, self-similar, developing flame, the burning velocity U_t can be determined by dividing the burning rate integrated across the flame brush by the area Σ of a spherical surface, equal to (Lipatnikov & Chomiak 2002a)

$$\Sigma = \frac{8\pi}{\varrho_b} \int_0^\infty \bar{\varrho}\tilde{c} r \, dr, \tag{1}$$

where $\varrho = \rho/\rho_u$ is the normalized density, subscripts u and b designate unburned and burned mixture, respectively, $\tilde{c}$ is the Favre-averaged combustion progress variable (Bray & Moss 1977), and overbars denote the Reynolds averages.

Second, for a statistically planar, self-similar, developing flame that moves from the left to the right, the following general balance equation

$$\frac{\partial}{\partial t}(\bar{\varrho}\tilde{c}) + \frac{\partial}{\partial x}(\bar{\varrho}\tilde{u}\tilde{c}) = \frac{\partial}{\partial x}\left(\bar{\varrho} D_t \frac{\partial \tilde{c}}{\partial x}\right) + \mathcal{V}\frac{\partial f}{\partial x} + \frac{w}{\tau_f}, \tag{2}$$

which subsumes a number of contemporary models of premixed turbulent combustion, can be rewritten in the following simple form (Lipatnikov & Chomiak 2004b)

$$\frac{\partial}{\partial t}(\bar{\varrho}\tilde{c}) + \frac{\partial}{\partial x}(\bar{\varrho}\tilde{u}\tilde{c}) = \frac{\partial}{\partial x}\left(\bar{\varrho}\mathcal{D}\frac{\partial \tilde{c}}{\partial x}\right) - U_t^o\frac{\partial \tilde{c}}{\partial x}. \tag{3}$$

Here, $f(\tilde{c},\gamma)$ and $w(\tilde{c},\gamma)$ are arbitrary, positive, bounded functions such that $f(\tilde{c}=0) = f(\tilde{c}=1) = w(\tilde{c}=0) = w(\tilde{c}=1) = 0$; $\gamma = \rho_u/\rho_b$ is the density ratio; $\mathcal{V}$ and τ_f are arbitrary velocity and time scales, respectively, introduced for dimensional reasons; D_t and $\mathcal{D} = D_t d_t(t)$ are the turbulent and generalized diffusivities, respectively, with the function $0 \le d_t(t) \le 1$ being defined elsewhere (Lipatnikov & Chomiak 2004b); and

$$U_t^o = \frac{1}{\tau_f}\int_{-\infty}^{\infty} w dx \tag{4}$$

is the unperturbed turbulent burning velocity.

Let us generalize Eq. 3 for expanding, statistically spherical flames as follows

$$\bar{\varrho}\frac{\partial \tilde{c}}{\partial t} + \bar{\varrho}\tilde{u}\frac{\partial \tilde{c}}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\bar{\varrho}\mathcal{D}\frac{\partial \tilde{c}}{\partial r}\right) - U_t^o\frac{\partial \tilde{c}}{\partial r}. \tag{5}$$

Using Eq. 1 and integrating Eq. 5, we obtain

$$U_t = -\frac{4\pi U_t^o}{\Sigma}\int_{-\infty}^{\infty}\frac{\partial \tilde{c}}{\partial r}r^2 dr = U_t^o \varrho_b \int_{-\infty}^{\infty}\tilde{c} r dr\left(\int_{-\infty}^{\infty}\bar{\varrho}\tilde{c} r dr\right)^{-1}. \tag{6}$$

If the well-known Bray-Moss (1977) model is invoked, then

$$\bar{\varrho}\tilde{c} = \varrho_b\bar{c} = \frac{1-\bar{\varrho}}{\gamma-1} \tag{7}$$

and the burning velocity is equal to

$$\begin{aligned}\frac{U_t}{U_t^o} &= \int_{-\infty}^{\infty}\tilde{c} r dr\left(\int_{-\infty}^{\infty}\bar{c} r dr\right)^{-1} = 1 - \int_{-\infty}^{\infty}\overline{c''} r dr\left(\int_{-\infty}^{\infty}\bar{c} r dr\right)^{-1} \\ &= 1 - (\gamma-1)\int_{-\infty}^{\infty}\bar{\varrho}\tilde{c}(1-\tilde{c}) r dr\left(\int_{-\infty}^{\infty}\bar{c} r dr\right)^{-1}. \end{aligned} \tag{8}$$

If a ratio δ_t/R_f of the mean flame brush thickness to a flame radius is sufficiently low, the last term on the right hand side of Eq. 8 scales as δ_t/R_f. To the leading order, we have

$$\frac{U_t}{U_t^o} = 1 - \frac{2(\gamma-1)\delta_t}{R_f}\int_{-\infty}^{\infty}\bar{\varrho}\tilde{c}(1-\tilde{c})\frac{dx}{\delta_t} + o\left(\frac{\delta_t}{R_f}\right), \tag{9}$$

where the integration is performed across the mean flame brush. Thus, the burning velocity in statistically spherical, premixed, turbulent flames is lower as compared with statistically planar flames, all other things being equal. The reduction effect scales as the normalized curvature $2\delta_t/R_f$ of the mean flame brush to the leading order as $\delta_t/R_f \to 0$.

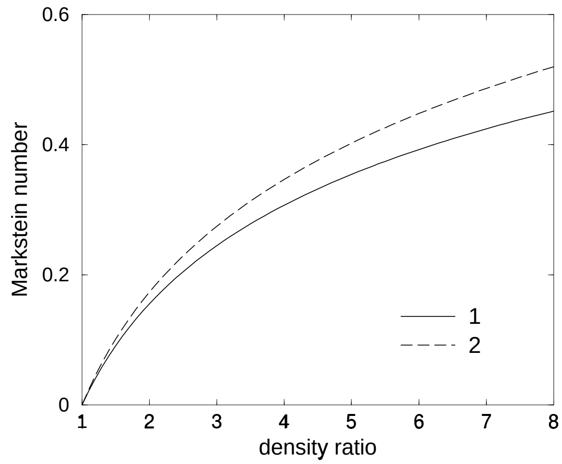

Figure 1: Turbulent Markstein number vs. the density ratio. Curves 1 and 2 have been computed using Eqs. 11 and 13, respectively.

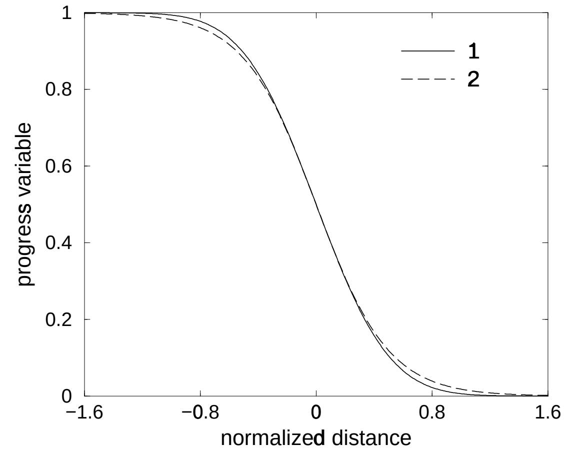

Figure 2: Profiles of the Reynolds-averaged combustion progress variable, given by Eqs. 11 (curve 1) and 13 (curve 2).

This reduction in U_t is similar to the well-known reduction (Clavin 1985) in the burning velocity of expanding, spherical, laminar flames with the Lewis number equal to unity.

For the turbulent flames, the burning velocity reduction effect can be described using the turbulent Markstein number, which is equal to

$$Ma_t = (\gamma - 1) \int_{-\infty}^{\infty} \bar{\varrho}\tilde{c}(1-\tilde{c})\frac{dx}{\delta_t}. \tag{10}$$

To the leading order, this integral can be calculated by invoking the following progress variable profile

$$\bar{c} = \frac{1}{2}\mathrm{erfc}\left(\xi\sqrt{\pi}\right) = \frac{1}{\sqrt{\pi}}\int_{\xi\sqrt{\pi}}^{\infty} \mathrm{e}^{-\zeta^2} d\zeta, \tag{11}$$

where

$$\xi = \frac{x - x_f(t)}{\delta_t(t)}, \qquad x_f = \int U_t^o dt, \qquad \delta_t \equiv \max{}^{-1}\left|\frac{\partial \bar{c}}{\partial x}\right| = \left(4\pi\int \mathcal{D}dt\right)^{1/2}. \tag{12}$$

It can be shown that Eqs. 11 and 12 satisfy Eq. 3 supplemented with Eq. 7 and the mass balance equation in the statistically planar, 1-D case. It is worth noting that the solution given by Eq. 11 is written for the Reynolds-averaged progress variable, whereas Eq. 3 is written for the Favre-averaged one. Although Eq. 11 does not satisfy Eq. 5 in the spherical case, the error associated with the substitution of Eq. 11 into Eq. 10 vanishes as $\delta_t/R_f \to 0$ and the corresponding error in Eq. 9 is on the order of $o(\delta_t/R_f)$.

The solid curve in Fig. 1 shows the dependence of the turbulent Markstein number calculated using Eqs. 7, 10-12 on the density ratio. The Markstein number grows with γ but depends neither on turbulence characteristics nor on other mixture properties. A simple analytical expression for Ma_t can be obtained by invoking the following alternative self-similar profile

$$\bar{c} = \left(1 + \mathrm{e}^{4\xi}\right)^{-1}, \tag{13}$$

which is very close (cf. curves 1 and 2 in Fig. 2) to the aforementioned analytical solution to Eq. 3. Using Eqs. 7 and 13, the integral in Eq. 10 can analytically be obtained

$$Ma_t = \frac{\ln(\gamma)}{4}. \tag{14}$$

Curve 2 in Fig. 1 shows the Markstein number calculated using Eq. 14. Although the two profiles given by Eqs. 11 and 13 are very close, the corresponding Ma_t differs from one another, the difference being increased by γ (cf. curves 1 and 2 in Fig. 1).

The turbulent Markstein numbers calculated above are typically larger than the values of Ma_t, obtained (Lipatnikov & Chomiak 2004a) by processing experimental data. Moreover, the "experimental" numbers decrease when the rms turbulent velocity u' increases, whereas the theoretical Ma_t does not depend on u'. These differences can be associated with the fact that the experimental numbers characterize the flame speed S_b with respect to burned mixture, whereas the Ma_t determined by Eq. 10 characterizes the burning velocity U_t. For the flames studied, the speed S_b is controlled not only by the burning rate but also by the growth of the mean flame brush thickness (Lipatnikov & Chomiak 2002a), the growth of δ_t being increased by u' (Lipatnikov & Chomiak 2002b).

Conclusions

For expanding, spherical, self-similar, premixed flames, a new turbulent Markstein number Ma_t regarding the burning velocity is determined by Eq. 10. Equation 14 provides a simple analytical estimate of Ma_t. The Markstein number grows with the density ratio, but depends neither on turbulence characteristics nor on other mixture properties.

Acknowledgments

This work has been supported by the Swedish Research Council.

References

KNC Bray and JB Moss 1977 "A Unified Statistical Model for the Premixed Turbulent Flame" *Acta Astronautica* 4: 291-319.

P Clavin 1985 "Dynamical Behavior of Premixed Flame Fronts in Laminar and Turbulent Flows" *Progress in Energy and Combustion Science* 11 (1): 1-59.

AN Lipatnikov and J Chomiak 2002a "Turbulent Burning Velocity and Speed of Developing, Curved, and Strained Flames" *Proceedings of the Combustion Institute* 29: 2113-2121.

AN Lipatnikov and J Chomiak 2002b "Turbulent Flame Speed and Thickness: Phenomenology, Evaluation, and Application in Multi-Dimensional Simulations" *Progress in Energy and Combustion Science* 28 (1): 1-73.

AN Lipatnikov and J Chomiak 2004a "Application of the Markstein Number Concept to Curved Turbulent Flames" *Combustion Science and Technology* 176: 331-358.

AN Lipatnikov and J Chomiak 2004b "A Theoretical Study of Premixed Turbulent Flame Development" *Proceedings of the Combustion Institute* 30: in press.