Effect of an oscillatory small scale flow on flame propagation

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Summary

The present investigation describes new results addressing the validity of Damköhler's second hypothesis in parallel small scale flows. Two main contributions have been made. The first is analytical based on matched asymptotics, and leads to a formula for the effective speed of a premixed flame in the presence of an *oscillatory* parallel flow, valid when the flow scale is much smaller than the laminar flame thickness. The second contribution, which is numerical, provides a significant set of two-dimensional calculations aimed at assessing the range of validity of the asymptotic findings. The calculations are based on a finite-volume multigrid approach and account in particular for volumetric heat-loss and differential diffusion effects. A good agreement between the numerics and asymptotics is found in all cases, both for steady and oscillatory flows, at least in the expected range of validity of the asymptotics. Additional related aspects such as the difference in the response of thin and thick flames to the combined effect of heat-loss and fluid flow are also discussed. It is found for example that the sensitivity of thick flames to volumetric heat-loss is negligibly affected by the flow intensity, in marked contrast to the sensitivity of thin flames. Interestingly, thin flames are found to be more resistant to heat-loss when a flow is present, even for unit Lewis number; this ceases to be the case, however, when the Lewis number is large enough.

Introduction

According to Damköhler's second hypothesis in turbulent combustion, the small scales of the flow-field do not cause any significant flame wrinkling but do change the flame structure by enhancing the diffusive processes. An original analytical contribution aimed at testing this hypothesis was carried out in [1] in the framework of prescribed *steady parallel flows*. Its main result is the formula

$$\frac{U_T}{U_0} \sim 1 + \frac{\ell^2}{2}\int_0^1 \left[\int_0^\eta u(\eta_1)\,\mathrm{d}\eta_1\right]^2 \mathrm{d}\eta\,,$$

valid for small values of the scale ℓ of the flow u. Here ℓ and u are measured against the thickness δ_L and speed S_L of the *adiabatic* planar flame. U_T and U_0 represent the effective flame speed and the *planar* laminar flame speed, also measured with S_L. In the absence of heat losses $U_0 = 1$, but more generally U_0 is the larger root of $U_0^2 \ln U_0 = -\kappa$, where κ represent the intensity of heat-loss . In this equation the argument of u must lie in $[0, 1]$ and its spatial mean must be equal to zero, which is always possible by an appropriate choice of the origin and scale on the transverse axis and of the reference frame. The formula describes the increase in the effective flame speed U_T which is seen to depend quadratically on both the scale and intensity of the flow while being independent of the Lewis number. It is useful to extend this result to more realistic situations, e.g. by accounting for flow unsteadiness and for more complex flows. As a step towards this goal, the present work generalises the analytical formula to *time-periodic* parallel flows. It provides also a numerical assessment of the range of validity of the analytical findings, both for steady and unsteady flows, and takes into account heat-loss and preferential diffusion effects.

Asymptotic analysis

The analysis is carried out using the asymptotic limit of small flow scale $\ell \to 0$ and large Zeldovich number β (with $\beta^{-1} \ll \ell$), and assuming the parallel periodic flow $u(y, t)$ to have a zero spatial mean in an appropriately chosen frame of reference.

We simply record the main finding for the effective propagation speed U_T:

$$\frac{U_T}{U_0} \sim 1 + \frac{\ell^2}{2}\int_0^1 \overline{\left[\int_0^\eta u(t,\eta_1)\,\mathrm{d}\eta_1\right]^2} \mathrm{d}\eta\,,$$

with $U_0 = 1$ in the adiabatic equidiffusional case, but more generally $U_0^2 \ln U_0 = -\kappa$ in presence weak heat-losses and preferential diffusion effects. The validity of the derivation in term of the frequency of the oscillatory flow will be discussed.

Computations and comparison with asymptotics

An extensive set of numerical calculations has been carried out, mainly in order to assess the validity of the asymptotic findings, both for time-independent and oscillatory flows. A finite volume discretisation combined with an algebraic multigrid solver has been used along with a non-uniform grid.

For time-independent flows, the following non-dimensional form is adopted:

$$u = A \cos \frac{\pi y}{\ell} \, .$$

Here A is the flow amplitude (measured with S_L) and ℓ the flow scale (measured with δ_L). For these flows, the asymptotic formula predicts the result

$$\frac{U_T}{U_0} = 1 + \frac{A^2 \ell^2}{4\pi^2} \, ,$$

against which the numerics are compared.

For time-dependent situations, the following harmonic form is adopted:

$$u = A \cos \frac{2\pi t}{\tau} \cos \frac{\pi y}{\ell}$$

where τ is the time-period measured against δ_L / S_L. The corresponding asymptotic prediction is

$$\frac{U_T}{U_0} = 1 + \frac{A^2 \ell^2}{8\pi^2} \, ,$$

and is compared to the numerical results.

For sake of illustration for the time-dependent case, shown in Fig. 1, corresponding to $\ell = 1$, $A = 2$ and $\tau = 1$ are the instantaneous amplitude, $\hat{A} \equiv A \cos 2\pi t/\tau$, and the total burning rate Ω versus time t, after an initial transient. From the plot, $U_T = \overline{\Omega}$, the time average of Ω, can be extracted and is found to be approximately equal to 0.92 in this case. Repeating the calculation for several values of ℓ and extracting $\overline{\Omega}$ generates Fig. 2, where U_T is plotted versus ℓ along with the curve based on the asymptotics.

Conclusion

In this investigation, two contributions have been made. Firstly, we have derived an analytical formula for the effective flame speed in the presence

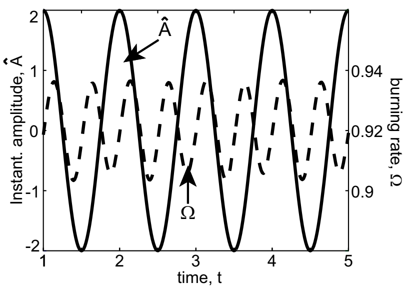

Figure 1: Instantaneous amplitude $\hat{A}$ (solid line) and total burning rate Ω (dashed line) versus time t.

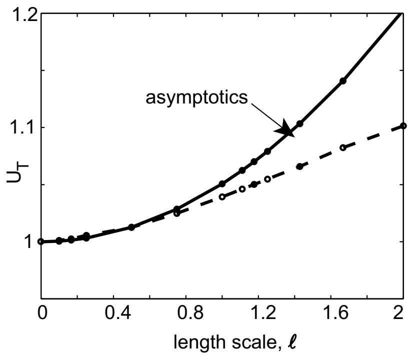

Figure 2: U_T versus ℓ predicted by asymptotics (solid line) and numerics (dashed line).

of a prescribed, oscillating, parallel flow, whose scale is small. Secondly, we have carried out a large number of numerical calculations as a systematic test of the asymptotic findings for both stationary and time-dependent flows. These accounted for the effects of volumetric heat-loss as well as differential diffusion. Additional aspects, such as the difference in the response of thin and thick flames to heat-loss in the presence of a flow, were also discussed.

References

[1] Daou J., Dold J., and Matalon, M. The thick flame asymptotic limit and Damköhler's hypothesis. *Combustion Theory and Modelling*, **6**, 141-153 (2002).