# **Stability Analysis of Low-NOx Gas Turbine Combustors**

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## Introduction

Lean premixed combustors can effectively reduce NOx emissions but they suffer of the arising of undesirable self-sustained oscillations. Extensive experimental and theoretical work has been done in order to identify the physiochemical mechanisms responsible for the gas turbine combustors instability [1-5], and to predict the critical operating conditions [1, 6-10]. However, these studies do not allow to definitively clarify the causes of the occurrence of the pressure oscillations experimentally observed. Some of them clearly point out the role of coupling between acoustic and heat release from the flame [2-5]. But the experimental results carried out by Richards et al. [1] show that heat losses can have a great influence on the observed oscillations, thus suggesting a thermo-kinetic nature.

To understand the role played by each phenomenon and to predict the occurrence of dynamic regimes and its characteristics (amplitudes and frequencies) as function of the operating conditions, simplified models can help. In these models the combustor is represented as a network of ideal reactors [11, 12]. The combustor is divided in several zones (mixing and recycling, combustion and post-combustion), each of them being modeled by relatively simple reactors (WSR and PFR). In particular, the combustor zone has been extensively modeled as a well stirred reactor [13, 14], also to study the combustor dynamic behavior [3, 4, 15-16].

Results of such a simple model, whose solutions and their stability properties can be investigated in the framework of the bifurcation theory, are presented. Continuation techniques are employed to perform the stability analysis in operating conditions typical of a lean premixed combustor.

### Mathematical model development

The network of reactors here adopted was proposed by [12] to model a real combustor. The scheme is shown in Fig. 1. Each reactor describes a particular zone of the combustor: WSR1 is where mixing of reactants occurs, WSR2 is where the flame mainly develops, and finally WSR3 is the second flame zone where combustion is completed. A recycle is established between the exit of WSR2 and the enter of the mixer (WSR1): a part of the hot combustion products is recycled in the mixer to describe the effect of recirculation vortices establishing at the entrance region of the combustion chamber.



Figure 1. Reactors network describing the combustor model.

The model has been developed under the assumptions of ideal gas behavior and constant specific heats. The model equations are the unsteady balances for total mass, fuel and oxygen mass fractions and energy, written for a constant volume and assuming variable pressure:

$$\frac{d\rho^{(i)}}{dt} = \frac{1}{V^{(i)}} \left( \mathcal{Q}^{E_{i}} \rho^{E_{i}} + \sum_{j=1}^{3} R^{j,i} \mathcal{Q}^{(j)} \rho^{(j)} - \mathcal{Q}^{(i)} \rho^{(i)} \right) 
\frac{dY_{f}^{(i)}}{dt} = \frac{1}{V^{(i)} \rho^{(i)}} \left[ \mathcal{Q}^{E_{i}} \rho^{E_{i}} \left(Y_{f}^{E_{i}} - Y_{f}^{(i)}\right) + \sum_{j=1}^{3} R^{j,i} \mathcal{Q}^{(j)} \rho^{(j)} \left(Y_{f}^{(j)} - Y_{f}^{(i)}\right) + V^{(i)} v_{f} \omega^{(i)} \right] 
\frac{dY_{o}^{(i)}}{dt} = \frac{1}{V^{(i)} \rho^{(i)}} \left[ \mathcal{Q}^{E_{i}} \rho^{E_{i}} \left(Y_{o}^{E_{i}} - Y_{o}^{(i)}\right) + \sum_{j=1}^{3} R^{j,i} \mathcal{Q}^{(j)} \rho^{(j)} \left(Y_{o}^{(j)} - Y_{o}^{(i)}\right) + V^{(i)} v_{o} \omega^{(i)} \right] 
\frac{dT^{(i)}}{dt} = \frac{1}{V^{(i)} \rho^{(i)} \left(c_{p} - \frac{R_{u}}{W}\right)} \cdot \\
\cdot \left[ \mathcal{Q}^{E_{i}} \rho^{E_{i}} c_{p} \left(T^{E_{i}} - T^{(i)}\right) + \sum_{j=1}^{3} R^{j,i} \mathcal{Q}^{(j)} \rho^{(j)} \left(T^{(j)} - T^{(i)}\right) + V^{(i)} \frac{R_{u}}{W} T^{(i)} \frac{d\rho^{(i)}}{dt} - V^{(i)} \sum_{k=1}^{5} \left(h_{f,k}^{0} - \frac{R_{u}}{W}T\right) v_{k} \omega^{(i)} + V^{(i)} q^{(i)} \right] \right]$$

where the superscript (*i*) refers the reactor number *i*,  $R^{j,i}$  is the fraction of outlet stream from reactor *j* towards reactor *i* (the only recycle present is  $R^{2,1}$ ), and the superscript  $E_i$  is an external inlet stream in reactor *i* (here only  $E_1 \neq 0$ ). The variables  $\rho$ , *V*, *Q*, *T*,  $y_f$ ,  $y_O$  are gas density, reactor volume, flow rate, temperature and fraction mass of fuel and oxygen in each reactor respectively.  $c_p$ , v,  $h^0_f$ , *W*,  $R_u$  are the constant pressure specific heat, the species stoichiometric coefficient, the standard enthalpy of formation, the mean molecular weight of the mixture and the universal gas constant respectively. The model reaction is the propane combustion. The reaction rate used in the model is a one step kinetic equation evaluated as [17]:

$$\omega^{(i)} = 0 \qquad \qquad \omega^{(i)} = k_0 \left( \rho^{(i)} \right)^{1.75} \exp\left( -\frac{Ta}{T^{(i)}} \right) (Y_f^{(i)})^{0.1} (Y_o^{(i)})^{1.65} \qquad i = 2,3$$

where  $k_0 = 2.8e8 \text{ kg/m}^3 \text{s}$  and Ta = 15098 l/K. The heat exchange terms are written as:  $q^{(i)} = h^{(i)} A^{(i)} (T^{(i)} - T_c^{(i)})$  i = 1, 2, 3

We have not included the momentum balance equations to express the rate of change of the outlet volume fluxes and we have assumed a constant flow velocity at the outlet section of each reactor assigning the values of the volume fluxes. The effect of this simplification will be outlined in the following.

#### Results

The bifurcation analysis has been performed by means of the software AUTO97, based on the continuation method [18]. Heat transfer coefficient in reactor 2 and recycle fraction were assumed as bifurcation parameters. A fixed value of wall temperature ( $T_c = 450$  K) was set, as well as the volume fluxes in each reactor. Their value ( $Q^{(1)} = 0.005895$ ,  $Q^{(2)} = 0.02587$ ,  $Q^{(3)} = 0.02364$ , m<sup>3</sup>/s) were computed for a typical operating condition of lean combustor, applying the discharge low (thereafter referred as  $Q^{(i)}$  computed).

The bifurcation diagram in terms of the dimensionless temperature of the reactor ( $\Theta^{(2)} = T^{(2)}/Ta$ ) versus the heat transfer coefficient in the WSR2 ( $h^{(2)}$ ) is shown in Figure 2 (left). Typical operating conditions for a lean combustor were assumed( $\Phi = 0.59$ ;  $Q^{EI} = 0.0055 \text{ m}^3/\text{s}$ ). A multiplicity of steady states exists: stable hot steady solutions (solid line), unstable steady states (dashed line) and cold stable steady solutions ( $\Theta^{(2)} \approx 0$ , not reported). On increasing  $h^{(2)}$ , the temperature in the ignited steady state decreases and just before the blow out condition, a stable Hopf bifurcation (HB) point arises (**■**), leading to stable oscillations ( $h^{(2)} \approx 300 \text{ W/m}^2\text{K}$ , a reasonable characteristic values of a real combustor [1]). Amplitudes and frequencies of the limit

cycle are shown in the upper corner of the same figure: the black points represent the maximum dimensionless temperature during oscillations. On increasing heat losses, amplitudes increase and frequencies decrease, in agreement with the trends experimentally observed [1].



**Figure 2.** Bifurcation diagram as function of the heat transfer coefficient in WSR2 (left), and recycle amount (right);  $\Phi = 0.59$ ;  $Q^{EI} = 0.0055$  m<sup>3</sup>/s.

The effect of changing the recycle is shown in Figure 2 (right), at a fixed values of  $h^{(2)} = 323$ . At this value, the ignited state cannot be sustained if the recycle is less then a 10% of the burnt product, blow-out still occurring in an oscillating mode. The trend is similar to that found at R = 0. The region of existence of oscillations before blow-out shifts to higher values of the heat transfer coefficient, with an extended range of stable oscillations (309.21 <  $h^{(2)}$  < 309.35 for R = 0 and 323.37 <  $h^{(2)}$  < 323.59 for R = 10%). Another region of existence of stable oscillations is found between R = 82 and R = 92%, characterized by very high amplitudes (about 20%).

To quantify the effect of uncoupling the momentum balance (constant volume fluxes as computed in a specific operating condition), the bifurcation diagram has been computed also under the assumption of homogeneous flow rates. When  $Q^{(i)} = Q^{EI}$ , besides the Hopf bifurcation point just before the blow out condition ( $h^{(2)} = 574 \text{ W/m}^2\text{K}$ ), other two HB points arise, singling out a new and wider region of stable oscillations ( $164 < h^{(2)} < 294$ , W/m<sup>2</sup>K). For the HB point just before extinction, the trend is the same of that found for computed flow rates (amplitude increases and frequency decrease on increasing  $h^{(2)}$ ). A different behavior of oscillations arises for the intermediate range of  $h^{(2)}$  (a not monotone trend).

A 2 parameters bifurcation analysis allows to show the interaction between heat losses and recycle. With  $Q^{(i)}$  computed (Figure 3 left), stable steady states exist at low values of  $h^{(2)}$  for each value of the recycle. At higher values of  $h^{(2)}$  ( $h^{(2)} = 170$ ) and high values of the recycles, a region of stable oscillations is present. On increasing  $h^{(2)}$ , the region of existence of oscillations shifts to higher values of the recycle. Starting from  $h^{(2)} = 323$  blow-out may occur at increasing values of *R*. At  $h^{(2)} > 600$  W/m<sup>2</sup>K, only not ignited solutions exist. In summary, with  $h^{(2)}$  between 170 and 300 W/m<sup>2</sup>K, only very high recycle makes the system unstable and oscillations arise; increasing  $h^{(2)}$ , low values of *R* cause blow-out preceded by stable oscillations. When  $Q^{(i)} = Q^{EI}$ , (Figure 3, right) the region of ignited states extends, as well as that of stable oscillating solutions that now represent a large stripe dividing the stable steady solution region for low and high values of  $h^{(2)}$ .

#### Conclusions

By modeling the combustor as a network of WSR, including recycle, it is shown that the

occurrence of an oscillating behavior of a single combustor's zone, found into a narrow region of the space of parameters without any recycle, extends to a wider region of self-sustained oscillations for the whole combustor and that new regions of existence of oscillations are found. The flow rates strongly affect the dynamic features of the combustor and then it appears advisable to couple the momentum balance equations.



**Figure 3.** Stability map as function of the heat transfer coefficient in WSR2 and the recycle. On left,  $Q^{(i)}$  computed; on right  $Q^{(i)} = Q^{EI}$ ;  $\Phi = 0.59$ ;  $Q^{EI} = 0.0055 \text{ m}^3/\text{s}$ .

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