Confined Burke-Schumann Flames with Small Stoichiometric Mixture Fraction and Small Fuel Radius

M. Sánchez-Sanz†, A. Revuelta‡, A.L. Sánchez†, A Liñán ¶

† Area de Mecánica de Fluidos, Universidad Carlos III de Madrid, 28911 Leganés, Spain
‡ Departamento de Combustibles Fósiles, CIEMAT, 28040 Madrid, Spain
¶ E. T. S. I. Aeronáuticos, Pl. Cardenal Cisneros 3, 28040 Madrid, Spain

Corresponding Author: M. Sánchez-Sanz (mssanz@ing.uc3m.es)

The Burke-Schumann limit of infinitely fast reaction rate [1, 2] is used to investigate laminar diffusion flames formed when a fuel jet of radius εa, with $\varepsilon \ll 1$, discharges into a coaxial air stream confined by an outer duct of radius a. Turbulent flow conditions have been considered in a number of classical studies, including that of Thring and Newby [3] for reacting flow and that of Craya and Curtet [4] for nonreacting flow. Our study focuses on laminar flames corresponding to moderately large values of the jet Reynolds number $Re = [\rho_o J_{\rm F}/(\pi\mu_o^2)]^{1/2}$, with $J_{\rm F}$ denoting the momentum flux of the fuel jet, whose density and viscosity are represented by ρ_o and μ_o, respectively. The steady slender flows that form under such conditions can be described with small relative errors of order Re^{-2} in the boundary-layer approximation, which was adopted in earlier investigations of unconfined jet diffusion flames [5, 6]. The nonreacting confined flow was recently addressed in [7]. The solution was seen to depend on a single parameter, the so-called Craya-Curtet number $C = (J_{\rm O}/J_{\rm F})^{1/2}$, where $J_{\rm O}$ denotes the momentum flux of the coflow oxidizer stream. For values of C above a critical value C_c the streamlines are seen to remain aligned with the axis, while for $C < C_c$ the entrainment demands of the jet cannot be satisfied, giving rise to the appearance of a toroidal recirculating region. For the nonreacting case, the value of C_c depends only on the shape of the coflow velocity profile, giving, for instance, $C_c = (0.65, 0.77)$ for uniform and parabolic coflow, respectively.

This study extends our previous work [7] by considering the chemical reaction between the inner fuel jet, where the fuel mass fraction is $Y_{{\rm F}_o}$ and the oxidizer coflow, where the oxygen mass fraction is $Y_{{\rm O}_o}$. An irreversible overall reaction

$$\mathrm{F} + s\mathrm{O} \rightarrow (1+s)\mathrm{Products} + (q), \tag{1}$$

is assumed, where s and q represent, respectively, the mass of oxygen burnt and the amount of heat released per unit mass of fuel consumed. When the diffusivities of both reactants are equal, the mixture fraction Z emerges as a convenient scalar to compute the resulting flow field [2]. In the Burke-Schumann limit of infinitely fast combustion the reactants cannot coexist in the first approximation, and the flame, located where $Z = Z_s = Y_{{\rm O}_o}/(Y_{{\rm O}_o} + sY_{{\rm F}_o})$, separates an oxidizer region for $Z < Z_s$ from a fuel region for $Z > Z_s$. If the Lewis number of the reactants is unity and heat losses to the confining duct are neglected, then the temperature field can be readily related to Z according to $T = 1 + \gamma Z/Z_s$ for $Z < Z_s$ and $T = 1 + \gamma(1 - Z)/(1 - Z_s)$ for $Z > Z_s$, where T is the temperature scaled with the initial temperature, T_o, assumed to be equal for both streams, and $\gamma = qY_{{\rm F}_o}/[c_p T_o(1 + S)]$ is the dimensionless stoichiometric flame temperature increase, with $S = sY_{{\rm F}_o}/Y_{{\rm O}_o}$ and c_p being the specific heat at constant pressure, assumed to be constant in the following development.

The solution can be simplified when, as often occurs in applications, ε and Z_s are both small quantities, while C remains of order unity. In that case, the velocity of the fuel jet is a factor ε^{-1} larger than that of the coflow, so that the fuel jet develops near the entrance like a free jet discharging into a stagnant oxidizer atmosphere. The flame lies initially in the outer edge of the annular mixing layer that grows from the orifice rim, which separates the fuel jet where $Z = 1$ from the outer oxidizer, where $Z = 0$. The thickness of this mixing layer increases with distance to become of the order of the fuel jet radius at distances of order $Re\,\varepsilon\,a$, where transverse mixing starts to reduce significantly the value of Z at the axis. Mixing of the fuel jet with the coaxial air stream continues downstream from this jet developing region, in a mixing region of characteristic length of order $Re\,a$ where Z is of order ε, with a final asymptotic value $Z = Z_\infty$, also of order ε. The flame lies in this mixing region when $Z_s \sim \varepsilon$, which emerges as the appropriate distinguished limit in our analysis.

To describe the resulting flame it is convenient to use as scales those corresponding to the region where the mixing between the fuel and oxidizer streams occurs across the whole confining duct. Thus, we choose to scale the radial coordinate with a and the axial coordinate with the characteristic length $Re\,a$, yielding the dimensionless coordinates r and x. In this mixing region, the characteristic values of the axial and radial velocity components are $u_c = [J_{\rm F}/(\pi a^2 \rho_o)]^{1/2}$ and $\mu_o/(\rho_o a)$, which can be used to define dimensionless velocity components, u and v, respectively, while the difference of hydrostatic pressure from the entrance value is scaled with its characteristic value $\rho_o u_c^2$ to give p. Furthermore, it is convenient to define a normalized mixture fraction of order unity by introducing the alternative variable $z = Z/Z_s$ as a replacement for Z. With this set of rescaled variables, the boundary-layer equations for an upward-facing combustor become

$$\frac{\partial}{\partial x}(r\rho u) + \frac{\partial}{\partial r}(r\rho v) = 0 \tag{2}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial r} = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial u}{\partial r}\right) + G\,\frac{T-1}{T} \tag{3}$$

$$\rho u \frac{\partial z}{\partial x} + \rho v \frac{\partial z}{\partial r} = \frac{1}{Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu \frac{\partial z}{\partial r}\right) \tag{4}$$

to be integrated with initial conditions

$$x = 0, \quad \begin{cases} 0 \le r < \varepsilon: & u = \varepsilon^{-1} U_{\rm F}, \quad z = Z_s^{-1} \\ \varepsilon < r < 1: & u = C U_{\rm O}, \quad z = 0 \end{cases} \tag{5}$$

and with boundary conditions

$$x > 0, \quad \begin{cases} r = 0: & \partial u/\partial r = v = \partial z/\partial r = 0 \\ r = 1: & u = v = \partial z/\partial r = 0 \end{cases} \tag{6}$$

Here, $Pr = 0.72$ represents the constant Prandtl number for the gas mixture and $U_{\rm F}(r/\varepsilon)$ and $U_{\rm O}(r)$, are shape functions of order unity for the initial velocity distributions at $x = 0$ (e.g., $U_{\rm F} = 1$ and $U_{\rm O} = (1-\varepsilon^2)^{-1/2}$ when the velocity profiles are uniform). Buoyancy is accounted for in the formulation through the dimensionless gravity $G = (gRea)/u_c^2$, the inverse of the relevant Froude number, which enters as a factor in (3), leading to the acceleration of the light fluid with $T > 1$.

To simplify the formulation, the density and viscosity are assumed to be independent of the mixture composition. Their values, conveniently scaled with their initial values ρ_o and μ_o, are given by

$$\rho = T^{-1} \quad \text{and} \quad \mu = T^{\sigma}, \tag{7}$$

where the former corresponds to the ideal gas law and the latter is a presumed power-law for the temperature dependence of the viscosity, with the exponent $\sigma = 0.7$ being used in the integrations below.

In the initial region of jet development, corresponding to $x \sim \varepsilon$ and $r \sim \varepsilon$, the variables u and z are of order $\varepsilon^{-1} \sim Z_s^{-1} \gg 1$. Their values continuously decrease as the jet develops downstream, yielding values of order unity in the mixing region corresponding to distances $x \sim O(1)$, where the flame admits a simplified description depending on a reduced number of parameters. The first simplification concerns the temperature on the fuel side of the flame, which becomes approximately equal to its adiabatic value, so that

$$\begin{aligned} T &= 1 + \gamma z \quad \text{for} \quad z < 1 \\ T &= 1 + \gamma \quad \ \text{for} \quad z > 1. \end{aligned} \tag{8}$$

gives with errors of order Z_s the temperature field in terms of the rescaled mixture fraction. Furthermore, the initial conditions (5) can be replaced in the integrations with the profiles of u and z that emerge in the intermediate region corresponding to $\varepsilon \ll x \ll 1$, where the jet radius is already much larger than the discharge orifice but still much smaller that the confining duct ($\varepsilon \ll r \sim x \ll 1$). In this intermediate region, the velocity and the mixture fraction are of order $\varepsilon^{-1} \gg u \gg 1$ and $Z_s^{-1} \gg z \gg 1$, respectively. Since the flame lies far from the axis, the temperature across the jet corresponds according to (8) to the adiabatic flame temperature $T = 1 + \gamma$, and consequently the velocity

$$u = \frac{3}{8(1+\gamma)^\sigma x}\left[1 + \frac{3}{64}\frac{1}{(1+\gamma)^{2\sigma+1}}\left(\frac{r}{x}\right)^2\right]^{-2} + CU_{\rm O}(r), \tag{9}$$

is obtained by adding the Schlichting self-similar solution [8] with $\rho = (1+\gamma)^{-1}$ and $\mu = (1+\gamma)^\sigma$ to the unperturbed coflow profile given in (5). Similarly, Squire solution [9]

$$z = \frac{\dot{m}_z(2Pr+1)}{8(1+\gamma)^\sigma x}\left[1 + \frac{3}{64}\frac{1}{(1+\gamma)^{2\sigma+1}}\left(\frac{r}{x}\right)^2\right]^{-2Pr}, \tag{10}$$

describes the mixture-fraction field. In 10, $\dot{m}_z = \int_0^{r \gg x} 2\rho r u z \mathrm{d}r = Z_s^{-1} \int_0^\varepsilon 2r\varepsilon^{-1} U_F \mathrm{d}r$ represents the constant flux of mixture fraction, which can be written in terms of the global equivalence ratio of the coflow system $\phi = (S\int_0^\varepsilon 2r\varepsilon^{-1}U_{\rm F}\mathrm{d}r)/(\int_\varepsilon^1 2rCU_{\rm O}dr) = \dot{m}_z/(Cg_{\rm O})$, where $g_{\rm O} = \int_0^1 2rU_{\rm O}\mathrm{d}r$ (e.g., $g_{\rm O} = 1$ for uniform coflow). With the selected scaling, the rescaled mixture fraction z evolves in the mixing region to approach the final asymptotic value $z = \phi$ as $x \gg 1$. Values of $\phi < 1$ correspond to overventilated flames that reach the axis, while values of $\phi > 1$ correspond to underventilated flames reaching the confining duct.

The solution, which depends on γ, C, G, and $\dot{m}_z = Cg_{\rm O}\phi$, is obtained by integrating (2)–(4) supplemented with (7) and (8) with boundary conditions (6) and with (9) and (10) used to evaluate the initial profiles at $x \ll 1$. Sample results of integrations are given below for a non-buoyant flame. As can be seen in the plots obtained for increasing γ, the acceleration due to heat release tends to reduce the size of the mixing region. Correspondingly, flow recirculation is achieved for values of the critical Craya-Curter number (e.g., $C_c = 0.28$ for $U_{\rm O} = 1$ and $\gamma = 5$) much smaller than those of the frozen flow ($C_c = 0.65$). A decreasing flux of oxidizer is used for the integrations shown on the right-hand side of the figure. As can be seen, the effect is twofold: the reduced oxidizer feed causes an increase in the global equivalence ratio, so that underventilated flames eventually appear, whereas the reduced coflow momentum flux leads to the emergence of a recirculating region. Because of the enhanced mixing rate associated with the recirculation, the rich flames that appear for small C tend to migrate towards the rear end of the recirculating region.

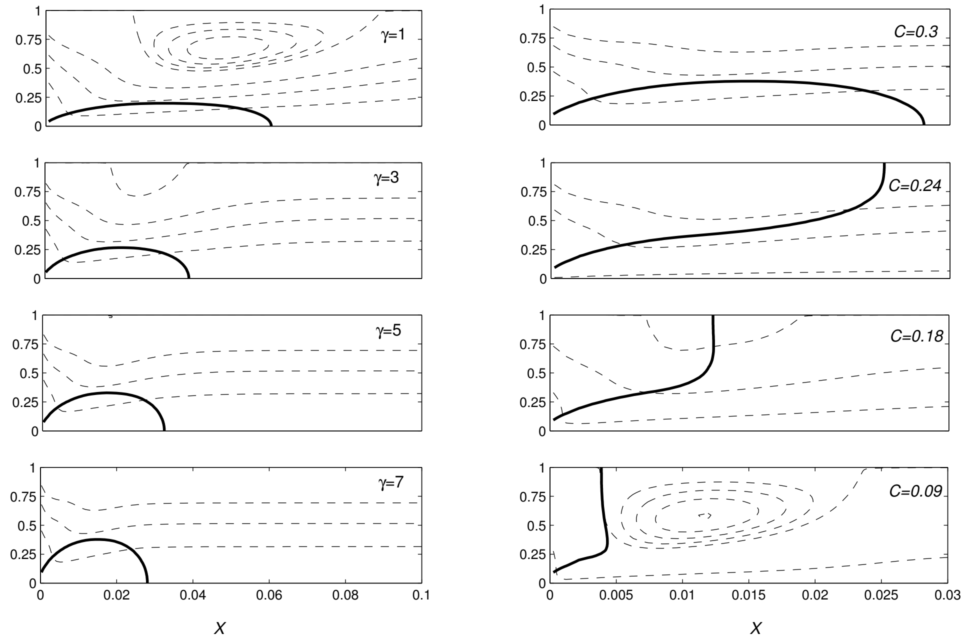

Figure 1: Flame location (solid line) and streamlines (dashed lines) corresponding to $\dot{m}_z = 0.27$, $G = 0$, and $U_O = 1$ for different values of γ (left-hand-side plots, $C = 0.3$) and for different values of C (right-hand-side plots, $\gamma = 7$). Equal increments $\delta\psi = 0.04$ are used for the downstream flowing streamlines, while $\delta\psi = 0.005$ is selected for the streamlines of recirculating flow.

References

[1] Burke SP and Schumann TEW 1928 Diffusion flames *Ind-Eng. Chem.* **20** 998–1004

[2] Williams FA 1985 *Combustion Theory* 2nd ed. (Benjamin Cummings, Menlo Park, CA)

[3] Thring MW and Newby MP 1953 Combustion length of enclosed turbulent jet flames *Proc. Combust. Inst.* **4** 789–796

[4] Craya A and Curtet R 1955 Sur l'évolution d'un jet en espace confiné *C. R. Acad. Sci.* Paris **241** 621–622

[5] Li SC, Gordon AS and Williams FA 1995 A simplified method for the computation of Burke-Schumann flames in infinite atmospheres *Combust. Sci. Technol.* **104** 75–91

[6] Vázquez-Espi C 2001 Analysis of axisymmetric laminar jet diffusion flames for small values of the stoichiometric mixture fraction *Combust. Sci. Technol.* **171** 1–38

[7] Revuelta A, Martinez-Bazán C, Sánchez AL and Liñán A 2004 Laminar Craya-Curtet jets *Phys. Fluids* **16** 208–211

[8] Schlichting H 1933 Laminare strahlausbreitung *Z. Angew. Math. Mech.* 13 260–263

[9] Squire HB 1951 The round laminar jet *Quart. J. Mech.* 4 321–329