Unsteady One-Dimensional Modeling of Ram Accelerator at Elevated Pressure in the Subdetonative Velocity Regime

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Introduction

Successful prediction of the thrust-Mach number relationship for the thermally choked ram accelerator propulsive mode has been accomplished in a straightforward manner by applying the quasi-steady conservation equations to a control volume attached to the projectile (Hertzberg et al. 1988, Bruckner et al. 1991).

Performance calculations of thermally choked operation at low fill pressure, i.e., at less than 3 MPa, using the ideal gas equation of state (EOS) for both the unreacted propellant and products were in good agreement with experimental results. In the fill pressure range of 3-8 MPa, however, the products are at such high pressure, i.e., of the order of several tens of MPa, that the ideal gas assumption was no longer valid. A more appropriate EOS; i.e., a virial-type EOS (Bauer et al. 1994), namely the formulation of Boltzmann EOS has been successfully applied (Bauer et al. 1998). It is based on simplified molecular interactions (Heuzé 1986). Moreover, the QUATUOR code (Heuzé et al. 1987) was used for determining the thermodynamic parameters in the initial state, as well as for the calculation of the chemical equilibrium composition.

At initial pressures beyond the range of 10 to 12 MPa, a further refinement of the modeling consists of taking into account the real gas corrections for the initial state. However, it turns out to provide a poor improvement in terms of the agreement with experimental data but the analytical method allows the initial-state real gas corrections to be obtained using any equation of state (Bauer and Knowlen 2003). Previous analytical studies showed that at moderate accelerations; i.e., on the order of 15,000 g, the unsteady terms in the conservation equations scarcely exceed a few percent of the magnitudes of the steady convective terms, which allows them to be neglected (Bruckner et al. 1991, Brouillette et al. 1995). Under conditions where the ratio of propellant to projectile density is sufficient to generate acceleration exceeding 30,000 g, an unsteady analysis is required (Bundy et al. 2000). A more accurate prediction of the thrust at high acceleration was obtained by Bundy et al. (2000) by accounting for the unsteady flow effects that are disregarded in the quasi-steady control volume model. The aim of the present study is to examine the influence of these unsteady effects in this one-dimensional modeling in
combination with the use of a real gas equation of state (EOS) in order to calculate the thrust characteristics of the thermally choked ram accelerator.

**Calculation procedure**

The main basis of this unsteady model is to describe the effects of the flow around the projectile as a global process between the state of the propellant entering the control volume and the state of the thermally choked exit flow. The model is based on the standard set of one-dimensional conservation equations. The flow properties are modified through a control volume by the rate of accumulation of mass, momentum, and energy in the control volume between the entrance and exit planes. Moreover, heat release from combustion and the rate of change of axial momentum are characterized by the chemical heat release ($\Delta q$) and net axial force on the projectile ($F$), respectively (Bauer et al. 2004).

In order to evaluate the extent of the unsteady state assumption, a first calculation was conducted for mixtures that are currently used as propellants in the UW ram accelerator experiments at elevated initial pressures; i.e., $2.6\text{CH}_4 + 2\text{O}_2 + 9.2\text{N}_2$. The non-dimensional thrust was calculated as a function of the Mach number values for 15 MPa and 20 MPa propellant fill pressure. The calculations were based on a control volume length equal to two projectile lengths ($L_{CV}=2L_p$). The evidence of this ratio is based on luminosity and pressure records showing that the termination of the combustion zone occurs approximately one projectile length behind its base (Bruckner et al. 1991). This aspect is discussed more extensively in the paper.

**Comparison with experimental data and discussion**

Comparing the resulting thrust vs. $M$ curves of the quasi-steady and unsteady assumptions, it is evident that the unsteadiness gives rise to lower values of the non-dimensional thrust. For both the quasi-steady and unsteady assumptions, the predicted thrust goes to zero as the Mach number of the flow approaches that of the CJ conditions, as expected.

The experimental data used for the following theoretical comparisons were those from representative experiments conducted at the ram accelerator facility of UW (Bundy et al. 2004). Velocity-distance data experiments, along with the results of the theoretical modeling using both the steady and unsteady state assumptions show that the unsteady model is in much better agreement with experimental data over the range of ram accelerator operation being considered here. This improvement in modeling is primarily attributed to the inclusion of the effects of accelerating a substantial mass of propellant along with the projectile.

The length of the control volume, $L_{CV}$, is a key element in the unsteady modeling because it appears in all of the conservation equations. The influence of this parameter on the theoretical non-dimensional thrust versus Mach number behavior is investigated. Moreover, a chemical kinetics investigation indicates that the length of the combustion zone decreases significantly as the Mach number increases. Consequently, the influence on non-dimensional thrust of a linear variation of $L_{CV}$ from both $4L_p$ to $1L_p$ and $6L_p$ to $1L_p$ over the Mach number range 2.5 to $M_{CJ}$ was explored. The larger the value of $L_{cv}$ at lower Mach number, the greater the reduction in thrust is from that of the steady-state prediction. As the Mach number approaches $M_{CJ}$, the
significance of the assumption for \( L_{cv} \) diminishes, as expected, since the acceleration approaches zero.

These trends are evident in the results of the unsteady theoretical modeling of both the 15 MPa and the 20 MPa experiments. The greater the rate of change of \( L_{cv} \), the more “flat” the non-dimensional thrust curves are at low Mach numbers. This seems to be relatively consistent with the experimental data shown. The experimental thrust remains relatively constant over this Mach number range. This thrust behavior at high Mach numbers is often attributed to cessation of thermal choking; however, the large uncertainty in the experimental non-dimensional thrust data precludes drawing any definitive conclusions about the relevance of the various assumptions for the behavior of the control volume length as a function of Mach number.

The actual value of the control volume length that would be most appropriate is very much related to the chemical kinetics of the process. The induction length behind an incident shock for this propellant mixture was calculated using the Chemkin code (Mitchell and Kee, 1992) based on the GRI kinetic scheme for methane combustion using the ideal gas EOS (Smith et al. 2000). Although these calculated induction lengths are only crude estimates at this point, they provide, at least, a qualitative explanation of the pertinent use of the variation of \( L_{CV} \) over the Mach number range. The calculation shows that in the Mach number range of 3.7 to 4, \( L_{CV} \) can be chosen between \( 4L_p \) to \( 1L_p \). At higher Mach number, this choice becomes questionable. Ultimately, an exponential variation in \( L_{CV} \) with Mach number, based on appropriate chemical kinetics, will be incorporated in the theoretical modeling in the future. In addition to real gas effects, the influence on induction time of turbulence, shock-boundary layer interaction, multiple shocks heating the flow and other factors should be accounted for.

**Conclusion**

The present investigation was aimed at demonstrating the pertinent use of an unsteady, real gas assessment in the 1D modeling of the ram accelerator process. A computer code was used to solve the conservation equations incorporating the unsteady terms while utilizing a virial equation of state. The calculated non-dimensional thrust turned out to be less than that derived from a steady-state assumption at all sub-detonative Mach numbers. The first series of calculations were based on a constant value of the control volume length; i.e., twice the projectile length. This length was also varied linearly as a function of Mach number and turned out to better match the experimental data. The comparison of calculations with experimental data showed that the unsteady, real gas assumption provides a much better agreement than the steady-state assumption. A further refinement is now possible by adjusting the length of the control volume. Several ways of doing this can be considered: either a linear variation (as done here) or an exponential variation, which would be more consistent with the chemical kinetics behavior (as shown by calculating the induction distance for the propellant mixture by means of chemical kinetics modeling). The induction length was obtained by solving the 1D conservation equations for the normal shock wave, coupled with the detailed evaluation of the chemical source terms, and forming a system of stiff ODEs. This length was defined as the distance at which the flow again becomes sonic. This yields an approximate exponential dependence of the control volume with respect to the Mach number.
Assuming a constant value for the length of the control volume may be adequate at Mach numbers greater than 4. This, together with the available experimental data and an investigation of the effects of turbulence and shock interactions, would be the most appropriate way to improve the 1 D modeling of the process and provide accurate predictions of ram accelerator performance for high acceleration conditions.

References