Pressure History at the Thrust Wall of a Simplified Pulse Detonation Engine

T. Endo[1,*], J. Kasahara[2,†], Y. Tanahashi[2], T. Numata[2], A. Matsuo[3], S. Sato[3], and T. Fujiwara[4]

[1] Nagoya University, Nagoya, Aichi 464-8603, Japan

[2] Muroran Institute of Technology, Muroran, Hokkaido 050-8585, Japan

[3] Keio University, Yokohama, Kanagawa 223-8522, Japan

[4] ATES Corporation, Toyota, Aichi 471-0845, Japan

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1. Introduction

Formulation of pressure history at the thrust wall is an important issue in researches on pulse detonation engines (PDEs). In this paper, we present theoretical formulation of the pressure history at the thrust wall of a simplified PDE. In addition, the derived pressure history is compared with numerical and experimental results.

2. Theoretical Analysis

A PDE is modeled as a straight tube with fixed cross section. One end of the tube is closed (thrust wall) and the other end is open. For simplicity, gases are treated as polytropic gases and as ideal fluids. Flow is assumed to be one-dimensional. In this paper, x is coordinate along the axis of the PDE tube where $x = 0$ and $x = L$ correspond to the closed and open ends, respectively. We analyze one cycle of PDE operation below.

Figure 1 shows a schematic *x*-*t* diagram of characteristics in the PDE tube. The tube is initially filled with a uniform detonable gas at rest,

* Present address: Hiroshima University, Higashi-Hiroshima, Hiroshima 739-8527, Japan; takuma@mec.hiroshima-u.ac.jp.

† Present address: University of Tsukuba, Tsukuba, Ibaraki 305-8573, Japan.

characterized by γ_1, p_1, a_1, and $u_1 (= 0)$ where γ, p, a, and u stand for the ratio of specific heats, pressure, sound speed, and flow velocity in the x coordinate, respectively. At the time $t = 0$, a Chapman-Jouguet detonation wave is ignited at the closed end and simultaneously starts to propagate toward the open end. The gas at the rear surface of the detonation wave is characterized by γ_2, p_2, a_2, and u_2. The location of the rear surface of the detonation wave (x_2) is given by $x_2 = D_{\mathrm{CJ}} t$ where D_{CJ} is the Chapman-Jouguet detonation speed.

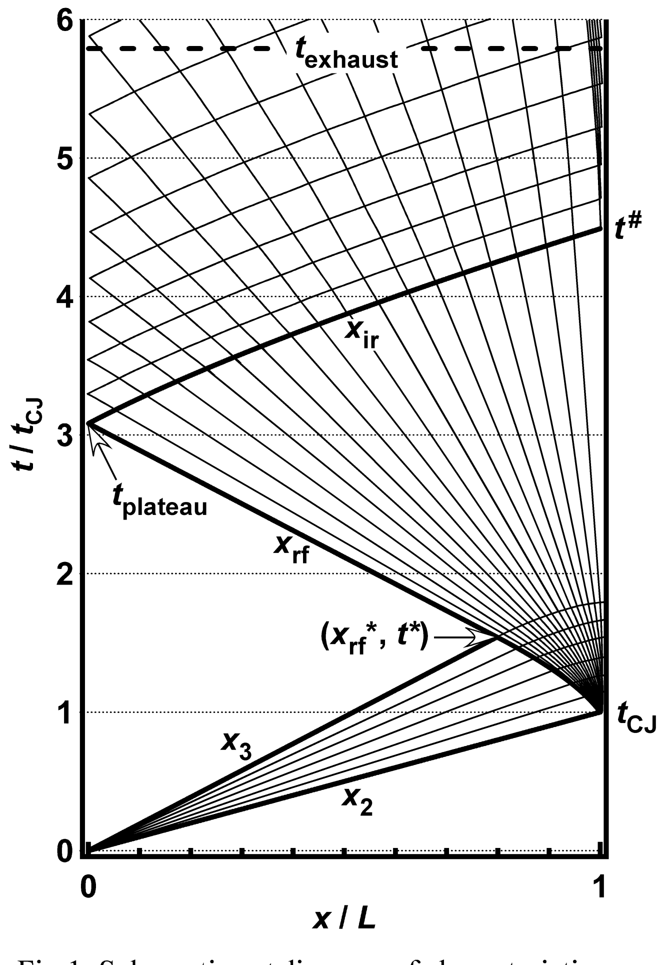

Fig.1 Schematic *x*-*t* diagram of characteristics.

Because the gas on the closed end is at rest, the detonation wave is followed by a self-similar rarefaction wave, which is called the decelerating rarefaction wave hereafter. The gas at the rear boundary of the decelerating rarefaction wave is characterized by $\gamma_3 (= \gamma_2)$, p_3, a_3, and $u_3 (= 0)$, which characterize all gas in the region $0 \le x \le x_3$ as well, where $x_3 (= a_3 t)$ is the location of the rear boundary of the decelerating rarefaction wave.

The detonation wave reaches the open end at the time t_{CJ}, and simultaneously, another rarefaction wave starts to propagate from the open end toward the closed end, which is called the exhausting rarefaction wave hereafter. We can calculate the location of the front boundary of the exhausting rarefaction wave (x_{rf}) since x_{rf} is a characteristic $(u - a)$ in the decelerating rarefaction wave. At the time t^*, the front boundary of the exhausting rarefaction wave intersects with the rear boundary of the decelerating rarefaction wave $\left(x_{\mathrm{rf}}^* = x_{\mathrm{rf}} \big|_{t=t^*} \right)$. After the time t^*, the front boundary of the exhausting rarefaction wave propagates in the uniform gas characterized by $\gamma_3(=\gamma_2)$, p_3, a_3, and $u_3(=0)$. The pressure plateau at the thrust wall lasts until the time t_{plateau} at which the front boundary of the exhausting rarefaction wave reaches the closed end. In the time region $0 \le t \le t_{\text{plateau}}$, pressure at the thrust wall (p_{w}) is kept as $p_{\text{w}} = p_3$.

The exhausting rarefaction wave just before the time t_{plateau} can be approximated as a self-similar rarefaction wave propagating from the open end toward the closed end in a uniform gas characterized by $\gamma_3(=\gamma_2)$, p_3, a_3, and $u_3(=0)$. By using this approximation, pressure at the thrust wall (p_{w}) in the time region $t_{\text{plateau}} \le t$ is given by an implicit function of t:[1]

$$a_3\left(t+\frac{L}{a_3}-t_{\text{plateau}}\right)\Big/L = \frac{3}{8}\left(\frac{p_{\text{w}}}{p_3}\right)^{-\frac{1}{7}} + \frac{1}{4}\left(\frac{p_{\text{w}}}{p_3}\right)^{-\frac{3}{7}} + \frac{3}{8}\left(\frac{p_{\text{w}}}{p_3}\right)^{-\frac{5}{7}},$$

which is a result of interference between the self-similar exhausting rarefaction wave and its reflection from the solid wall. Pressure at the thrust wall (p_{w}) decays down to the initial pressure (p_1) at the time t_{exhaust}. For usefulness, we fitted the above implicit function by an explicit function:

$$\frac{p_{\text{w}}}{p_3} = k_A \exp\left[-k_B\frac{a_3}{L}\left(t+\frac{L}{a_3}-t_{\text{plateau}}\right)\right] + k_C \exp\left[-k_D\frac{a_3}{L}\left(t+\frac{L}{a_3}-t_{\text{plateau}}\right)\right]$$
$$\begin{pmatrix} k_A = 0.77158, & k_B = 0.65293, \\ k_C = 14.955, & k_D = 3.2277 \end{pmatrix}.$$

The boundary of the region of interference between the exhausting rarefaction wave and its reflection from the solid wall (x_{ir}) reaches the open end at the time $t^{\#}$. Since x_{ir} is a characteristic $(u+a)$ in the exhausting rarefaction wave, we can calculate x_{ir}.

Summarizing the theoretical analysis, the pressure history at the thrust wall of a simplified PDE is given by the following formula:

$$p_{\text{w}} = \begin{cases} p_3 & \left(0 \le t \le t_{\text{plateau}}\right) \\ p_3\left\{\begin{array}{l} k_A \exp\left[-k_B\frac{a_3}{L}\left(t+\frac{L}{a_3}-t_{\text{plateau}}\right)\right] \\ +k_C \exp\left[-k_D\frac{a_3}{L}\left(t+\frac{L}{a_3}-t_{\text{plateau}}\right)\right] \end{array}\right\} & \left(t_{\text{plateau}} \le t\right) \end{cases}$$

and $p_{\text{w}}\big|_{t=t_{\text{exhaust}}} = p_1$,

where

$$p_3 = \delta_A p_1,\quad a_3 = \frac{\gamma_1 M_{\text{CJ}}{}^2+\gamma_2}{\gamma_1 M_{\text{CJ}}{}^2}\frac{1}{2}D_{\text{CJ}},\quad t_{\text{plateau}} = \delta_B t_{\text{CJ}},$$

$$t_{\text{exhaust}} = \delta_C t_{\text{CJ}}\,,\quad t_{\text{CJ}} = L/D_{\text{CJ}}\,,\quad M_{\text{CJ}} = D_{\text{CJ}}/a_1\,,$$

$$\delta_A = \frac{\gamma_1 {M_{\text{CJ}}}^2 + \gamma_2}{2\gamma_2}\left(\frac{\gamma_1 {M_{\text{CJ}}}^2 + \gamma_2}{\gamma_1 {M_{\text{CJ}}}^2 + 1}\frac{\gamma_2 + 1}{2\gamma_2}\right)^{\frac{\gamma_2+1}{\gamma_2-1}},$$

$$\delta_B = 2\left(\frac{\gamma_1 {M_{\text{CJ}}}^2 + \gamma_2}{\gamma_1 {M_{\text{CJ}}}^2 + 1}\frac{\gamma_2 + 1}{2\gamma_2}\right)^{-\frac{\gamma_2+1}{2(\gamma_2-1)}},$$

and

$$\delta_C = 2\frac{\gamma_1 {M_{\text{CJ}}}^2}{\gamma_1 {M_{\text{CJ}}}^2 + \gamma_2}\left(\frac{3}{8}\delta_A^{\frac{1}{7}} + \frac{1}{4}\delta_A^{\frac{3}{7}} + \frac{3}{8}\delta_A^{\frac{5}{7}} - 1\right) + \delta_B\,.$$

3. Comparison with Simulation and Experiment

Figure 2(a) shows the results of a numerical simulation and the theoretical analysis in the case that the initial gas was a stoichiometric hydrogen-oxygen mixture of the room temperature and of $p_1 = 1$ atm, and the tube length was $L = 0.9$ m. In this case, $\gamma_1 = \gamma_2 = 7/5$ and $M_{\text{CJ}} = 5.31$ (obtained from the experiment) were used for the analytical formula, and a two-step reaction model was adopted in the numerical simulation. As shown in Fig. 2(a), the theoretical analysis agreed very well with the numerical simulation.

Figure 2(b) shows the results of an experiment and the theoretical analysis in the same case as Fig. 2(a). As shown in Fig. 2(b), the theoretical analysis agreed with the experiment successfully.

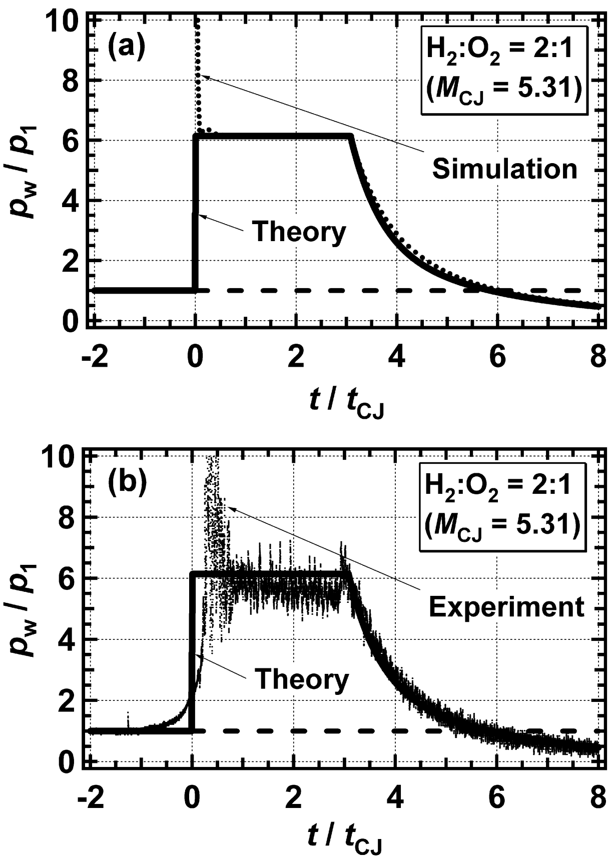

Fig. 2 Comparison among the analytical, numerical, and experimental results.

4. Conclusions

We theoretically derived a formula describing pressure history at the thrust wall of a simplified pulse detonation engine (PDE). The derived formula was compared with numerical and experimental results, and agreed with them very well.

References

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