Dependence of the laminar flame speed on flame stretch in the presence of heat loss

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Abstract

We reexamine the laminar flame speed dependence on a week stretch in the presence of a volumetric heat loss, using a Near-Equidiffusion-Flame asymptotic analysis similar to that of [1]. For simplicity a thermo-diffusive approximation with an arbitrarily prescribed flow field is adopted. We show that the flame speed-stretch relation remains valid in non-adiabatic situations, provided the speed and stretch are rescaled *and* an effective Lewis number is used. For illustration, the case of two-dimensional flame edges in the counterflow configuration is examined and a comparison with numerical calculations is made. Detailed study of the latter case extending the asymptotic results will also be provided.

Formulation

The non-dimensional governing equations for our model are

$$\begin{aligned} \frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta &= \epsilon \Delta \theta + \epsilon^{-1} \omega - \frac{\epsilon^{-1}}{\beta} \kappa \theta \\ \frac{\partial y_F}{\partial t} + \mathbf{v} \cdot \nabla y_F &= \epsilon Le^{-1} \Delta y_F - \epsilon^{-1} \omega \,. \end{aligned}$$

Here y_F and θ are the scaled fuel mass fraction and temperature. $\mathbf{v}$ is the velocity field measured with the laminar speed S_L^0 of an adiabatic planar flame. The parameter $\epsilon \equiv l_{Fl}^0/L$ measures the thickness of the planar adiabatic flame l_{Fl}^0 relative to the reference length L (typically the flow scale or the flame front radius of curvature). κ is a scaled heat-loss coefficient and Le is the Lewis number. Finally, ω is the reaction rate given by

$$\omega = \frac{\beta^2}{2} y_F \exp\left[\frac{\beta(\theta-1)}{1+\alpha(\theta-1)}\right] ,$$

where β is the Zeldovich number, and α a thermal expansion coefficient.

As boundary conditions needed for the asymptotic analysis we take

$$\theta = 0 , \quad y_F = 1 \quad \text{as} \quad x \to -\infty,$$

corresponding to the conditions in the frozen mixture along with the requirement that the solutions remain free from exponentially growing terms as $x \to \infty$.

Asymptotic analysis and illustration

We first consider the limit $\beta \to \infty$ with $l_F \equiv \beta(Le-1)$ and κ being of order one. The problem can thus be reformulated in terms of the leading order temperature θ^0 and the excess enthalpy $h \equiv \theta^1 + y_F^1$, where superscripts indicate orders of expansions in β^{-1}. In the reformulated problem, which is free from β, we then examine the thin flame limit $\epsilon \to 0$. Skipping details, we simply record the two-term expansion obtained for the laminar flame speed S_L in the form $S_L = S_0 + \epsilon S_1$. S_0 is found to obey

$$S_0 \exp(\kappa/S_0^2) = 1 .$$

Thus, to leading order, at each location of the flame front, S_L is equal to the speed of the non-adiabatic planar flame, as one may expect. To second order, the result can be written as

$$\frac{S_L}{S_0} = 1 - \left(1 + \frac{\tilde{l}_F}{2}\right) \tilde{K}\epsilon \, .$$

Here we have introduced the *effective reduced Lewis number* defined by

$$\tilde{l}_F = \frac{l_F}{1 - 2\kappa/S_0^2}$$

and the non-dimensional flame stretch $\tilde{K}$. The non-dimensional flame stretch $\tilde{K}$ is such that it reduces in the adiabatic case $\kappa = 0$ to the well known expression

$$\tilde{K} = -\mathbf{n} \cdot \nabla \times (\mathbf{v} \times \mathbf{n}) + (\mathbf{v}_{flame} \cdot \mathbf{n})(\nabla \cdot \mathbf{n}) \, ,$$

where $\mathbf{v}$ and $\mathbf{v}_{flame}$ are the fluid and flame velocities, respectively, and $\mathbf{n}$ a unit vector normal to the flame front. Our results show that the flame-speed stretch relationship known in the literature may be applied in non-adiabatic situations provided that the laminar flame speed and stretch are scaled with the laminar speed and time of the non-adiabatic flame *and* an effective Lewis number is used. The fact that the presence of heat-loss modifies the effective Lewis number has important implications on flame stability, and is consistent with the conclusions of stability studies in the literature.

As illustration, we use the asymptotic results to examine the propagation of two-dimensional edge-flames in the counterflow configurations (see [2] for details). The propagation speed U of the structure as a whole in weak strain situations is found to be given by

$$\frac{U}{S_0} = 1 - \left[1 + \frac{1}{2}\frac{l_F}{1 - 2\kappa/S_0^2}\right] \frac{4\epsilon^2}{S_0^2}$$

Comparison with numerical calculations are reported in Figure 1. Here, to be consistent with the notation of [2], we have used ϵ^2, rather than ϵ, to characterise the thickness of the front relative

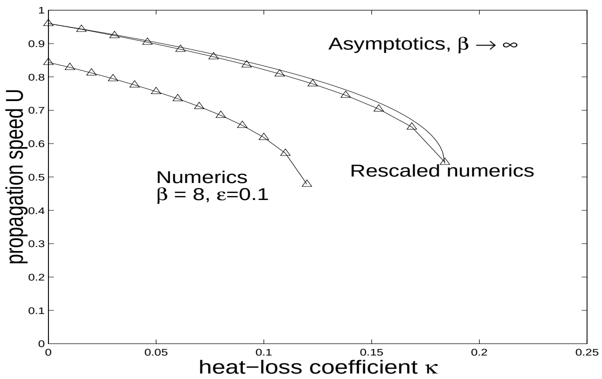

Figure 1: Comparison between the asymptotic and numerical results.

to its radius of curvature (typically the distance between the trailing planar wings of the flame front).

Plotted is U versus κ for $\epsilon = 0.1$, $\beta = 8$, and $l_F = 0$. It is seen that the qualitative agreement is good. The quantitative discrepancy observed is a consequence of the finite activation energy used in the computations which underestimate the value of κ at extinction ($\kappa_{ext}^{num} \approx 0.122$ to be compared with the asymptotic value $\kappa_{ext}^{asy} \approx 0.184$), as well as the value of U corresponding to $\kappa = 0$, say $\hat{U}$ ($\hat{U}^{num} \approx 0.84$ to be compared with $\hat{U}^{asy} \approx 0.96$). However, a linear rescaling of the numerical results ($\kappa \to \kappa\kappa_{ext}^{asy}/\kappa_{ext}^{num}$, $U \to U\hat{U}^{asy}/\hat{U}^{num}$), shows that the rescaled numerics compare well with the asymptotics, even in near-extinction conditions. Additional results on flame-edges in the last configuration, not restricted to the thin flame limit, will be also presented.

References

[1] Matalon M. and Matkowsky B.J., *JFM*, Vol. 124, pp. 239-259, 1982.

[2] Daou, R., Daou, J. and Dold, J. The effect of heat loss on flame edges in a premixed counterflow, *Combustion Theory and Modelling* 7, 221-242, 2003.