A weak solution of point source blast wave problem

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The "point source model" consists of a system of gasdynamics equations that describes the motion of ideal gas initiated by an instantaneous release of finite amount of energy at a point (or line or plane) in a uniform state. This model has originally been introduced to describe the blast wave propagation caused by an intense explosion[1)]. The original system of equations has conveniently been converted into a system of equations for four unknowns $f(x,y), g(x,y), h(x,y), \lambda(y)$ on the region $0 \le x, y \le 1$ by the use of blast wave transformation[1)].

Here we introduce the variables $\xi = -\log x$ and $\eta = \log R$, R being related to λ by $\lambda = (R/y)dy/dR$. With $T = (xR)^{\alpha+1}/y$, we also set $F = f/x,\ G = gh^{-\gamma+1}T,\ H = hyT,\ I = (\gamma h f^2/2 + g/(\gamma-1))T$, where $\alpha = 0, 1$ and 2 correspond respectively to plain, line, and point sources, and γ is the ratio of the specific heats of the gas. We convert the of equations for f, g, h, λ[1)] into the form

$$-U_\xi + V_\eta = 0,\ U(0,\eta) = -R^{\alpha+1}K, \quad 0 \le \xi < \infty,\ -\infty < \eta < \infty, \tag{1}$$

where

$$U = \begin{pmatrix} (F-1)I + FG(\frac{H}{Ty})^{\gamma-1} \\ (F-1)G \\ (F-1)H \end{pmatrix},\ V = \begin{pmatrix} I \\ G \\ H \end{pmatrix},\ K = \begin{pmatrix} \frac{1}{\gamma-1} \\ (1+\frac{\gamma-1}{\gamma+1}(1-y))(1-\frac{2}{\gamma+1}(1-y))^\gamma y^{-1} \\ 1 \end{pmatrix}. \tag{2}$$

This system of equations is supplemented by the condition (conservation of energy)

$$\int_0^1 \frac{I}{x} dx = R_0^{\alpha+1} + \frac{1}{(\alpha+1)(\gamma-1)} R^{\alpha+1},$$

where R_0 is the characteristic length of the explosion defined by the explosion energy and the initial pressure.

We consider a weak solution of (1), that is, (U, V) satisfying

$$\int_{-\infty}^{\infty} U_i(0,\eta)\Phi(0,\eta)d\eta + \int_0^\infty \int_{-\infty}^{\infty} (U_i\Phi_\xi - V_i\Phi_\eta)d\eta d\xi = 0 \quad (i = 1,2,3) \tag{3}$$

for all smooth functions Φ which are identically zero outside some bounded set[2)]. If $\eta = \varphi(\xi, s)$ is a solution, depending on i, of the ordinary differential equation $d\eta/d\xi = V_i/U_i$ with $\eta|_{\xi=0} = s$ for U, V defined in (2), then changing the variables ξ, η in (3) to s, t determined by $\xi = t, \eta = \varphi(\xi, s)$ shows that $U_i = U_i(0,s)/\varphi_s$ and $V_i = U_i\varphi_\xi$ satisfy (3). In practice, we may construct a solution from an explicit expression for V_i/U_i in variables ξ and η.

References

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2) J. Glimm: Comm. Pure. Appl. Math. 18 (1975) 697