

Pulsating Propagation of Two-dimensional Flame Fronts

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It is well known that flammable regions of a combustible mixture are not only affected by flame propagation rates, but also by their stability properties. The stability characteristics of premixed flames and the onset of instabilities are therefore important both for fundamental research as well as for the development of new combustion technologies.

Flame oscillations is a problem that has been studied extensively in the literature. The spontaneous oscillation of a plane premixed flame is known to result from the disparity between the thermal diffusivity of the mixture and the molecular diffusivity of the deficient component. Asymptotic theories [1] predict that oscillations occur when the effective Lewis number Le is sufficiently bigger than one or, more specifically, when $Le > Le^*$ with $Le^* = 1 + 4(1 + \sqrt{3})\beta^{-1}$; here β is the Zeldovich number assumed large. This has also been verified by numerical calculations [2] which show that, as $\beta \rightarrow \infty$, the critical Lewis number for the onset of instability Le_c approaches Le^* . Calculations also show the sensitivity of the results to the specified value of β ; for example, with $\beta = 10$ the critical Le_c is increased by nearly 70%. In any case, the predicted Lewis number beyond which oscillations occur is quite large and not easily accessible in ordinary mixtures. For this reason experimental observations have been limited to burner-stabilized flames where the influence of conductive losses to the burner rim act to lower Le_c .

In this presentation we examine the propagation of premixed flame fronts in channels of finite widths and in the presence of convection that either supports or opposes the propagation. The channel's width is treated as a parameter so that the analysis spans the whole range from narrow to wide channels. As a result of the imposed flow and of conductive heat loss to the walls, the flame is curved and propagates at a speed different than the laminar flame speed [3]. Unlike planar fronts their stability properties have not been previously examined. Our objective is to investigate the conditions for steady propagation and for the onset of oscillations leading to pulsating-propagation. The relevance of this work extends beyond its fundamental importance; the results are of interest to flammability studies and to the evolving technological interest in micro-scale combustion.

We consider a two-dimensional deflagration wave that travels in an infinitely long channel of width $2a$, separating the fresh cold mixture from the hot combustion products. A diffusive-thermal model, which allows examining the propagation of a flame in a prescribed flow while neglecting the effect that the flame has on the flow field, is considered. The flow here is assumed to be a Poiseuille flow with a centerline velocity u_0 taken to be positive when the flow is directed from the unburned towards the burned gas, and negative when directed from the burned towards the unburned gas; see Figs. 1 and 2. The chemical activity is described by a one-step overall reaction and proceeds at a rate given by an

Arrhenius law with activation energy E and pre-exponential factor B .

We select as a unit length the half-width of the channel a , and as a unit speed the laminar flame speed S_L . The ratio of the laminar flame thickness l_{th} to the width of the channel, denoted by $\epsilon \equiv l_{th}/a$, is treated as a parameter. The mass fraction of the deficient reactant is normalized by its value in the fresh mixture Y_u and a non-dimensional temperature θ is defined via $\theta = (T - T_u)/(T_a - T_u)$, where T_u is the temperature of the unburned gas and T_a is the adiabatic flame temperature. Thus, the dimensionless governing equations in a frame attached to the propagating flame become

$$\frac{\partial \theta}{\partial t} + [U + u_0(1 - y^2)] \frac{\partial \theta}{\partial x} - \epsilon \nabla^2 \theta = \epsilon^{-1} \varpi \quad (1)$$

$$\frac{\partial Y}{\partial t} + [U + u_0(1 - y^2)] \frac{\partial Y}{\partial x} - \epsilon Le^{-1} \nabla^2 Y = -\epsilon^{-1} \varpi \quad (2)$$

where Le is the Lewis number and

$$\varpi = \frac{1}{2Le} \beta^2 Y \exp \left\{ \frac{\beta(\theta - 1)}{1 + \alpha(\theta - 1)} \right\} \quad (3)$$

is the reaction rate with $\beta = E(T_a - T_u)/R^\circ T_a^2$ the Zeldovich number, R° the gas constant, and $\alpha = (T_a - T_u)/T_a$ the heat-release parameter. The large activation energy expression for S_L has been used in expression (3) for ω . Finally $U = U(t)$ is the flame propagation speed measured with respect to the walls.

Far to the left, the chemistry is frozen due to a sufficiently large β , so that $Y = 1$ and $\theta = 0$ as $x \rightarrow -\infty$. Far to the right all properties are assumed uniform and $\partial Y/\partial x = \partial \theta/\partial x = 0$ as $x \rightarrow \infty$. The symmetry condition along the axis of the channel implies that

$$\frac{\partial Y}{\partial y} = \frac{\partial \theta}{\partial y} = 0 \quad \text{at } y = 0 \quad (4)$$

and the conditions corresponding to impermeable and non-adiabatic walls are

$$\frac{\partial Y}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = -\frac{k\theta}{\beta} \quad \text{at } y = 1 \quad (5)$$

where k/β measures the intensity of the heat loss by conduction (the insertion of the scaling factor β has been made simply for convenience). $k = 0$ corresponds to adiabatic walls and $k = \infty$ to cold walls held at the same temperature as that of the fresh gases.

The unsteady problem consisting equation (1)-(2) with the stated boundary conditions has been solved numerically, starting with arbitrary initial data. The computations were carried out in a finite domain on a rectangular grid, uniform in y but variable in x with more points distributed near the reaction zone location. Typically 200×40 points were used in the x, y directions respectively. The number of points were doubled in order to test the independence of the calculations to that choice as well as to the length of the domain. An explicit marching procedure was used with first or fourth order discretization in time. The presence of the reaction rate requires the time step to be chosen sufficiently small so as to ensure numerical stability. The propagation speed $U(t)$ has been determined at each time step by holding the location $(x_*, 0)$ where $Y = Y_*$ with the value Y_* chosen so that x_* well approximates the location of the reaction zone.

The results presented here use the moderate values $\beta = 10$ for the Zeldovich number. For this value of β , the laminar flame speed S_L differs from the asymptotic value used in the nondimensionalization. Velocities thus have been re-normalized using the calculated steady propagation speed $U = 0.918$

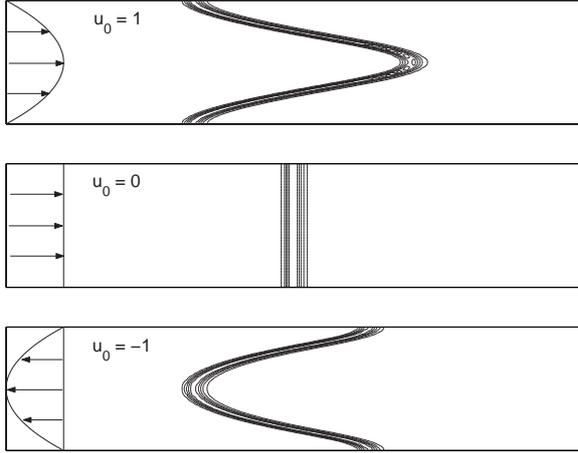


Figure 1: Reaction rate contours for adiabatic walls with unity Lewis number ($\epsilon = 0.1$).

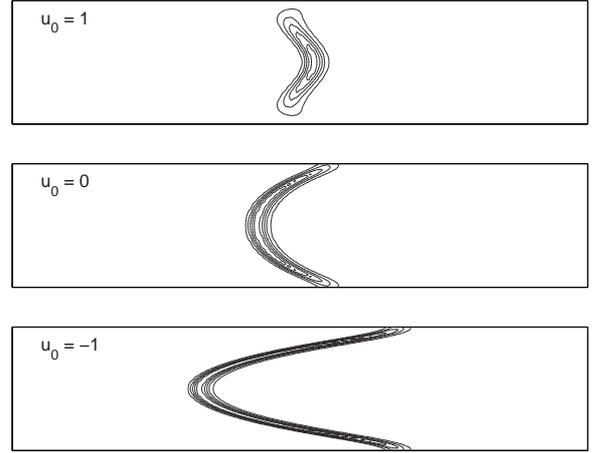


Figure 2: Reaction rate contours for non-adiabatic walls ($k = 8$) with unity Lewis number ($\epsilon = 0.1$).

obtained for an adiabatic ($k = 0$) wide channel ($\epsilon = 0.1$) with unity Lewis number and no flow ($u_0 = 0$). Thus, the value $0.918 S_L$ constitutes the reference speed, in agreement with [2].

Fig. 1 shows representative calculations for the adiabatic case $k = 0$ and unity Lewis number. This case corresponds to a *wide* channel, $\epsilon = 0.1$. The curves in this figure correspond to reaction rate contours. In the absence of a flow ($u_0 = 0$) the flame is planar and propagates to the left at a speed $U = 1$ (after the renormalization). When the flow is directed from the unburned towards the burned gas ($u_0 > 0$), the flame is curved and for $u_0 = 1$ it propagates to the left at a speed $U = 0.53$. This case corresponds to flame flashback. At higher speeds the flame is blown-off, propagating to the right with

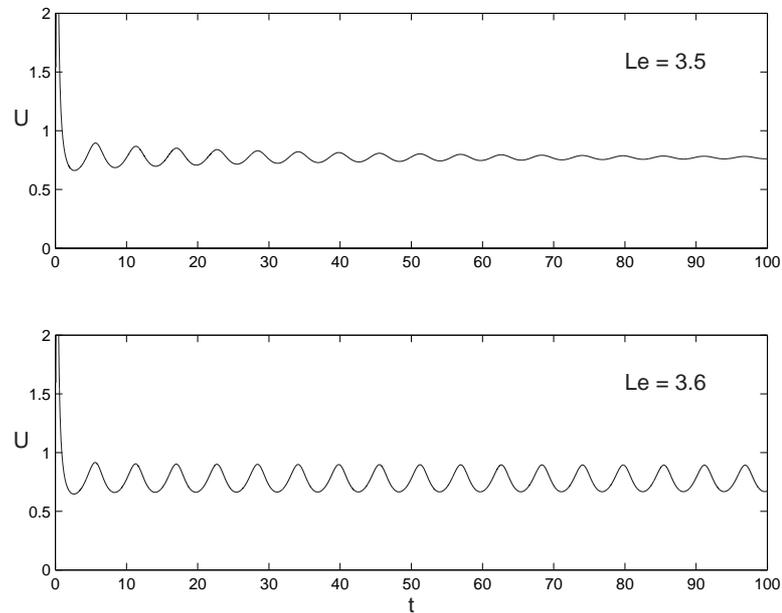


Figure 3: The propagation speed U as function of time for two values of the Lewis number. The top figure corresponds to $Le = 3.5$ and the bottom figure to $Le = 3.6$. Calculated for $k = 0$ and $\epsilon = 0.5$

	$Le = 3.5$	$Le=3.6$	$Le=4.0$
$\epsilon = 0.1$	$U = 0.84$	$A = 0.16, \omega = 5.5, \langle U \rangle = 0.83$	$A = 1.0, \omega = 5.4, \langle U \rangle = 0.79$
$\epsilon = 0.5$	$U = 0.84$	$A = 0.23, \omega = 1.1, \langle U \rangle = 0.83$	$A = 1.0, \omega = 1.1, \langle U \rangle = 0.79$
$\epsilon = 1.0$	$U = 0.84$	$A = 0.22, \omega = 0.55, \langle U \rangle = 0.83$	$A = 1.0, \omega = 0.54, \langle U \rangle = 0.79$

Table 1: Numerical results for selected ϵ and Le with $u_0 = 0$ and $k = 0$. Here A , ω and U represent respectively the amplitude, frequency and propagation speed, and $\langle U \rangle$ is the average propagation speed of the pulsating flame.

$U < 0$. When the flow is directed from the burned towards the unburned gas, the flame is always blown off by the flow and, for $u_0 = -1$, propagates to the left at a speed $U = 1.94$. Figure 2 shows similar results but in the presence of conductive losses at the walls with $k = 8$. We note that even without a flow the flame is curved and is more sensitive to heat losses when it propagates against the flow. As a result of the conductive losses at the walls, the reaction intensity weakens there and a dead-space (clearly seen when $u_0 = 1$) develops.

Concerning flame oscillations, we first compare our predictions with previously known results. For a *wide* channel ($\epsilon = 0.1$) and no flow ($u_0 = 0$) our calculations show that the critical Lewis number under adiabatic conditions is $Le_c \approx 3.6$, and this compares well with the value 3.58 obtained in [2] for planar unbounded flames. For adiabatic walls this result appears insensitive to the channel's width. Fig. 3 shows the propagation speed as a function of time for the two values $Le = 3.5$ and 3.6 with $\epsilon = 0.5$. In the first case, the flame is seen to reach a steady-state with $U \rightarrow 0.84$ as $t \rightarrow \infty$; in the latter a limit cycle develops and pulsating propagation occurs with the flame moving at an average speed $\langle U \rangle = 0.83$. By further increasing the Lewis number for a given channel width the mean propagation speed decreases, the amplitude of oscillations increases while the frequency ω remains nearly constant, see Table 1. It should be noted, however, that the propagation speed has been scaled with respect to $S_L \sim \sqrt{Le}$. Therefore, in dimensional form the propagation speed which increases with increasing Le up to the onset of oscillations, decreases slightly (on the average) near the bifurcation point and then continues to increase but at a very slow rate. Finally, comparing the results for wide and narrow channels, we see that in narrow channels the critical Lewis number Le_c remains nearly the same and, for a given Le , the mean propagation speed and the frequency of oscillations (in dimensional form) remain nearly constant.

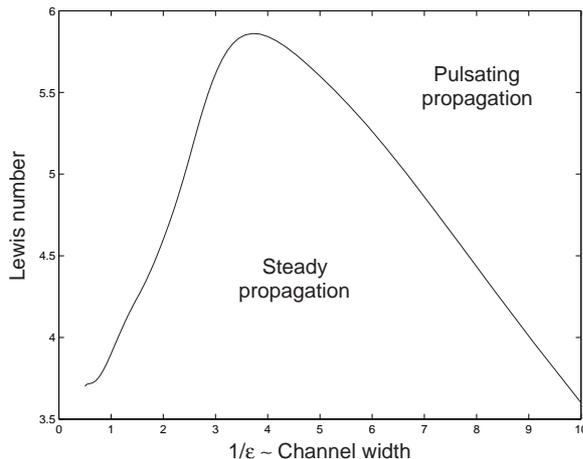


Figure 4: Critical Lewis number Le_c plotted as a function of channel width for $k = 0$ and $u_0 = -1$.

As expected, conductive heat losses lower the critical Lewis number Le_c . More significantly, however, is the effect of convection and of the channel's width on criticality. This is shown in Fig. 4 where Le_c has been plotted as function of $1/\epsilon$, which is proportional to the channel width, for the case $k = 0$ and $u_0 = -1$. We note that oscillations are more likely to occur in wide and narrow channels, since Le_c in both cases are relatively low and accessible when appropriately diluting the combustible mixture. Oscillations are less likely to occur in channels of moderate widths where the predicted Le_c is large enough and outside the range of common combustible mixtures. These results extend to the case with $u_0 = \pm 1$ and when heat losses are present.

References

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