MODEL BASED COMBUSTION CONTROL; THEORY TO APPLICATION

Ahmed F. Ghoniem
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

Model based control of combustion dynamics has been identified as an important enabling technology for operating combustion systems outside their "natural" performance envelope using a network of sensors that monitor the state, and control systems and actuators that supply feedback signals in forms that can impact the combustion mechanisms. One challenging area in which active control has been effective is in the control of combustion instability, also known as a thermoacoustic instability. A prerequisite to the design of optimal control strategies for these high amplitude and bandwidth phenomena is the availability of accurate models that describe the mechanism of the phenomenon and the system's response to external actuation. While models that describe the acoustic response of combustion systems to unsteady heat release have been known for sometime now, input-output relations between the heat release dynamics and the unsteady flow have become available only recently. These combustion dynamics models formulated through the solution of the governing equations in the appropriate flow-combustion interaction regime, or through data reduction of available results, low dimensional projection and system identification approaches. Techniques that exploit the predictive capability of these models, or just their overall structure in the design optimal active control techniques for suppressing growing pressure oscillations have been developed and several designs have actually been implemented in bench top and intermediate scale systems with encouraging results. In this talk, I will focus on heat release dynamics models that capture the essence of the combustion phenomena in several systems, and how model based active control has led to superior results when applied to mitigate combustion dynamics.

I. THERMOACOUTIC INSTABILITY

Positive coupling of heat release oscillations and acoustics in weakly damped systems, which has been observed in many applications ranging from low powered systems designed for low emissions to high powered systems designed for performance, is known to lead to thermo acoustic instability, manifested primarily by large-amplitude pressure oscillations. In all these systems the combustion process in perturbed periodically by the acoustic field through one of many mechanisms, some of which will be described shortly, leading to sustained time dependent heat release that acts as a forcing term in the acoustic field. A simple but powerful statement for the conditions that lead to positive coupling between the combustion process and the pressure oscillations is the Rayleigh criterion:[1]

$$\frac{d}{dt}\left(\int_0^L e' A\,dx\right) = \frac{\gamma-1}{\bar\rho \bar c^2}\int_0^L p' q' A\,dt - \left(E'A\right)\Big|_0^L - \dot\Psi > 0\,, \qquad (1)$$

where the heat release oscillation is *q'*, pressure oscillation is *p'*, *e'* is the acoustic energy, *E'* is the acoustic energy flux, *A* is the cross sectional area normal to the *x* direction, *t* is time, *L* is the duct length, Ψ is the dissipation of the acoustic energy. This statement indicates that *during the growing phase of the instability*, the phase between pressure and heat release oscillations must be less than 90°. It does not however explain the reason for the phase to reach the requisite values, or the mechanism by which acoustic oscillations stimulate heat release response. These fall under the area of heat dynamics models that must be developed by combustion analysts.

Before reviewing some of the progress in developing these models, it is instructive to review the differential equations that capture the feedback between acoustics and heat release in a bit more detail. These are derived by linearizing the reactive Navier-Stokes equations, while neglecting dissipative effects, mean flow and mean heat release (only for simplicity, the more general case is slightly more complex):[2]

$$\frac{\partial^2 p'}{\partial t^2} - \bar c^2 \frac{\partial^2 p'}{\partial x^2} = \left(\gamma-1\right)\frac{\partial q'}{\partial t}. \qquad (2)$$

Where γ is the specific hear ration. Using a Galerkin expansion[3] for the pressure, $p' = \bar p \sum_i \psi_i(x)\eta_i(t)$, a finite-dimensional model for the combustion driven acoustic modes of the system can be written in the following form:

$$\frac{d^2\eta_i}{dt^2} + \omega_i^2\eta_i = K\frac{dq'}{dt},\ where\ \frac{\gamma-1}{\bar p}E^{-1}\psi_i(x_f)\ and\ \ E = \int_0^L \psi_i^2\,dx. \qquad (3)$$

Where w is the mode frequency, ψ is the mode shape and sub f indicates the coombustion. Now the problem has been reduced to expressing the heat release perturbation in terms of the dynamic flow variables, e.g., $q' = q'(\eta,\dot\eta)$, to find conditions under which the Rayleigh criterion is satisfied. Linearizing this relation to express the unsteady heat release in terms of the pressure and velocity perturbations;

$$\frac{dq}{dt} = a\eta + b\dot\eta, \qquad (4)$$

and substituting back into equation (3), we see that heat release dynamics adds damping through velocity perturbation, and changes the characteristic frequency of the acoustic mode as well. Clearly, through the influence of *K*, the shape of the acoustic mode and the location of the combustion zone within the acoustic mode are important. Through the influence of the sign and magnitude of *b*, the impact of the acoustic field on the combustion process is captured. The objective of a large class of combustion dynamics models is to determine this parameter and describe its dependence on the combustor design.

II. COMBUSTION DYNAMICS

It is well knows that turbulent combustion, the dominant mode of combustion in the vast majority of practical systems, takes on different forms depending on the turbulent intensity and length scales, and the combustion characteristics, e.g., laminar burning velocity, flame thickness, extinction strain, etc. For analytical tractability, the development combustion dynamics models that close the feedback loop described by equation (3) has taken advantage of the classification of turbulent combustion into different regimes in which burning rate is governed by one limiting mechanism, e.g., a wrinkled premixed flame, a well-mixed reactants and products, a flame convoluted by the flow, etc. The objective of these models is to derive equation (4), or a close alternative, and determine the dependence of its parameters on the operating conditions. Several attempts are review next.

II.1. Wrinkled Flame Models & Impact of Mixture Inhomogeneity

Wrinkled flame models apply in the case of low intensity, large-scale turbulence, when burning occurs in thin flames. Under conditions when the flow velocity is much larger than the burning velocity, the linearized equation describing the flame surface motion can be integrated and the heat release rate perturbation can be expressed as: [4]

$$\frac{d\dot{Q}'}{dt} + \frac{2S_u}{R}\dot{Q}' = \beta R S_u u', \qquad (5)$$

where $\dot{Q}'$ is the total heat release in the combustion zone, S_u is the laminar burning velocity, R is the radius of the anchoring zone, u is the flow velocity, and β is a characteristic constant of the mixture. In this case, the flame acts as a first order filter that attenuates oscillations with frequency higher than $2S_u / R$, with a phase dependent gain. The model shows that flames possesses *a frequency selection mechanism* that lead to the amplification of some acoustic modes, depending of the laminar burning velocity and the stabilization mechanism.[5] Note that, the dynamic properties do not depend on the mean flow or the average flame shape, only on the average equivalence ratio through the laminar burning velocity and enthalpy of reaction. The flame surface model has been successful in describing the conditions of instability in several experiments[6] and has been extended to cases in which the flow perturbations are more complex[7] and when boundary conditions impose certain constraints on the flame response.[8]

The flame dynamics model was extended to describe the combined effect of flame surface area oscillations and possible equivalence ratio fluctuations induced due to the presence of unchocked air or fuel flow delivery nozzles, or the utilization of oscillating fuel valves.[9] The extended model takes on the following form:

$$\frac{d\dot{Q}'}{dt} + \omega_f \dot{Q}' = b\left(\frac{u'}{\bar{u}} + b_2\frac{\phi'}{\bar{\phi}} + b_3\frac{d\phi'/dt}{\omega_f\bar{\phi}}\right), \qquad (6)$$

where ϕ is the mixture equivalence ratio at the burning zone and the overbar indicates mean quantities. The relationship between the equivalence ratio fluctuations at the source and the pressure/velocity oscillations at the combustion zone is often complicated by the convective time lag between these two locations, and is modeled as:[10]

$$\frac{\phi'}{\bar{\phi}} = -\frac{1}{\bar{u}}u'(t-\tau). \qquad (7)$$

The time lag τ is one of the determining factors in selecting the unstable modes, and presents a special challenge to active control strategies that will be addressed shortly. Mixture inhomogeneity arises due to different design characteristics in the systems, e.g., unchocked air or fuel supply, steady fuel supply in the middle of an acoustic field where the air flow is subjected to acoustic oscillations, etc.

II.2. Well-Stirred Reactor Model & Critical Operating Parameters

When turbulence intensity is strong and the mixing rates are sufficiently high, one may assume nearly homogeneous combustion zone, and use the well-stirred reactor theory to model combustion in this regime. While idealized, this model has been useful in defining the absolute maximum burning rate attainable in a given volume due to chemical kinetics rates, and as one of the building blocks of a network of reactors used to predict burning characteristics in practical systems. Assuming single step reaction and negligible heat loss, and constant mean pressure conditions[11], the linearized equations of this model show that:

$$\frac{d\dot{Q}'}{dt} + \alpha \dot{Q}' = \beta m', \qquad (8)$$

where, $\alpha(\bar{\dot{m}},\phi) = \frac{1}{\tau_r}\left[\ 1 + n\frac{(\bar{T} - T_i)}{\bar{T}} - \frac{(\bar{T} - T_i)}{\bar{T}^2}T_a + n\frac{(Y_{f,i} - \bar{Y}_f)}{\bar{Y}_f}\ \right]$, and,

$$\beta(\bar{\dot{m}},\phi) = A_f \rho \Delta h_r \rho^{n-1} \bar{Y}_f^n \exp\left(\frac{-T_a}{\bar{T}}\right)\left[n\frac{(\bar{T} - T_i)}{\bar{T}} - \frac{(\bar{T} - T_i)}{\bar{T}^2}T_a + n\frac{(Y_{f,i} - \bar{Y}_f)}{\bar{Y}_f}\right].$$

Here $\dot{m}$ is the mass flow rate, *m'* is the perturbation, T_i and T_a are the inlet gas tempertaure and the activation temperature, respectively, Y_f is the fuel mass fraction, τ_r is the mean residnece time, *n* is the order of reaction, ρ is density and Δh_r is the enthalpy of recation. Similar to the wrinkled flame model result, the combustion process acts as a first order filter. *However, the properties of the filter, contrary to the wrinkled flame model results, depend strongly on the mean flow, or residence time, and mean equivalence ratio.* The characteristic parameters α and β can become negative as the equivalence ratio decreases or the mass flow rate increases beyond certain critical values. Analysis shows that negative values of α correspond to unstable operation, or blow out.[12] Moreover, β reaches zero at the point of maximum average heat release rate, becoming negative for additional reduction in fuel concentration or increase in mean mass flow rate, before combustion extinction is observed. Within the small increment of either the equivalence ratio or the mean flow rate, between maximum power and blow-out, the phase between the flow rate and the heat release perturbation changes by 180 degrees from its values for lower heat release rates. The model predictions match the trends observed near blow out.[13,14] Many practical combustors, especially those endowed with sufficiently strong swirl exhibit very similar characteristics in which transition from stable to unstable operation occurs close to either maximum heat release rate (maximum flow rate) or minimum equivalence ratio. The transition is often followed by blow out as the opertaing conditons change slightly, an observation that is borne out this simple model; a very encouraging conclusion. The model has been used to predict the stability frequency in several experiments.[15]

II.3. Shear Layer Stabilized Combustion

Many combustion systems, especially in premixed applications, utilize a separating shear layer downstream a bluff body as a mixing device to anchor flames.[16,17,18] In these systems, flames convoluted around large-scale structures are observed, under stable and unstable operating conditions, with the frequency of shedding on the convoluted structures matching that recorded by the pressure measuremnets. One of the most fascinating observations in some of the reported experiments is that some spectral modes under unstable operating conditions may not correspond to some of the resident acoustic modes of the combustion system, suggesting the presence of other fluid dynamic sustained oscillation mechanisms, besides acoustics, in the system.[19] To explain the origin of this frequency, in a recent study, we demonstrated that under certain conditions, shear layer instabilities can become absolute, i.e. self-sustained oscillations can be expected at the absolute mode frequency of the reacting shear layer. We showed that as the shear layer thickness and the backflow in the recirculation zone increase, the unstable modes become absolute and the Strouhal number of the oscillations, based on the step height, average flow velocity and oscillation frequency, are close to O(0.1), a value supported by many experimental and numerical simulation studies.[20,21] Analysis of absolute mode characteristics showed that increasing the equivelence ratio, which imposes a temperature distribution with higher tempertaure ratios on the shear layer, can delay the transition to absolute mode until the temperature ratio reaches higher values.

In cases when hydrodynamic mode play the role of the resonant oscillator, we showed that the heat release dynamics can be modeled using the following second order oscillator equation:

$$\frac{d^2 \dot{Q}'}{dt^2} + 2\zeta_o \omega_o \frac{d\dot{Q}'}{dt} + \omega_o^2 \dot{Q}' = 0, \qquad (9)$$

where ω_o is the frequency of the absolute mode and ζ_o is the associated damping.

II.4. Data Driven Models and POD Analysis

Combustion instability mechanisms in which several processes play competing roles, including large-scale structures, equivalence ratio inhomogeneity as well as probable local extinction and reignition, can not be easily modeled analytically and sometimes even numerically due to sevral uncertainities and analytical complexities . Instead, we have proposed using data driven reduced models, e.g. POD, in which the numerical results (or spatially resolved experimental data) are used to construct a space of optimal basis functions that describe the different modes in the flow and rank them according to the energy in each mode, for such purpose. The differnetial equations are then projected onto these modes using a Galerkin error reduction approach, to derive a set of differential equations governing the amplitudes of the modes. These equations are then utilized to predict the response of the flow to imposed perturbations, provided that the meanflow remains nearly the same and the dominant modes do not change their shapes under the impact of external forcing. These approaches are powerful, however they are system specific.

II.5. System Idenitifcation Models

As an alternative to physically based and data based models of combustion, system identification approaches have been used in connection with experimental data or numerical simulations of combustion instability. In this case, one is interested in input-output models that captures the overall response of the system. Typically these approaches are applied after the system has achieved a stationary state, such as a stable limit cycle, and the sustained oscillations are often represented using lightly damped linear models in the neighborhood of the limit cucle. Besides the model structure, one also needs an appropriate persistently exciting input to identify the system. Methods such as the ARMAX and N4SID have successfully been used to design models and model based control for combustion systems. Alternatively models based on representing the pressure signal using a Fourier series and captured through nonlinear observers have been used. For instance, taking advantage of the insight and physically-based modelling, averaging methods and system identification of limit cycle systems with time delays are used to determine the parameters of the model. Note however that SI models are both system and conditions specific and unless care is taken to select the proper structure, some subtle but important dynamics may be lost. A POD based system identification combines the advantages of traditional system identifcation and the power of POD based method in that they capture the spacial and temporal chartateristics of the phenomena simultansously.

III. ACTIVE CONTROL

Based on the inherent instability mechanism, that is the positive feedback between the pressure and heat release oscillations when their phase is less than 90, phase shift controllers have been extensively applied to stabilize combustion systems by measuring the pressure, adding the appropriate phase, and generating a new presure signal that cancels out the existing oscillation, while carefully adjusting the gain to achieve stabilization within a short settling time. Phase shift has also been implemented as pure time delay whose value is adjusted until best pressure suppression is achieved. Adaptive version, e.g. an extremum seeking controller, of the same strategy have also been attempted.[22] While successful, the scope of this strategy has been limited; in some cases secondary peaks were encountered following the application of the phase shift approach, and in many cases, perturbations and noise could upset the operation, and stability over a range of operating conditions can not be guaranteed. More powerful approaches that take advantage of detailed description of the phenomena in terms of accurate models are described next.

III.1. Linear Optimal Control

To reduce the pressure oscillations and minimize settling time, given an actuator with certain authority constraints, a control strategy which seeks to minimize a cost function of the form: $J = \int_o \left(p'^2 + \rho_c u_c^2\right) dt$, where u_c is the control input and ρ_c is chosen to represent the available control effort is used. One of the combustion models decribed above is used to determine the control input as a function of the pressure measurement and the model parameters. To minimize the effect of the modeling uncertainity, an LQG-LTR control procedure is used so that the estimator minimizes the effect of modeling errors by representing the latter as fictitous Gaussian errors. This controller has been used successfully[23] to suppress existing pressure oscillationsd by 50dB, without generating secondary peaks, whereas phase shift controller only achieved 20-30dB. In[24], an additional 10-15 dB reduction in pressure amplitude was realized when applying LQG-LTR to stabilize a swirl combustor over an empericial phase shift controller. On drawback here is that time delay is either neglected its effect is Pade approximated into a finite dimensional model, restricting the applicability of this control to cases in which time delay is small.

III.2. Time Delay Control

Control strategies designed to work with large time delays are used for systems in which the dynamics are dominated by multi scale processes, with the shortest being the acoustic scale and the longest arising perhaps due to convective effects, or in case of liquid fuels, due to evaporation and mixing. Using the models in equations (3, 6 and 7), we have shown that a variety of strategies can be applied to overcome the negative impact of time lags, depending on the possible locations of the actuator and the other contraints. Simple PI controllers that make optimal use of actuator locations can be used to cancel out or minimize the delay effects. However they may require placement that is not accessible or may lead to the formation of local hot spots and hence increase emissions. A more general strategy utilizes the Posicast controller which is based on the Smith controller. [25] The idea behind that controller is the use an accurate system model to forecast the future outcome of a certain current input, knowing that the outcome must contribute to a gradual but quick stabilization. The reader is referred to other work[26] for further details regarding the stability and robustness properties of this controller, and experimental and numerical results of the closed-loop performance that demonstrate its effectiveness. Application of time delay control in[27] achieved a reduction of 12 dB on a reheat buzz instability using less than 3% of the fuel in the control loop.

III.3. Model Based Self Tuning and Adaptive Time Delay Controllers

Adaptive control techniques are considered necessary in combustion system due to a variety of reasons including the range of opertaing condition a typical combustion system experiences, and the modeling uncertainity associated with either analysis or system identification. For this purpose, adaptive filters whose coefficients are adjusted to minimize the errors that represent a departure from the desired values of key parameters have been used in LMS techniques. An alternative approach to this impplementation of adaptive control is to exploit the structure of the dynamic model itself. For example, for certain actuator locations, the transfer functions can be shown to have relative degree smaller than two, with stable zeros and known high-frequency gain. For these cases, a simple adaptive phase-lead compensator can be shown to successfully suppress the pressure oscillations[28].

Time-delay control can be accommodated by adding a signal to the control input that attempts to anticipate the effects of the delay. The same approach can be adopted in an adaptive controller as well. [29] An experimental implementation of a PosiCast control algorithm to a 85 kW swirl stabilized combustor have been conducted.[30] As mentioned before, these implementations rely on the availability of models that capture the combustion dynamics inherent in the system.

IV. CONCLUSIONS

Recent results that have been summarized here show that it is possible to capture the esssential mechanisms in combustion dynamics in models that can be used to construct the complete feedback loop of thermoacoustic instability. The successful application of these models in the design of active control strategies for the suppression of the instability has also been shown. Current effort is focused on extending the control beyond instability suppression and into other performance parameters such as emission reduction, pattern factor and efficiency improvement.

Acknowledgement

This work has been supporetd over the years by grants from NSF, an STTR progran fron the AFOSR, and by The Propulsion Program of the Office on Naval Research, Dr. Gabriel Roy program manager.

References

[1] J.W.S. Rayleigh, *The Theory of Sound*, vol. 2. New York: Dover, 1945.
[2] M. Fleifil, J.P. Hathout, A.M. Annaswamy, and A.F. Ghoniem, "The origin of secondary peaks with active control of thermoacoustic instability," *Combust. Sci. Technol.,* vol. 133, pp. 227-265, 1998.
[3] B.T. Zinn and M.E. Lores, "Application of the Galerkin method in the solution of nonlinear axial combustion instability problems on liquid rockets," *Combust. Sci. Technol.* vol. 4, pp. 269-278, 1972.
[4] M. Fleifil, A.M. Annaswamy, Z. Ghoniem, and A.F. Ghoniem, "Response of a Laminar Premixed Flame to Flow Oscillations: A Kinematic Model and Thermoacoustic Instability Results," *Combust. Flame*, vol. 106, pp. 487-510, 1996.
[5] S. Durcuix, D. Durox and S. Candel, "Technical and Experimental Determination of the transfer function of a laminar premixed flame, " *Proc. of the Combustion Inst.,* vol. 28, 2000, pp.765-773.
[6] A.M. Annaswamy, M. Fleifil, J.P. Hathout, and A.F. Ghoniem, "Impact of linear coupling on the design of active controllers for the thermoacoustic instability," *Combust. Sci. Tech.,* vol. 128, pp. 131-180, 1997.
[7] A.A. Peracchio and W. Proscia, "Nonlinear heat release/acoustic model for thermoacoustic instability in lean premixed combustors," *ASME J. Eng. Gas Turbines Power,* vol. 121, no. 3, pp. 415-421, 1999.
[8] A.P. Dowling, "A kinematic model of a ducted flame," *J. Fluid Mech.,* vol. 394, pp. 51-72. 1999.
[9] J.P. Hathout, M. Fleifel, A.M. Annaswamy, and A.F. Ghoniem, "Heat-release actuation for control of mixture in homogeneity-driven combustion instability," *The Proceedings of the Combustion Institute,* vol 28, 2000, pp. 721-730.
[10] T. Lieuwen and B.T. Zinn, "The role of equivalence ratio oscillations in driving combustion instabilities in low NO_x gas turbines," in *Proc. of the Combustion Inst.*, vol. 27, 1998, pp. 1809-1816.
[11] S. Park, A.M. Annaswamy, and A.F. Ghoniem, "Heat release Dynamics Modeling of Kinetically Controlled Burning," *Combustion and Flame,* vol. 128, pp. 217-231, 2002.
[12] Y.B. Zeldovich, G.I. Barenblatt, V.B. Librovich, and G.M. Makhviladze, "The Mathematical Theory of Combustion and Explosion," Consultants Bureau, N.Y., 1985.
[13] G.A. Richards, G.J. Morris, D.W. Shaw, S.A. Keeley, and M.J. Walter, *Comb. Sci. Tech.,* vol. 94, pp. 57-85, 1993.
[14] J.M. Cohen and T.J. Anderson, "Experimental investigating near blow out instability in a lean premixed step combustor," AIAA-96-0819, 34th Aerospace Sciences Meeting, Reno, NV 15-18 Jan 1996.
[15] S. Murugappan, S. Acharya, E.J. Gutmark, and T. Messain, "Characteristics and Control of Combustion Instability in a Swirl Stabilized Spray Combustor," 35th AIAA/ASME/SAE/ASEE Joint Propulsion Conference, L.A., CA, 20-24 June, 1999.
[16] J.O. Keller, L. Vaneveld, D. Korschelt, G.L. Hubbard, A.F. Ghoniem, J.W. Daily, and A.K. Oppenheim, "Mechanism of Instabilities in Turbulent Combustion Leading to Flashback," *AIAA J.,* vol. 20, pp. 254-262, 1982.
[17] S.R.N. DeZilwa, J.H. Uhm and J.H. Whitelaw, "Combustion Oscillations Close to the Lean Flammability Limit," *Combust. Sci. and Tech.,* vol. 160, pp. 231-258, 2000.

[18] K.C. Schadow, E. Gutmark, T.P. Parr, D.M. Parr, K.J. Wilson, and J.E. Crump, "Large-Scale Coherent Structures as Drivers of Combustion Instability," *Combust. Sci. and Tech.,* vol. 64, pp. 167-186, 1989.
[19]A.F. Ghoniem, A.M. Annaswamy, D. Wee, T. Yi, and S. Park, "Shear Flow Driven Combustion Instability; Evidence, Simulation and Modeling," *The Proceedings of the Combustion Institute,* vol. 29, 2002.
[20] H.N. Najm and A.F. Ghoniem, "Numerical Simulation of the Convective Instability in a Dump," *AIAA J.,* vol. 29, pp. 911-919, 1991.
[21] H. Najm and A.F. Ghoniem, "Modeling Pulsating Combustion Due to the Flow-Flame Interactions In Vortex Stabilized Pre-Mixed Flames," *Combust. Sci Tech.,* vol. 94, pp. 259-278, 1993.
[22]S. Murugappan, E.J. Gutmark, S. Acharya, and M. Krstic, "Extremum seeking adaptive controller for swirl stabilized spray combustion," presented at the 28th Int. Combust. Symp., Edinburgh, U.K., July 2000.
[23] A.M. Annaswamy, M. Fleifil, J. Rumsey, J.P. Hathout, R. Prasanth, and A.F. Ghoniem, "Thermoacoustic instability: Model-based optimal control designs and experimental validation," *IEEE Trans. Contr. Syst. Technol.,* vol. 8, pp. 905-918, Nov. 2000.
[24] S. Murugappan, S. Park, A.M. Annaswamy, A.F. Ghoniem, S. Acharya, and D.C. Allgood, "Optimal Control of a Swirl Stabilized Spray Combustor Using System Identification Approach," *Combust. Sci. Tech.,* vol. 175, pp. 55-81, 2003
[25] O.J. Smith, "A controller to overcome dead time," *ISA J.,* vol. 6, pp. 28-33, 1959.
[26] S. Niculescu and A.M. Annaswamy, "A simple adaptive controller for positive-real systems with time-delay," in Proc. Amer. Controller Conf., Chicago, IL., 2000, pp. 3666-3670.
[27] P.J. Langhorne, A.P. Dowling, and N. Hooper, "Practical active control system for combustion oscillations," *J. Propulsion and Power,* vol. 6, no. 3, pp. 324-333, 1990.
[28] A.M. Annaswamy, O.M. El-Rifai, M. Fleifil, J.P. Hathout, and A.F. Ghoniem, "A Model-Based Self-Tuning Controller for Thermoacoustic Instability," *Combust. Sci. Tech.,* vol.135, pp. 213-240, 1998.
[29] S. Evesque, A.P. Dowling, and A.M. Annaswamy, "Adaptive algorithms for control of combustion," in NATO RTO/AVT Symp. Active Contr. Technol. Enhanced Performance in Land, Air, and Sea Vehicles, Braunschweig, Germany, May 2000.
[30] A. Riley, S. Park, A. P. Dowling, S. Evesque and A. M. Annaswamy, "Adaptive closed-loop control on an atmospheric gaseous lean-premixed combustor," ASME/IGTI Turbo Expo (accepted), Atlanta, GA, 2003.