INERTIA EFFECT ON A STRUCTURE OF PRESSURE DRIVEN FLAMES IN INERT POROUS MEDIA

Viatcheslav Bykov, Igor Goldfarb, Vladimir Gol'dshtein
Ben-Gurion University of the Negev, Department of Mathematics
P.O.B.653, Beer-Sheva 84105, Israel

There has been a recent surge of interest in the combustion of gaseous mixture within porous inert media [1, 2]. Flames stabilized within the porous matrix have higher burning speeds and distinct flammability limits than of open flames. There is a number of possible explanations of the phenomenon, namely: internal feedback of heat from the burned gases to unburned ones through radiative heat transfer, conventional heat conduction through a solid skeleton, diffusion of a heated gas because of a local pressure elevation, etc. We will focus on the last: a local pressure elevation.

Conventional approach to flame propagation modelling in open air ignores the pressure perturbations. It is reasonable because the flame propagation velocity in open air is significantly lower then the sound speed. This conclusion is not correct for gaseous combustion processes in inert porous media, because (for comparatively narrow pores) the speed of pressure perturbation may be significantly lower then the sound velocity in open air. Under such circumstances a local elevation of pressure may lead to a formation of the self-sustaining combustion wave controlled by pressure diffusion.

In the recent papers [2] this heuristic idea was converted into original model that describe a new physical mechanism of flames spreading through inert porous media. The model used an assumption that in comparatively narrow pores a propagation of combustion waves might be governed mainly by diffusion of pressure in porous media, rather then conventional heat transfer. The present paper continues the study in this direction and is concentrated on the impact of the inertia phenomena on the fine structure and main characteristics of the flame front.

Traditionally the study of the premixed gas flames in an open space makes use of asymptotic technique. In particular, a method of inner and outer asymptotics represents a powerful and effective tool for an analysis of the structure and velocity of flames in various media. Unlike the conventional deflagration, in pressure driven flame the temperature within reaction zone undergoes a nearly jumpwise increase, whereas the preheat zone is rather wide and the temperature growth relatively slowly in it. As a result the traditional multiple scale approach developed for analytical tackling of deflagrations meets here with formidable difficulties. The present piece of work is aimed to overcome this hurdle and to evaluate main characteristics of propagating flame. In this paper we demonstrate how a powerful method of integral manifolds (MIM) [3] can be successfully applied to relatively new field – problem of a pressure driven flame propagation through two-phase medium (inert solid skeleton filled with a flammable gaseous mixture).

We will use a model suggested in [4]. We restrict ourselves to a one-dimensional approach presuming that it gives us a conceptual qualitative information about the main dynamical properties of the process. The porous medium is considered as a set of the evenly spread parallel capillaries of the same inner radius (so-called cell model), filled with premixed combustible gas mixture (the solid matrix is inert). The presence of the porous medium is accounted for by the friction force (velocity-dependent) added to the momentum equation. We presume that the friction force is an inner one, which does not affect the total energy balance of the system.

To single out the pure impact of the pressure effect on the properties of self-sustained combustion wave driven by the local pressure elevation, the conventional mechanism of the combustion wave propagation (thermal diffusion) is regarded as negligibly small (compared with pressure diffusion – barodiffusion) and it is excluded from our consideration. Additionally, this allows us to simplify a mathematical description of the real extremely complex problem and to make it tractable analytically. Presumably, the approximated problem preserves most basic features of the originally full nonlinear system.

Within the above premises, the system of governing equations contains energy (1), concentration (2), momentum (3), continuity (4), state (5) equations, and it reads

$$\frac{d}{dx}\left(\rho(u-D)(c_v T+\frac{1}{2}u^2)+Pu\right)=QAC_f\,\rho\exp\left(-E/RT\right), \qquad (1)$$

$$\frac{d}{dx}\left(\rho(u-D)C_f\right)=-AC_f\,\rho\exp\left(-E/RT\right), \qquad (2)$$

$$\frac{d}{dx}\left(\rho(u-D)u+P\right)=-K_F\rho u|u|; \qquad (3)$$

$$\frac{d}{dx}\left(\rho(u-D)\right)=0 \qquad (4)$$

$$P=\frac{\rho}{\mu}RT=\left(c_p-c_v\right)\rho T \qquad (5)$$

The following notations were used: T - temperature (K); P – pressure (Pa); E - activation energy (J/kmol); D – velocity of the flame front in the laboratory system of coordinates (m/s); C - concentration of the deficient reactant; c - specific heat capacity (J/kg/K); u - gas velocity in the laboratory frame of reference (m/s); Q - combustion energy (J/kg); W - reaction rate (kg/(s m^3)); ρ- density (kg/m^3); K- permeability of the medium (m^2); ν - kinematical viscosity (m^2/s); A – pre-exponential (frequency) factor (1/s); R - universal gas constant. Subscripts mean: f - combustible component of the gas mixture; p - under constant pressure; v- under constant volume; 0 - undisturbed state; b - burnt (behind the combustion wave front), F – related to the case of quadratic dependence of the friction force on gas velocity. The system (1)-(5) is subject to boundary conditions (fresh mixture far before the front of the flame)

$$T(x\to+\infty)=T_0;\quad C_f(x\to+\infty)=C_{f0};\quad P(x\to+\infty)=P_0;\quad \rho(x\to+\infty)=\rho_0; \qquad (6)$$

The system (1)-(5) can be partially integrated and reduced to the form of the set of the two ODEs. A qualitative analysis of the reduced system using conventional methods is rendered extremely difficult due to the complexity of the right hand sides of the equations. Therefore, appropriate numerical procedures must be resorted to. Alternatively, the presence of the sufficiently different time scales raises the possibility of using some kind of asymptotic procedures. In the present piece of work, we exploit a powerful asymptotic technique, namely, a geometrical version of the integral (invariant) method (MIM) [3, 5, 7, 8]. The relevant mathematical background of this sort of analysis can be found in the theory of integral (invariant) manifolds [3, 7]. The MIM permits to decompose a phase-space analysis of an arbitrary multi-scale system into separate studies of its fast and slow subsystems. The advantage of this decomposition is that any subsystem has lower dimensions than the original one. Although numerical solution of the equations is straightforward, a general analytical parametric analysis of the system behaviour such as will be presented here, is unattainable by numerical means.

To make the reduced system tractable by MIM, we need to introduce a pair of new variables to re-write the system in the form of singularly perturbed system of ODEs. A physical rationale lying behind a choice of the new dimensionless variables is rather simple. Along the lines of Zel'dovich approach [9], we can single out two qualitatively different sub-zones within the flame front, which are used to refer to as preheat and reaction ones. Each of the two sub-zones is characterized by its own unique feature. Within the preheat zone an input of the exothermic chemical reaction is assumed to be negligible, whereas the friction force plays a dominant role in the system dynamics. As a result, the system energy remains almost constant and we can interpret this fact as an existence of an approximate law of energy conservation in this sub-zone. On the other hand, the momentum of the system changes essentially within preheat sub-zone. Hence, one of the new variables (v) can be introduced as a deviation from the approximate law of the energy conservation (in the absence of reaction). Having determined a new variable v in such a way, its *slow* alteration within the preheat sub-zone is expected (with respect to momentum, for example). Unlike the preheat sub-zone, the reaction one can be typified by another approximate conservation law - the system momentum conservation. Therefore, another new variable (u) can be chosen as a variation from this approximate law. On the basis of similar reasoning we expect that u is slow variable (with respect to energy or its modifications).

Thus, there is a good reason to consider a flame configuration in the following way: within the preheat sub-zone to use the pair of the variables (v, w) (v – slow, w – fast), within the reaction sub-zone - the pair (u, w) (u - slow, w - fast). This decomposition permits us to re-write the original system - as two separate systems for the different sub-zones of the flame. Each of these systems has the conventional form of singularly perturbed system of ODEs, which makes direct application of the MIM legitimate.

The analysis of the system trajectory (solution) in the phase space is naturally subdivided into two parts. The first stage of the trajectory is analyzed in the plane v-w. It has a starting point P_{in} and represents the fast motion from the initial point in the direction to the attractive branch of the slow curve. The equation of the slow curve is determined by the RHS of the equation for the variable w. While the fast motion takes place the slow variable v conserves its initial value. The first stage ends when the trajectory approaches the slow curve (matching point Q, connecting the two parts of the trajectory). The second stage starts at this point (the repulsive branch of the manifold rejects the trajectory in the plane u-w) and moves in the direction of the singular (final) point P_{fin}. The slow variable u conserves the value obtained at the matching point Q. Only for a special value of the flame velocity λ_F the trajectory can reach the singular (final) point P_{fin} (all other trajectories moves above or below the final point P_{fin}.). The matching point Q separating these two sub-zones plays the key role in the determination of the flame velocity and allows us to determine the details of the trajectory and to gain the expression for the flame velocity.

The suggested approach allowed us to get the analytical estimation of the inertia effect on the flame velocity. The desired estimation represents a solution of the algebraic equation (7).

$$E_0 + \varepsilon_{\text{inert}} E_1 + \varepsilon_{\text{inert}}^{3/2} E_{3/2} + \varepsilon_{\text{inert}}^{5/2} E_{5/2} = 0, \quad \varepsilon_{\text{inert}} = \frac{\beta D^2}{C_p T_0} \tag{7}$$

where the coefficients E_0, E_1, $E_{3/2}$, $E_{5/2}$ depend on the problem parameters. The equation (7) is an equation for inertia parameter $\varepsilon_{\text{inert}}$ and can be solved numerically.

In the absence of the inertia, the flame velocity is proportional to the cubic root of the Arrhenius exponent, which coincides with our earlier results. The impact of the inertia grows with increasing of the flame velocity. The inertia influence becomes more important in the parametric region closer to the sound velocity in a fresh mixture. The theoretical predictions demonstrated rather good agreement with the direct numerical simulations.

Figure 1 allows us to get a visual impression how the approximation built on the asymptotic approach (MIM) describes a real trajectory in the phase space. The figure is a projection of the real trajectory in the three-dimensional space Π-θ-η onto the (θ-Π) plane and the (θ-η) plane. The approximation $P_{in}Q$ of the preheat sub-zone corresponds to the stage $P_{in}W$ of the real trajectory, and approximation QP_{fin} of the reaction sub-zone to the WP_{fin}.

One can see that within the preheat sub-zone (θ in the interval 0-50) the energy of the system is almost constant – a difference between the two curves (theoretical and numerical) is not visible. Visually the pressure Π is strictly proportional to the temperature θ. A discrepancy begins to be observable only in the vicinity of the matching point Q.

In a similar way one can ensure that the momentum of the system conserves its final value within the reaction sub-zone with rather good accuracy. We are hampered to distinguish two curves (solid and dashed ones) within the interval [70, 200] of θ values. This testifies, that the suggestion made regarding approximate law of momentum conservation was rather reasonable. One can see, that the approximation looks reasonable and the most discrepancy is observed in the region of the matching point Q.

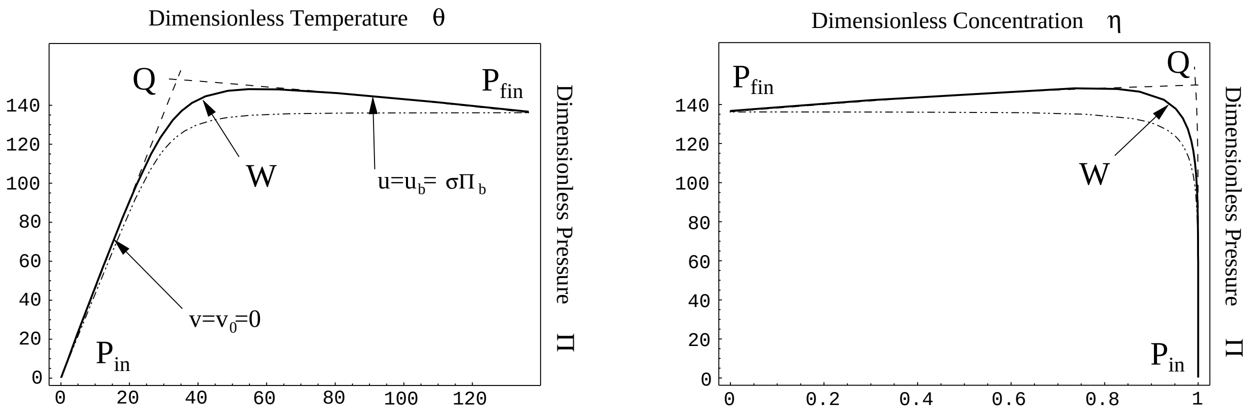

Figure 1. Projection of the system real trajectory on the planes θ-Π (left) and θ-η (right). Solid line $P_{in}WP_{fin}$ – result of numeric simulations, dashed lines $P_{in}Q_2$ (preheat zone) and Q_2P_{fin} (reaction zone) – our approximations. The system parameters read: $\beta = 0.0295;$ $\gamma = 1.3;$ $\varepsilon_1 = 6.43 \cdot 10^{-6};$ $\sigma = 0.23$ $\varepsilon_2 = 4.35 \cdot 10^{-8};$ $q = 5.66667$. The dimensionless flame velocity (used for dashed trajectory, numerical data) is $\lambda_F = 1.42$ the dimensional one is D=72.1 m/s.

Finally, we note that although the presented application of the modified MIM approach to the model of the pressure driven flames in porous media does provide a broader more accurate perspective than that previously obtained with non-inertial models, it is not without its own deficiencies. It is indisputable that a more realistic description should ideally include one or more additional effects: the wider region of the flame velocities (inertia impact is expected to be larger), thermal diffusion (conventional mechanism of flame propagation), more details of the chemistry such that initiation of the chemistry via a radical pool can be accounted for, etc. These directions of further improvement of our model are currently under investigation.

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