# Calculation of detonation waves in a column of a chemically active bubbly medium

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**Abstract**

The problem of a detonation wave propagating in a cylindrical column of a chemically active bubbly medium shielded by the liquid from the tube walls is formulated and numerically solved within the Iordanskii-Kogarko two-phase one-velocity model with allowance for energy dissipation due to acoustic irradiation of bubbles. The wave structure of the reaction zone and the detonation velocity of the bubbly medium column are calculated. It is found that a self-sustained wave can propagate with a velocity 1.5-2.5 times higher than the velocity of one-dimensional bubble detonation.

**Key words:** detonation waves, bubbly liquid, reacting gas bubbles.

## Introduction

An experimental and theoretical analysis of one-dimensional detonation waves (DW) in reacting bubbly media [1-3] shows that the one-dimensional wave structure is a soliton moving with a constant velocity D lower than the frozen velocity of sound in the mixture. Based on Iordanskii's model [4], the dynamics and structure of the two-dimensional reaction zone in a self-sustained DW propagating in a two-layer bubbly medium with one layer containing bubbles of an inert gas were numerically studied in [5]. It was found that the DW velocity in a two-layer bubbly medium is always smaller than in a one-layer medium. The question of possible propagation of detonation in a finite-radius cylindrical column of a chemically active bubbly medium surrounded by an inert liquid remained open. The dynamics of a heterogeneous DW in a two-layer system "active bubbly medium – inert liquid" is numerically studied in the present work.

## Statement of the problem

Let the central part (of radius $R_p$) in a cylindrical tube of radius $R_c$ be filled by a bubbly liquid with a uniform volume concentration $\alpha_{20}$ ($\alpha_{20} << 1$) and identical spherical (of radius $a_0$) bubbles of a reacting gas, and let a cylindrical layer of width $\Delta R = R_c - R_p$ be filled by a liquid. At the time $t = 0$, the pressure at the left end of the tube ($x = 0$) in a circle of radius $R_w = \min\{2R_p, R_c\}$ instantaneously increases from the initial value $p_0$ up to $p_w > p_0$ and remains constant during the time $t_w$. We have to determine the dynamics of the wave process in a column of the active bubbly medium for $t > 0$, depending on the scale parameters of the problem $R_c$, $R_p$, and $a_0$.

The liquid is assumed to be compressible and satisfy the Tait equation of state

$$p = p_0 + \rho_0 c_0^2[(\rho_1/\rho_0)^n - 1]/n,$$

wjere p and $\rho_1$ are the current pressure and density of the liquid, $\rho_0$ and $c_0$ are the initial density and velocity of sound in the liquid, and n is the polytropic index.

Heat transfer between the gas and the liquid is neglected; therefore, the adiabaticity condition is valid for the gas phase. When the bubbles reach the critical radius of ignition $a_* < a_0$, instantaneous release of energy occurs (explosion in constant volume). Then, the equation of state for the gas phase can be represented in the form [5]

$$p_2 = p_i (\rho_2/\rho_2^0)^{\gamma_i}, \quad (i = 0,1)$$

where $p_2$ is the gas pressure, $\rho_2$ and $\rho_2^0$ are the current and initial densities of the gas, and $\gamma_i$ is the ratio of specific heats; the subscripts refer to the initial gas ($i$ = 0) and to combustion products ($i$ = 1, $p_1 > p_0$).

The equations of two-dimensional unsteady motion of a monodisperse bubbly medium in the two-phase model with allowance for energy dissipation due to acoustic irradiation of bubbles are similar to those given in [5]. The bubble-detonation model considered with given thermophysical properties of the liquid is characterized by the following dimensionless parameters:

$$\gamma_0,\ \gamma_1,\ R_* = a_* / a_0,\ B = p_1/p_0,\ Re = a_0\rho_0\sqrt{p_0 / \rho_0}/\mu,\ \beta = \sqrt{p_0 / \rho_0}/c_0$$

($\mu$ is the dynamic viscosity of the liquid). It was found [6] that bubble-detonation models that ignore acoustic irradiation of bubbles in the liquid do not yield a steady-state solution in the form of a running wave (soliton). Evaluating the contribution of liquid viscosity and acoustic irradiation of bubbles to dissipative losses, we found that energy dissipation due to liquid-phase viscosity can be neglected for $\beta \cdot Re \gg 1$. Therefore, in modeling detonation in active bubbly media for which the condition $\beta \cdot Re \gg 1$ is satisfied, the Reynolds number Re is not included into the dimensionless parameters of the model.

For given physicochemical properties of the phases and initiation parameters ($p_w$, $t_w$, and $R_w$), the solution of the problem formulated above depends on three dimensionless factors:

$$L = R_p/a_0,\ \ \delta = R_p/R_c,\ \alpha_{20}.$$

The problem was solved numerically. For integration of the system of differential equations, we used the Godunov-Kolgan finite-difference scheme in movable grids with capturing of the shock front and bubble-ignition front.

**Calculation results**

The numerical study was performed for a column of an active bubbly medium (water with bubbles of $C_2H_2$ + 2.5 $O_2$) in water for the following constants: $p_0$ = 1 atm, $\rho_0$ = 1000 kg/m$^3$, $c_0$ = 1500 m/sec, n = 7.15; $\rho_2^0$ = 1.238 kg/m$^3$, $\gamma_0$ = 1.33, $\gamma_1$ = 1.136, $R_*$ = 0.25, and B = 10.97. In calculations for L = 10, the governing parameters varied within the ranges $\delta \in [0.1 \div 1]$ and $\alpha_{20} \in [0.01 \div 0.04]$. It was found that initiation parameters $p_w/p_0$ = 100, $T_w = t_w/t_0$ = 0.3, and $R_w = \min\{2R_p, R_c\}$ are sufficient to excite a detonation wave in a column of the bubbly medium with bubbles of $C_2H_2$ + 2.5 $O_2$. Here, $t_0 = a_0\sqrt{p_0 / \rho_0}$ . The bubble-detonation wave propagating over the column of the active bubbly mixture reaches a self-sustained regime with a constant velocity $D_{st}$ depending on the parameters $\delta$ and $\alpha_{20}$ at a distance $X = x/a_0 = 100 \div 500$ from the point of initiation. The calculated values of $D_{st}$ for $\alpha_{20}$ = 2% and several values of $\delta$ are listed in Table 1. The detonation velocity of the column $D_{st}$ increases monotonically with increasing thickness of the layer of the inert liquid. In particular, for $\delta$ = 1/6, the detonation velocity of the column $D_{st}$ = 1.37 km/sec is two times the wave velocity of one-dimensional bubble detonation $D_0$ = 0.68 km/sec. In a steady DW (see Table 1), the pressure maximum in the liquid on the axis of symmetry ($P_{max}$) increases with increasing width of the inert liquid layer, and the pressure maximum in the liquid near the tube wall ($P^c_{max}$) monotonically decreases.

Table 1.

| $\delta = R_p/R_c$ | 1 | 4/5 | 2/3 | 1/2 | 1/3 | 1/4 | 1/6 | 1/10 |
|---|---|---|---|---|---|---|---|---|
| $D_{st}$ (km/sec) | 0.68 | 0.78 | 0.87 | 1.02 | 1.19 | 1.27 | 1.37 | 1.41 |
| $P_{max}$ | 110 | 127 | 143 | 178 | 225 | 269 | 313 | 303 |
| $P^c_{max}$ | 110 | 85 | 76 | 66 | 57 | 53 | 45 | 33 |

Thus, in contrast to a two-layer system " active – passive bubbly mixture", where the presence of an inert bubbly layer leads to a decrease in DW velocity [5], the presence of an inert liquid layer in a two-layer system "active bubbly mixture – inert liquid" leads to an increase in velocity of a steady DW. By varying the independent parameters $\alpha_{20} \in [0.01 \div 0.04]$ and $\delta \in [0.1 \div 1]$, the velocity of the bubble-detonation wave in the column can be changed within wide limits $D_{st} \in [0.51 \div 1.44]$ km/sec, always remaining subsonic ($D_{st} < c_0$). The value of $D_{st}$ increases monotonically with increasing thickness of the inert liquid layer and decreasing volume concentration of bubbles in the column. It was found that the DW in a column of an active bubbly medium has the form of a soliton. The spatial structure of the soliton for a steady DW for $\alpha_{20}$ = 2 % is shown in Fig. 1 for $\delta = ½$ (a) and $\delta = ¼$ (b).

An analysis of $D_{st}$ calculated as a function of $\alpha_{20}$ and $\delta$ shows that the results admit physically clear interpretation if the generalizing parameter is assumed to be the mean volume concentration of bubbles in the tube $\overline{\alpha_{20}} = \alpha_{20}\delta^2$. The dependences of $D_{st}$ on $\overline{\alpha_{20}}$ are plotted in Fig. 2 (solid curves). Despite a tenfold change in the ratio $R_c/R_p$ and a fourfold change in the initial volume concentration of bubbles in the column $\alpha_{20}$, the detonation velocities as functions of the generalizing parameter $\overline{\alpha_{20}}$ are grouped within a 10 % range of values. For fixed $\overline{\alpha_{20}}$, a monotonic decrease in detonation velocity of the bubbly-medium column is observed with increasing $\alpha_{20}$. The calculation results for $D_{st}$ can be approximated by the following formula within 5 %:

$$D_{st}/c_0 = (1 + 6.44\,(\overline{\alpha_{20}})^{0.5} + 16.3\,\overline{\alpha_{20}})^{-1}.$$

**Conclusions**

It is found by means of mathematical simulation that a detonation wave may propagate in a cylindrical column of a chemically active bubbly medium shielded by the liquid from the tube walls. The velocity of this wave is greater that in a one-layer bubbly system with the same volume concentration of bubbles. The soliton-like two-dimensional structure of bubble detonation with a pressure profile decreasing toward the inert liquid layer is obtained and analyzed. It is demonstrated that introduction of a generalizing parameter $\overline{\alpha_{20}}$ (mean volume concentration of bubbles in the tube) allows one to estimate the detonation velocity of a chemically active bubbly medium in a liquid (within 10 %) using the one-dimensional model of bubble detonation.

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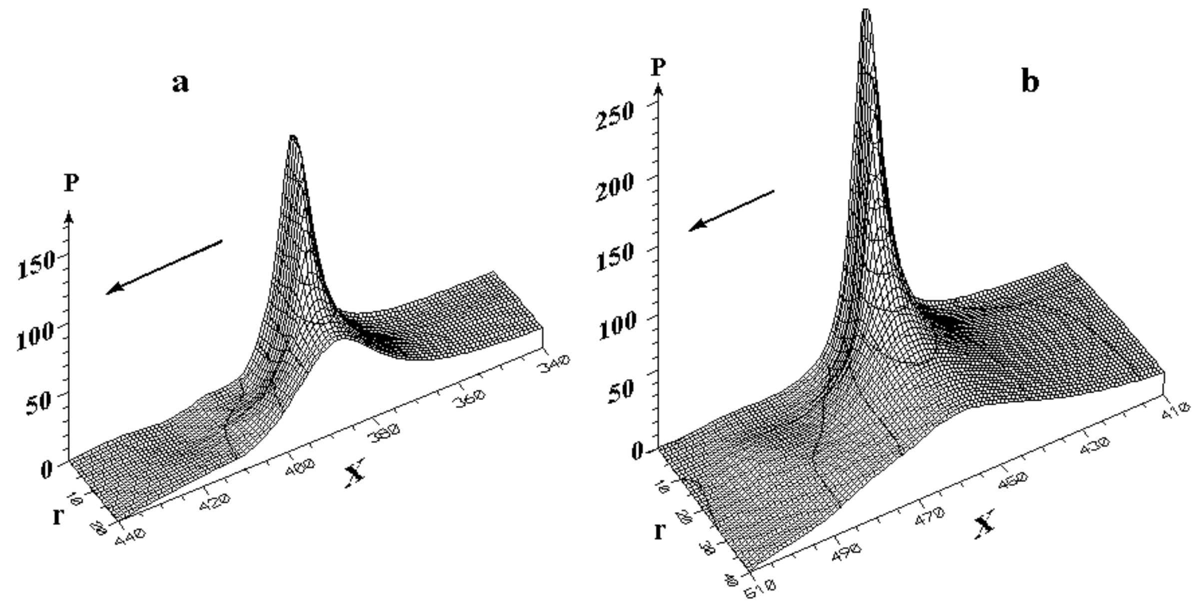


Fig. 1. Pressure profiles in the liquid for a steady DW. ($\alpha_{20}$ = 0.02; T = $t/t_0$ = 4): a) $\delta$ = 1/2; b) $\delta$ = 1/4.

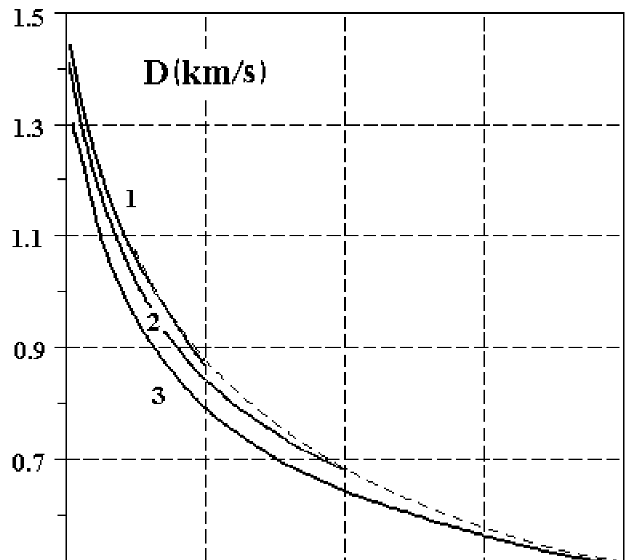


Fig. 2. Detonation velocities of the column $D_{st}$ as functions of $\overline{\alpha_{20}} = \alpha_{20}\delta^2$ (1 – $\alpha_{20}$ = 0.01; 2 – 0.02; 3 – 0.04). (the dashed curve shows the one-dimensional bubble detonation velocity for $\delta$ = 1).