Linear and Nonlinear Stability of Cylindrical and Spherical Detonations

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Keywords: detonations, pulsating and cellular instabilities, direct ignition.

The stability of curved detonation fronts, such as cylindrically or spherically expanding detonations, is investigated. Firstly, a linear stability analysis of weakly curved, quasi-steady detonations is performed for the standard one-step reaction model with Arrhenius kinetics. The main purpose of such an analysis is to determine how rapidly increasing curvature of the front stabilizes or destabilizes the detonation. Figure 1 shows the one-dimensional neutral stability boundary in an activation temperature (E)-curvature (κ) plane, for fixed heat of reaction and ratio of specific heats. This corresponds to a pulsating instability (i.e. to perturbations in the radial direction). The dashed line in the figure is the critical curvature for quasi-steady waves, i.e. the curvature above which there are no quasi-steady solutions.

Figure 1 shows that increasing curvature makes the detonation wave more unstable. Moreover, the figure also shows that the detonation is always unstable sufficiently near the critical quasi-steady curvature. Dispersion relations (growth rates versus curvature) are also given, which show that increasing curvature actually has a very rapid destabilizing effect on the wave, and hence curved detonations may be highly unstable even when the planar front is stable.

High-resolution one-dimensional numerical simulations of pulsating cylindrically and

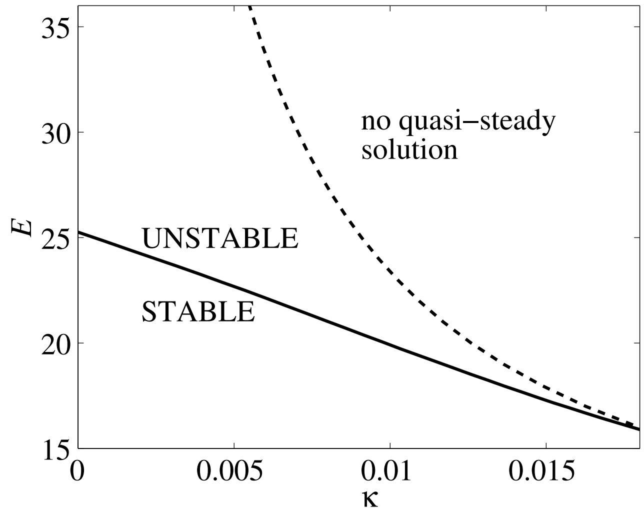

Figure 1: Neutral stability boundary for one-dimensional perturbations in an activation temperature-curvature diagram.

spherically expanding detonations produced by a high energy source (direct ignition) are then performed, and the results compared with the linear analysis. It is shown that the nonlinear dynamics can depend dramatically on the source (initiation) energy. Figure 2 shows the shock pressure versus radius for $E = 25$ and various initiation energies.

There are two stages in the evolution of the pulsating instability. Initially, the amplitude of the pulsations grow as the detonation expands outwards. The growth rate depends on the radius where the blast wave initially approaches the quasi-steady speed (i.e. on the initiation energy). The higher this radius, the more stable the detonation and the slower the growth of the amplitude, in agreement with the linear analysis. In the second stage, the pulsations reach a saturated nonlinear amplitude. However, as the detonation continues to expand and becomes less curved and hence more stable, this saturated amplitude decreases. Hence the fully developed nonlinear oscillations form an amplitude 'envelope' and once the pulsations reach this envelope, the subsequent propagation and evolution is independent of the initiation energy. However, if the front reaches the neutrally stable

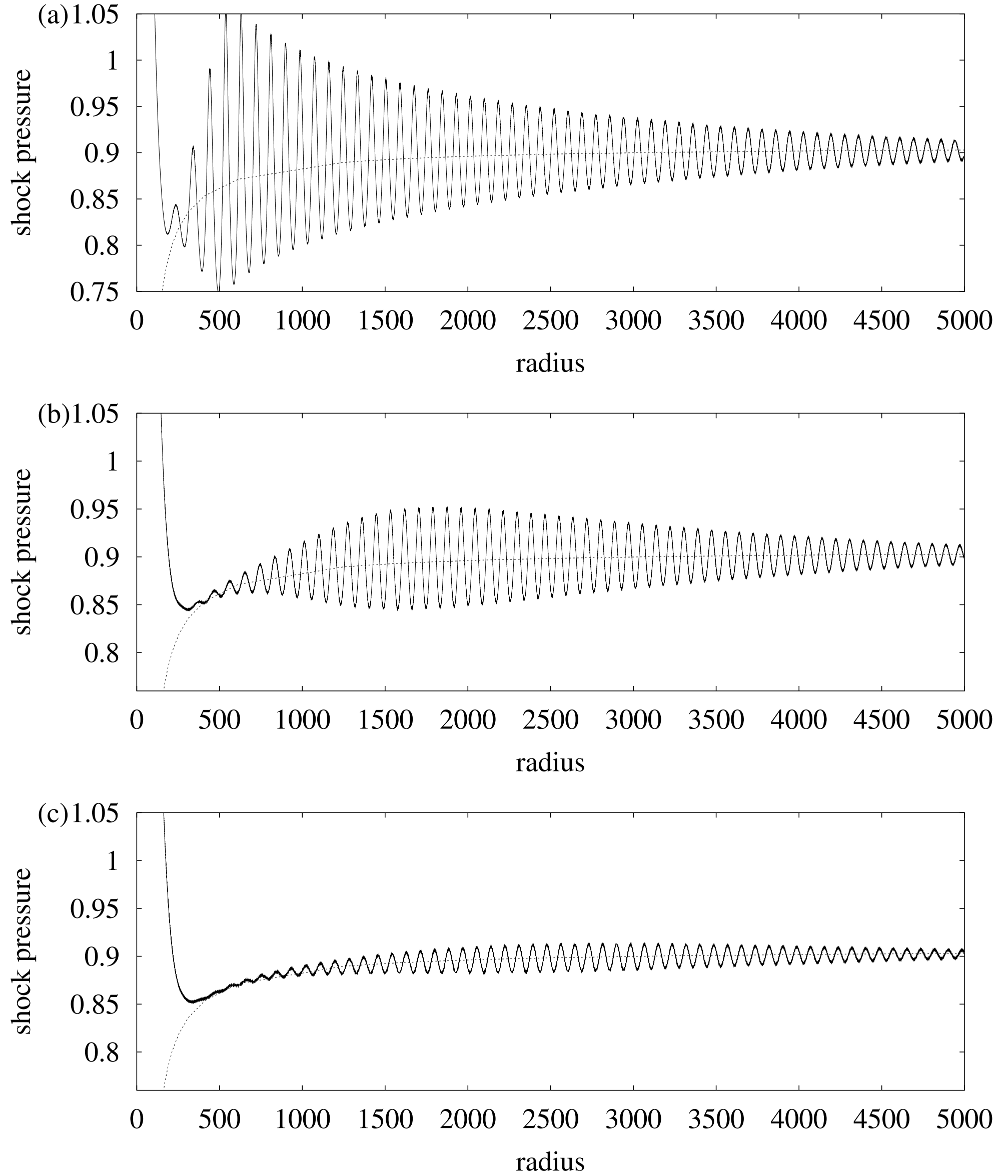

Figure 2: Shock pressure versus shock radius for cylindrically expanding detonations, corresponding to a dimensionless activation temperature $E = 25$ and (a)-(c):increasing source (initiation) energies. The dotted line is the quasi-steady shock pressure-shock radius relation.

curvature (as predicted by the linear analysis) before the pulsation amplitude reaches the saturated nonlinear envelope, then the amplitude begins to decay away from it (e.g. figure 2(c)). The spherical and cylindrical cases are compared and contrasted.

Finally, two-dimensional simulations of the direct ignition scenario are performed to investigate the evolution of the cellular detonation instability. It is shown, however, that if we consider only a quarter-plane, parametric studies of the two-dimensional problem are computationally prohibitive due to the large radius at which the detonation forms (and hence a very long circumference of the front) combined with the high resolution of the reaction zone required to properly capture the instability. Calculations with resolutions between 4 to 16 numerical grid points per reaction zone length of the steady, planar wave are performed and it is shown that the onset, evolution, size and regularity of the detonation cells, as well as the critical initiation energy required to produce a detonation, are highly grid dependent at such affordable resolutions. However, implications for the cellular instability based on the results of the one-dimensional calculations are discussed within the context of experimental results.

Acknowledgements: this work is jointly funded by Engineering and Physical Sciences Research Council (UK) and Defence Science and Technology Laboratory (UK).