Theoretical Analysis on Simplified Pulse Detonation Engine Model

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Nomenclature

a : velocity of sound

c_v : constant-volume specific heat

D_{CJ} : Chapman-Jouguet detonation wave speed of detonable mixture

f_{cyc} : frequency of pulse detonation engine operation

I_{cyc} : impulse density acting on thrust wall per one-cycle operation

L : length of pulse detonation engine tube

M_{CJ} : Mach number of Chapman-Jouguet detonation wave

p : pressure

p_{av} : time-averaged pressure on thrust wall

q : heat released in chemical reaction per unit mass

T : temperature

t : time relative to instant of ignition

t_{cyc} : period of pulse detonation engine operation

t_f : duration of filling process

t_I : time at which the detonation wave reaches thrust wall and x_2 becomes L

t_{II} : time at which exhaust of detonation products is completed

u : velocity of gas in x direction

x : coordinates along axis of pulse detonation engine

γ : ratio of specific heats at constant pressure and constant volume

ρ : density

Subscripts

1: condition of unburned detonable mixture

2: Chapman-Jouguet surface of detonation wave

ex : open-end boundary condition

N: von Neumann spike of detonation wave

1. Model

The performance of pulse detonation engines ignited at the open-end was theoretically analyzed by using a simple model. A pulse detonation engine was modeled as a one-dimensional straight tube with one end closed and the other end open. The detonation wave, the rarefaction wave and the PDE performance were simplified and theoretically studied by using Hugoniot relation for the Chapman-Jouguet detonation wave, flow relation for one-dimensional self-similar rarefaction wave and the momentum theorem for thrust estimation. One-dimensional numerical simulation was conducted and the theoretical results are in good agreement with numerical simulation.

2. Aanlysis

First, the quantities on the CJ surface p_2, ρ_2, u_2, D_{CJ} are as follows,[1,2]

$$p_2 = \frac{\gamma_1}{\gamma_2 + 1} M_{CJ}{}^2 p_1 \tag{1}$$

$$\rho_2 = \frac{\gamma_2 + 1}{\gamma_2} \rho_1 \tag{2}$$

$$u_2 = \frac{1}{\gamma_2} a_2 = \frac{1}{\gamma_2 + 1} D_{CJ} \tag{3}$$

$$D_{CJ} = \sqrt{2(\gamma_2{}^2 - 1) q} \tag{4}$$

$$M_{CJ} = \frac{D_{CJ}}{a_1} \tag{5}$$

$$t_I = \frac{L}{D_{CJ}} \tag{6}$$

the position of the CJ surface is written by

$$x_2 = D_{CJ} t \tag{7}$$

On the von Neumann spike, p_N and ρ_N are written as follows,[1,3]

$$p_N = \frac{2\gamma_1}{\gamma_1 + 1} M_{CJ}{}^2 p_1 \tag{8}$$

$$\rho_N = \frac{\gamma_1 + 1}{\gamma_1 - 1} \rho_1 \tag{9}$$

Second, the rarefaction wave following the CJ detonation wave is as follows,[1,4]

$$p = (\frac{1}{\gamma_2} + \frac{\gamma_2 - 1}{\gamma_2} \frac{x}{x_2})^{\frac{2\gamma_2}{\gamma_2 - 1}} p_2 \tag{10}$$

$$\rho = (\frac{1}{\gamma_2} + \frac{\gamma_2 - 1}{\gamma_2} \frac{x}{x_2})^{\frac{2}{\gamma_2 - 1}} \rho_2 \tag{11}$$

$$u = u_2 - \frac{2}{\gamma_2 + 1} \frac{x_2 - x}{t} \tag{12}$$

$$a = a_2 - \frac{\gamma_2 - 1}{\gamma_2 + 1} \frac{x_2 - x}{t} \tag{13}$$

Let x=0, then the open-end boundary of the rarefaction wave are given as follows,

$$p_{ex} = (\frac{1}{\gamma_2})^{\frac{2\gamma_2}{\gamma_2 - 1}} p_2 = (\frac{1}{\gamma_2})^{\frac{2\gamma_2}{\gamma_2 - 1}} \frac{\gamma_1}{\gamma_2 + 1} M_{CJ}{}^2 p_1 \tag{14}$$

$$\rho_{ex} = (\frac{1}{\gamma_2})^{\frac{2}{\gamma_2 - 1}} \rho_2 = (\frac{1}{\gamma_2})^{\frac{2}{\gamma_2 - 1}} \frac{\gamma_2 + 1}{\gamma_2} \rho_1 \tag{15}$$

$$u_{ex} = u_2 - \frac{2}{\gamma_2 + 1} \frac{x_2}{t} = -u_2 = -\frac{1}{\gamma_2 + 1} D_{CJ} \tag{16}$$

Third, suppose $\rho_{ex} u_{ex}$ is the characteristic mass exhaust rate per unit area and keeps constant during exhaust phase, the time-averaged pressure on the thrust wall p_{av} and other important characteristics are given as follows,

$$p_{av} = \rho_{ex} u_{ex}{}^2 + p_{ex} = (\frac{1}{\gamma_2})^{\frac{2\gamma_2}{\gamma_2 - 1}} \rho_1 D_{CJ}{}^2 \tag{17}$$

$$t_{II} = \frac{\rho_1 L}{\rho_{ex} u_{ex}} = \gamma_2^{\frac{\gamma_2 + 1}{\gamma_2 - 1}} \frac{L}{D_{CJ}} \tag{18}$$

$$t_{cyc} = t_{II} + t_f > \gamma_2^{\frac{\gamma_2+1}{\gamma_2-1}} \frac{L}{D_{CJ}} \tag{19}$$

$$f_{cyc} = \frac{1}{t_{cyc}} < \frac{1}{\gamma_2^{\frac{\gamma_2+1}{\gamma_2-1}} \frac{L}{D_{CJ}}} \tag{20}$$

$$I_{cyc} = p_{av} t_{cyc} = p_{av} t_{II} = \frac{1}{\gamma_2} \rho_1 D_{CJ} L \tag{21}$$

Finally, we compare the open-end ignition with the closed-end ignition, as shown in table 1.

Table 1 Comparison between open-end ignition and closed-end

	Open-end ignition	Closed-end ignition[1]
p_{av}	$0.10\rho_1 D_{CJ}{}^2$	$0.11\rho_1 D_{CJ}{}^2$
I_{cyc}	$0.71\rho_1 D_{CJ} L$	$0.85\rho_1 D_{CJ} L$
t_{cyc}	$> 7.6\frac{L}{D_{CJ}}$	$> 8.0\frac{L}{D_{CJ}}$

3. Conclusions

We analytically obtained simple formulas for the performance of pulse detonation engine ignited at the open-end. Comparison with the analysis results for PDE ignited at the closed-end indicates that both initiation schemes have almost the same operation performance. One-dimensional computation has been conducted to verify the theoretical results. Study shows that theoretical results are in good agreement with computational results.

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References

[1] Endo, T. and Fujiwara, T., " A Simplified Analysis on a Pulse Detonation Engine Model," Transaction of Japan Society for Aeronautical and Space Sciences, 2002: vol 44, No. 146. pp. 217-222.

[2] Landau, L. D. and Lifshitz, E. M., "Fluid Mechanics, 2nd ed., Butterworth-Heinemann, Oxford, 1987, Ch. 14.

[3] Landau, L. D. and Lifshitz, E. M., "Fluid Mechanics, 2nd ed., Butterworth-Heinemann, Oxford, 1987, Ch. 9.

[4] Landau, L. D. and Lifshitz, E. M., "Fluid Mechanics, 2nd ed., Butterworth-Heinemann, Oxford, 1987, Ch. 10.