Mathematical modelling of triple flame ring behaviour

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ABSTRACT

We study axisymmetrical triple flame that is observed at the base of lifted laminar diffusion jet flame. Dynamics of the flame base is described in terms of laminar flamelet, propagating along the stoichiometric iso-surface in the direction opposite to those of the jet. Due to this "opposed flow" mechanism the round jet flame is stabilized over the fuel exit. The lift-off height, i.e. the distance between the burner and the flame is consequently determined by the jet velocity and the flame response. This "lifted" regime, or triple flame ring, exists for a limited range of Reynolds numbers only: for small values of the jet velocity the flame is attached to the burner, and for velocities exceeding some critical value, blow-out occurs. In its turn, the impact of the mixture fraction gradient on the stabilization over the fuel exit is important: the smaller the gradient, the higher the triple flame propagation velocity.

It was shown (see [4]), that heat release has two main effects on the flame characteristics: first, the local velocity of the triple flame propagation is increased due to the deviation of the velocity vector field ahead of the flame front; secondly, the mixture fraction gradient is reduced and thus the flame velocity is increased. Consequently, these effects have to be included in our analysis.

The laminar diffusion jet flame in the present work is described with help of axisymmetric zero Mach number combustion equations for unbounded domains. Chemical process is chosen to be a

single-step one, with reaction rate described by Arrhenius law.

The jet velocity is delivered by Squire-Landau self-similar solution for round jet.

In the previous studies (see [2], [3]) two-dimensional approximation for triple flame velocity was used in order to predict the lift-off height of round jet. By this, all the non-local characteristics of the flame base propagation was neglected. We fill up this gap, introducing the leading order dependence on the flame base radius.

In the present paper we consider the situation, where heat release is small, so that the density changes are incorporated through perturbative analysis of the reactive flow characteristics. Moreover, the activation energy supposed to be large, mixture fraction gradient is small, and their product is of the order one (i.e. only well-developed triple flames are considered).

Solution to the passive scalar equation for mixture fraction in our analysis is approximated by linear profile with gradient, prescribed at the inlet.

The flame front is assumed to be parabolic, the curvature of it is found as a part of the solution. This simplifies considerably the original free-boundary problem and allows us to introduce local parabolic-cylinder coordinates, matching the flame front. Afterwards, standard techniques of the activation-energy asymptotics can be applied.

Solutions for both temperature and mixture fraction are found in the form of expansions with respect to small parameter α characterizing the heat released into the flame.

The resulting singular perturbative problems are solved in local parabolic-cylinder coordinates with help of matched asymptotic expansions. The equations for temperature together with prescribed boundary values define the eigenvalue problems at each order, the eigenvalue inversely proportional to the square of triple flamelet propagation velocity.

Closed expressions for the flame base propagation velocity in dependence on heat release, mixture fraction gradient and radius of the flame base, are found.

Comparisons with direct numerical simulations (DNS) and theoretical predictions of the lift-off height by Boulanger, Vervisch, Reveillon and Ghosal are presented below.

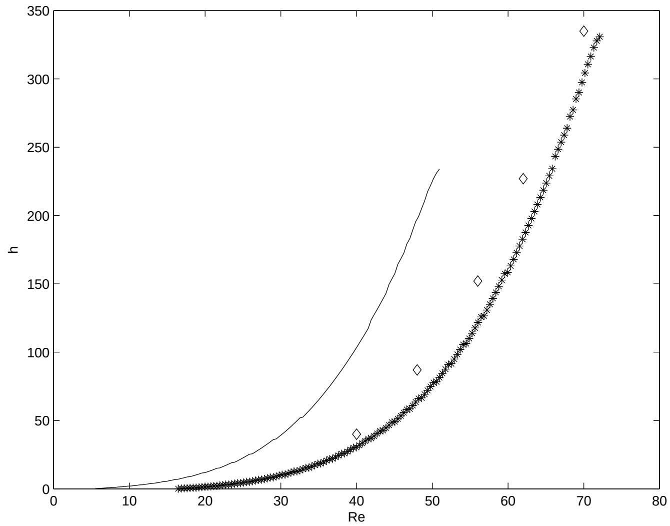

Figure 1: *Comparison of our theoretical results for lift-off heights (in units of nozzle radius) as function of Reynolds number with DNS and theoretical approximation by J. Boulanger et al. for* $\alpha = 0.3$. *Line: plane case, diamonds: DNS results, asterisks: results with correction due to the radius of the flame base.*

Constant c denotes the parameter, characterizing dependence on the radius of the flame base, r_0:

$$c = 1 - \frac{1}{r_0^2}.$$

On Figures 1 and 2 the lift-off heights versus Reynolds numbers are depicted for two values of the parameter, characterizing the heat release: $\alpha = 0.3$ and $\alpha = 0.8$.

In both cases our theoretical results fit well the DNS data. We can conclude then that an important amelioration in theoretical prediction of the lift-off heights compare to the planar velocity approximation [2] is achieved.

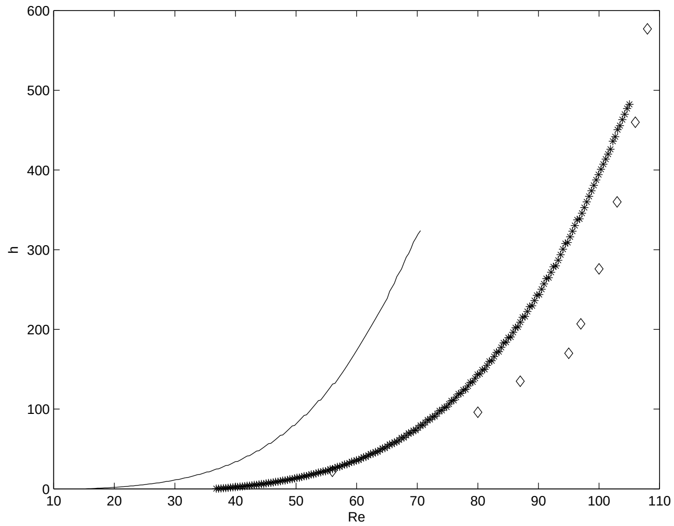

Figure 2: *Comparison of our theoretical results for lift-off heights (in units of nozzle radius) as function of Reynolds number with DNS and theoretical approximation by J. Boulanger et al. for* $\alpha = 0.8$*. Line: plane case, diamonds: DNS results, asterisks: results with correction due to the radius of the flame base.*

References

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