A Numerical Study of Premixed Turbulent Flame Dynamics

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Many experimental observations show that premixed turbulent flame speed and thickness grow in time (or with distance from flame-holder) in most flames. The goal of this work is to numerically study the effects of pressure-driven transport on the development of premixed turbulent flame structure, thickness, and speed by solving the following generalized flamelet closure

\[
\frac{\partial}{\partial t} \left( \bar{p}\bar{c} \right) \quad + \frac{\partial}{\partial x} \left( \bar{p}\bar{u}\bar{c} \right) = \frac{\partial}{\partial x} \left( \bar{p}D_t \frac{\partial \bar{c}}{\partial x} \right) + U \frac{\partial}{\partial x} \left[ \bar{p}\bar{c}(1 - \bar{c}) \right] + \frac{\rho_u}{\tau_f} \left( \frac{\bar{p}}{\rho_u} \right)^q \bar{c}(1 - \bar{c}),
\]

of the mean combustion progress variable balance equation

\[
\frac{\partial}{\partial t} \left( \bar{p}\bar{c} \right) \quad + \frac{\partial}{\partial x} \left( \bar{p}\bar{u}\bar{c} \right) = - \frac{\partial}{\partial x} \left( \rho u' c' \right) + \bar{W}.
\]

Term III in Eq. 1 is a typical closure of the mean rate of product creation, \( \bar{W} \), provided by various flamelet models [1,2]. Different models result in different expressions for the flame time scale, \( \tau_f \), but the specification of such an expression is not needed here, because a closure for \( \tau_f \) does not affect the numerical results presented in a normalized form if \( \tau_f \) is not varied in space and time.

Terms I and II in Eq. 1 model turbulent diffusion and pressure-driven transport [2], respectively, and together represent a generalized closure of the transport term IV in Eq. 2. Here, \( D_t \) is the turbulent diffusivity and \( U \) is a velocity scale. Term II may be associated with the submodel of pressure-driven transport, \( \gamma S_L \bar{p}\bar{c}(1 - \bar{c})/2 \), developed by Bray et al. [3] for stagnating flames. Then, \( U = \gamma S_L/2 \), where \( \gamma = \rho_u/\rho_b - 1 \) is the heat release factor, and \( S_L \) is the laminar flame speed.

We have kept the turbulent diffusion term I in Eq. 1; despite the fact that, in many laboratory flames, this term is much smaller than the pressure-driven transport term II almost in the whole flame brush (\( 0 < c_1 < \bar{c} < c_2 \leq 1, \ c_1 \ll 1, \ 1 - c_2 \ll 1 \)), for instance, term I was omitted by Bray et al. [3]
when modeling stagnating flames. One reason for keeping this term in simulations of a planar flame moving in a statistically stationary and uniform mixture is as follows. If one omits term I in this case, then the asymptotically steady solution of Eq. 1 should satisfy the following equation

\[
S_0 \frac{dc}{dx} = U \frac{d}{dx} \left[ \frac{\tilde{b}}{\rho_u} \tilde{c}(1 - \tilde{c}) \right] + \frac{1}{\tau_f} \left( \frac{\tilde{b}}{\rho_u} \right)^q c(1 - \tilde{c}).
\]  

(3)

However, Eq. 3 includes only three dimensional parameters, \( S_0 \), \( U \), and \( \tau_f \), and, due to dimensional reasons, \( S_0 = U f(\gamma) \) for an arbitrary \( \tau_f \), i.e., the fully-developed turbulent flame speed, \( S_0 \), does not depend on the mean rate of product creation. The absurdity of this conclusion\(^1\) justifies keeping of term I in Eq. 2 even if this term is much less than term II at \( c_1 < \tilde{c} < c_2 \).

To simulate the propagation of a statistically planar 1D flame in statistically stationary and uniform mixture from the left to the right, Eq. 1 has been normalized

\[
\frac{\partial}{\partial \tilde{t}} (\tilde{b} \tilde{c}) + \frac{\partial}{\partial z} (\tilde{b} \tilde{w} \tilde{c}) = \frac{\partial}{\partial z} \left( \frac{\partial \tilde{c}}{\partial z} \right) + P \frac{\partial}{\partial z} [\tilde{b} \tilde{c}(1 - \tilde{c})] + \frac{\tilde{b}^q}{4} \tilde{c}(1 - \tilde{c})
\]  

(4)

by invoking the following velocity, \( u_o = 2(D_t / \tau_f)^{1/2} \), length, \( l_o = (D_t \tau_f)^{1/2} \), time, \( t_o = l_o / u_o \), and density, \( \rho_u \), scales and, then, solved together with the normalized mass balance equation and with the following state equation, \( \tilde{\tilde{c}} = (1 + \gamma \tilde{c})^{-1} \) [1,2]. The focus of this work is placed on the effects of \( P \) on the unsteady solution of Eq. 4, or, in other words, on the role played by pressure-driven transport, since we have already studied the dynamic behavior of the solution of Eq. 4 with \( P = 0 \) [4,5].

Shown in Fig. 1 are the effects of \( P \) on the self-similarity of the structure of developing flames. Numerous experimental data discussed elsewhere [4,5] show that the structure of various premixed turbulent flames is self-similar, i.e., the spatial profiles of the progress variable, normal to the flame brush, are described by the same function at different instants \( t \) after ignition when using the developing flame brush thickness, \( \delta(t) \), in order to normalize the spatial coordinate. Our previous simulations [4,5] have shown that combustion models associated with \( q = 0 \) in Eq. 4 predict this property if \( P = 0 \), whereas the models associated with \( q = 1 \) are not capable for doing so (Fig. 1b). An increase in \( P \) makes the self-similarity of the profiles more pronounced if \( q = 0 \) (Fig. 1a) and almost self-similar

\(^1\)The comment cannot be applied to stagnating flames studied by Bray et al. [3], because one more dimensional parameter, the flame strain rate, should be taken into account in the latter case.
solutions can be obtained even with $q = 1$ if $P$ is sufficiently large (Fig. 1c). Thus, term II in Eq. 1, associated with the pressure-driven transport, enhances the trend to self-similarity.

Figure 2 shows that both normalized burning velocity, $u_t = \int_{-\infty}^{\infty} \hat{W} dx / (\rho a u_0)$, and flame thickness, $\delta^{-1}_t (t) = \max |d\hat{e}/dz|$, are decreased by $P$. However, Fig. 3 indicates that the effects of $P$ on the development of $u_t$ and $\delta_t$ can be substantially reduced by re-normalizing the results using new velocity, $u_{t,\infty} \equiv u_t (t' \to \infty)$, length, $\delta_{t,\infty} \equiv \delta_t (t' \to \infty)$, and two time, $\delta_{t,\infty}/u_{t,\infty}$ or $P^{-2}$, scales. Indeed, curves drawn with the former time scale are close one to another if $P \leq 0.5$, whereas the curves computed at $P = 1$ differ substantially from the other curves (Fig. 3a); and curves drawn with the latter time scale are close one to another if $P \geq 0.5$, whereas the curves computed at $P = 0.2$ differ substantially from the other curves (Fig. 3b) Moreover, the obtained results show that $u_{t,\infty} \sim 1/P$.
Figure 3: Development of burning velocity (fine curves) and flame thickness (bold curves), computed with $q = 0$ at various $P$, shown in legends, and re-normalized with the following velocity, $u_{t,\infty}$, length, $\delta_{t,\infty}$, and time scales, $\delta_{t,\infty}/u_{t,\infty}$ (a) or $P^{-2}$ (b).

and $\delta_{t,\infty} \sim 1/P$ if $P \geq 0.5$. These observations imply that the flame dynamics changes substantially at $P \approx 0.5$ (or $U \approx (D_t/\tau_f)^{1/2}$). We may note also that the development of re-normalized burning velocities and flame thicknesses is similar to one another in the whole range of $P$ studied (cf. fine and bold curves in Fig. 3), in line with the results of a theoretical analysis of the self-similar solutions of Eq. 2, discussed in Refs. [5].

When considering the ranges of weak ($P < 0.5$) and strong ($P > 0.5$) pressure-driven transport separately, the role played by pressure-driven transport is mainly reduced to a decrease in $u_{t,\infty}$ and $\delta_{t,\infty}$ by $P$; whereas the development of re-normalized burning velocities and flame thicknesses is weakly affected by $P$. Moreover, if the submodel of $\hat{W}$ is able to yield a self-similar flame structure ($q = 0$), then, the structure is weakly affected by $P$. If the submodel of $\hat{W}$ is not capable for doing so ($q = 1$), then, the pressure-driven transport can make the structure self-similar (cf., Figs. 1b and 1c).


