Propagation of detonation wave in rotated gas flows

Levin V.A.,* Skopina G.A.**

Institute for automation and control processes the Far Eastern Branch of RAS,
Vladivostok, Russia, E-mail: levin@imec.msu.ru

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The propagation of detonation of shock waves in vortex flows of gases is studied. The axes symmetric flow is esteemed, when except for longitudinal velocity, having an inhomogeneous profile, is present transverse velocity, i.e. the stream is twisted. In this case there is a stationary equations solution of gas dynamic depicting ideal, perfect gas flow, which one looks like

\[ U = U_0(r), \ V = 0, \ W = W_0(r), \ \rho = \rho_0(r), \ \tilde{P}_0 = \int \rho W^2/r \ dr, \]

and the vortex vector has components

\[ \omega_{0x} = 1/2r \frac{\partial r W_0}{\partial r}, \ \omega_{or} = 0, \ \omega_{0\phi} = -1/2 \frac{\partial U_0}{\partial r}. \]

Here U, V, W - corresponding components of velocity in a cylindrical coordinate system (x, r, \(\phi\)), \(\rho\) - density, \(\tilde{P}_0\) - pressure. Further on an instant \(t = 0\) on a symmetry axis there is a explosion or initiation of detonation. Then across a stream the shock or detonation wave will be diffused, which one are a discontinuity surface, thus the symmetry of flow to an axis x is saved.
Using non-stationary Euler equations and conservation laws on a discontinuity surfaces, is demonstrated, that at transferring through a discontinuity surface the vector \( \sigma / \rho \) is continuous, though each of magnitudes \( \sigma \) and \( \rho \) suffer a breaking. Thus, for the given class of flows on a discontinuity surface there is a additional conservation law

\[
\frac{\sigma_1}{\rho_1} = \frac{\sigma_0}{\rho_0}.
\]

From this proportion at once follows that the vorticity for detonation and shock wave increases proportionally to ratio of densities. At identical rate of propagation waves the vorticity behind a detonation wave is less, than for a shock wave. This deduction is fair and for plane shearing flow.

Further is studied a capability of a detonation wave propagation in a Chapman-Jouguet regime (C.J.) in rotated streams. From analysis of the solution of equations in neighborhood of a detonation C.J. wave the necessary condition of existence of such regime is obtained:

\[
\frac{1}{\gamma + 1} \frac{\partial \ln \rho_0}{\partial \ln r} + \frac{\gamma^2}{(\gamma + 1)^2} \frac{W_0}{D_J^2} \geq 0, \quad D_J^2 = C.J.\text{velocity}.
\]

At implementation of an inverse inequality the detonation wave is over compressed and its propagation velocity will be more then velocity of a C.J. wave.

* - Professor, Deputy director IACP FER RAS

**-Senior Scientist