Propagation and Extinction of Unsteady Spherical Spray Flame Fronts

J. B. Greenberg
Faculty of Aerospace Engineering
Technion - Israel Institute of Technology
Haifa 32000, Israel
email: aer9801@aerodyne.technion.ac.il

Extended Abstract

Introduction

Recent experimental research [1] has considered the propagation of a spherical laminar flame front through a mixture of fuel droplets and air. It was found that the presence of the droplets was responsible for cellular and pulsating flame fronts that were clearly observed. In view of these findings Near Equidiffusional Flame (NEF) analyses of planar laminar spray flames were carried out [2,3] to try to pinpoint the mechanism responsible for the onset of the observed instabilities. It was shown that heat loss suffered by the system, as a result of the absorption of heat by the droplets for evaporation, triggered the behavior of the spray flame front.

Although it is well known that an NEF analysis is to be preferred when examining the question of flame instabilities, a slowly varying flame (SVF) analysis is more appropriate if a relatively simple evolution equation for the flame front is sought [4].

The question of the ignition of a mist of droplets is of prime importance in the context of the re-light problem in aircraft. Despite this fact there is rather sparse experimental evidence available in the literature (see, for example, [5,6]) and few theoretical works. In the case of the latter, derivation of rules of thumb, based on overall balance of energy considerations, seems to be the general trend [7-9]. In a more detailed numerical model [10] ignition of a polydisperse mist of droplets in a tube, due to heating at one of its ends, was considered. It was shown that, in the configuration that was investigated, the Sauter mean diameter is not suitable for describing the ignition characteristics of a polydisperse spray.

In previous work [11] numerical simulations of a spherical flame front propagating through a mixture of fuel droplets and air were presented. The emphasis was on the complex chemical and thermal structure of the evolving flame so that the operating conditions always ensured the continued existence of the flame front. In the current paper the same problem is examined but the treatment is analytic within the framework of an SVF model. Unlike the aforementioned numerical research the main aim here is to look into conditions for the possible extinction of the flame front and the way they are influenced by the presence of the spray of droplets.

The Model

Consider an unconfined domain containing a mixture of fuel droplets, fuel vapor, oxygen and an inert gas. At time $t = 0$ this mixture is ignited and, under appropriate conditions, a flame front begins to propagate outwards with spherical symmetry through the mixture. The main assumptions of the model are as follows:

(One) Velocities are small compared to the speed of sound.
(Two) Viscous dissipation and the work done by the pressure are negligible.
(Three) Constant transport properties determined primarily by those of the gas phase.
(Four) Dufour and Soret effects are negligible.
(Five) One step first order chemical reaction with Arrhenius kinetics and a large dimensionless activation energy $\boldsymbol{\theta}$.
(Six) Reactant composition fuel rich and far from stoichiometric.

(Seven) A slowly varying flame with $O(\theta^{-1})$ heat losses.

(Eight) Lewis number not too close to one, $|Le-1| \approx O(1)$.

(Nine) Variable gas density.

(Ten) A vaporization front exists downstream of the flame front.

(Eleven) Droplets in the spray have approximately the same average velocity as their host environment .

(Twelve) The droplets are taken to have the same temperature as the host environment.

The situation under consideration is sketched in Fig. 1.

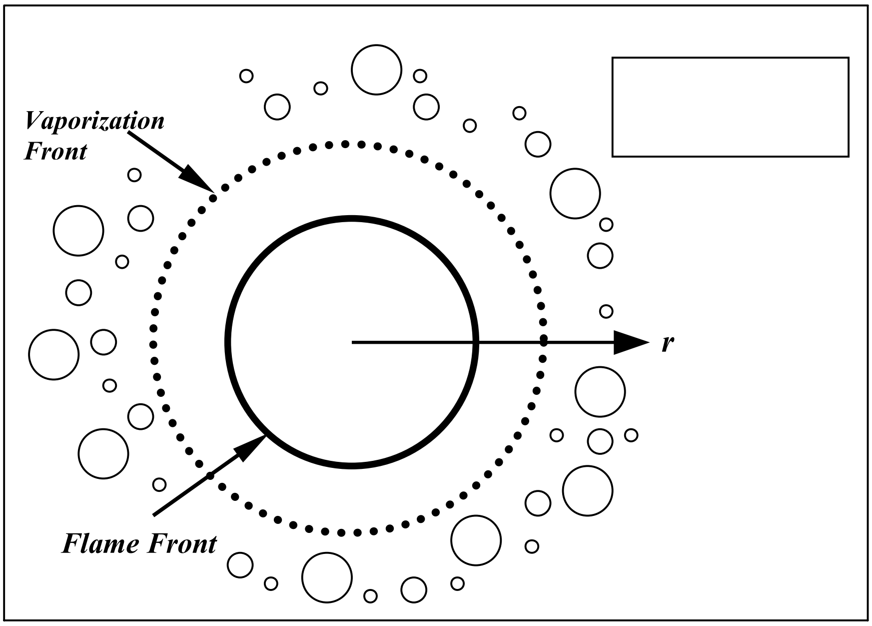

Figure 1: Spherical spray flame configuration

Under these assumptions the governing equations assume the following form

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho u\right)=0$$

$$\rho\frac{\partial T}{\partial t}+\rho u\frac{\partial T}{\partial r}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+(1-\alpha)exp(\frac{1}{2}(T-1))\hat{\delta}(r-F(t))-h(T^4-\alpha^4)-\eta S_v$$

$$\rho\frac{\partial m_O}{\partial t}+\rho u\frac{\partial m_O}{\partial r}=\frac{1}{Le}\frac{\partial}{\partial r}\left(r^2\frac{\partial m_O}{\partial r}\right)-exp(\frac{1}{2}\theta(T-1))\hat{\delta}(r-F(t))$$

$$\rho\frac{\partial m_d}{\partial t}+\rho u\frac{\partial m_d}{\partial r}=-S_v$$

$$\rho=\frac{1}{T}$$

in which ρ is the mixture density, u is the velocity, T is the temperature, α is ratio of the unburned gas temperature to the adiabatic burned gas temperature, $F(t)$ is the location of the flame front, $\hat{\delta}$ is the delta function, h is the heat loss coefficient, η is the latent heat of vaporization of the droplets in the spray, S_v is the rate of vaporization, m_O is the mass fraction of oxygen, m_d is the mass fraction of liquid fuel in the spray, r is the radial coordinate and t is time. Note that these quantities have been normalized in the usual fashion.

The Solution

Introducing a coordinate system linked to the flame front via $x = r - F(t)$ and scaling the variables in accordance with an SVF analysis leads to a set of equations that can then be solved by exploiting asymptotic expansions of the dependent variables in power series in θ^{-1}. In view of assumption (j) all droplets evaporate at an infinite rate at the location of the vaporization front. This is taken to be where they attain the (dimensionless) boiling temperature of the liquid fuel, T_v, so that for $x > x_v$ the mass fraction of liquid fuel in the droplets assumes its value in the unburned mixture $m_d = \delta$ (constant), whereas for $x < x_v$ it is 0.

Then, after a lengthy analysis, it can be shown that the evolution equation for the flame front in terms of the flame velocity normalized by that of an adiabatic plane flame is

$$\frac{dS}{dR} + S^2 \ln S^2 = \frac{2S}{R} - l - \Gamma\delta S^2$$

where $S = \frac{dF}{dt}$, R relates to the flame front location, l = the heat loss term and $\Gamma = \eta\theta$, where $\Gamma = O(1)$.

Similarly, an equation for the evolution of the vaporization front's location is derived

$$x_v = F(t) + \frac{1}{S} \ln\left(\frac{1-\alpha}{T_v - \alpha}\right).$$

Note that, in the absence of the liquid phase ($\delta = 0$), the flame front evolution equation reduces to the well-known gas phase equation (see [12]). It is clear from this equation that the influence of the spray on the flame front evolution is exhibited as an additional heat loss term resulting from the absorption of heat by the fuel droplets for vaporization.

Some Results

In Fig. 2 the flame velocity is drawn as a function of R. After the initial ignition the flame velocity settles down to a constant value in the case when no droplets are present. Although increasing the droplet load reduces the flame velocity, beyond some critical value the heat loss due to the spray extinguishes the flame (S drops to zero).

In Fig. 3 the critical extinction radius is plotted as a function of the droplet loading for different values of the radiative heat loss parameter l. The asymptotic trend of the curves for $l=0.3$ and $l=0.35$ are indicative of the droplet loading limit for which extinction does *not* occur. For example, when $l = 0.35$, a droplet loading of less than about 1.8 implies that extinction will not occur. As the droplet loading increases the critical extinction radius of the flame decreases. For a radiative heat loss of $l = 0.4$ the flame extinguishes even when the fuel is completely gaseous in the unburned mixture. Due to their additional endothermicity the presence of droplets in the unburned mixture serves to attenuate extinction at a more premature critical radius.

Acknowledgements

The author gratefully acknowledges the support of the Lady Davis Chair in Aerospace Engineering and the Technion Fund for the Promotion of Research. Thanks are due to A. Zayde for dedicated technical assistance.

References

1. F. Atzler, *Fundamental Studies of Aerosol Combustion,* Ph.D. Thesis, School of Mechanical Engineering, University of Leeds, U.K. 1999.
2. J.B. Greenberg, A.C. McIntosh and J. Brindley, Combustion Theory and Modelling, Vol. 3, pp. 567-584, 1999.
3. J.B. Greenberg, A.C. McIntosh and J. Brindley, Proceedings of the Royal Society of London, Series A, in press, 2000.
4. J.D. Buckmaster and G.S.S. Ludford, *Theory of Laminar Flames,* Cambridge Press, 1982.
5. K. Miyasaka and Y. Mizutani, Combustion and Flame, Vol. 25, pp. 177-186, 1975.
6. K. Miyasaka and Y. Mizutani, Proceedings of the Combustion Institute, Vol. 16, pp. 639-645, 1976.
7. D.R. Ballal and A.H. Lefebvre, Proceedings of the Royal Society of London, Series A, Vol. 364, pp. 277-294, 1978.
8. J.E. Peters and A.M. Mellor, Combustion and Flame, Vol. 38, pp. 65-74, 1980.
9. D.R. Ballal and A.H. Lefebvre, Proceedings of the Combustion Institute, Vol. 18, pp. 1737-1746, 1981.
10. S.K. Aggarwal and W.A. Sirignano, Combustion Science and Technology, Vol. 46, pp. 289-300, 1986.
11. A. Kalma and J.B. Greenberg, International Journal of Turbo- and Jet Engines, Vol. 14, pp.201-216, 1997.
12. P.D. Ronney and G.I. Sivashinsky, SIAM Journal of Applied Mathematics, Vol.49, pp. 1029-1046, 1989.

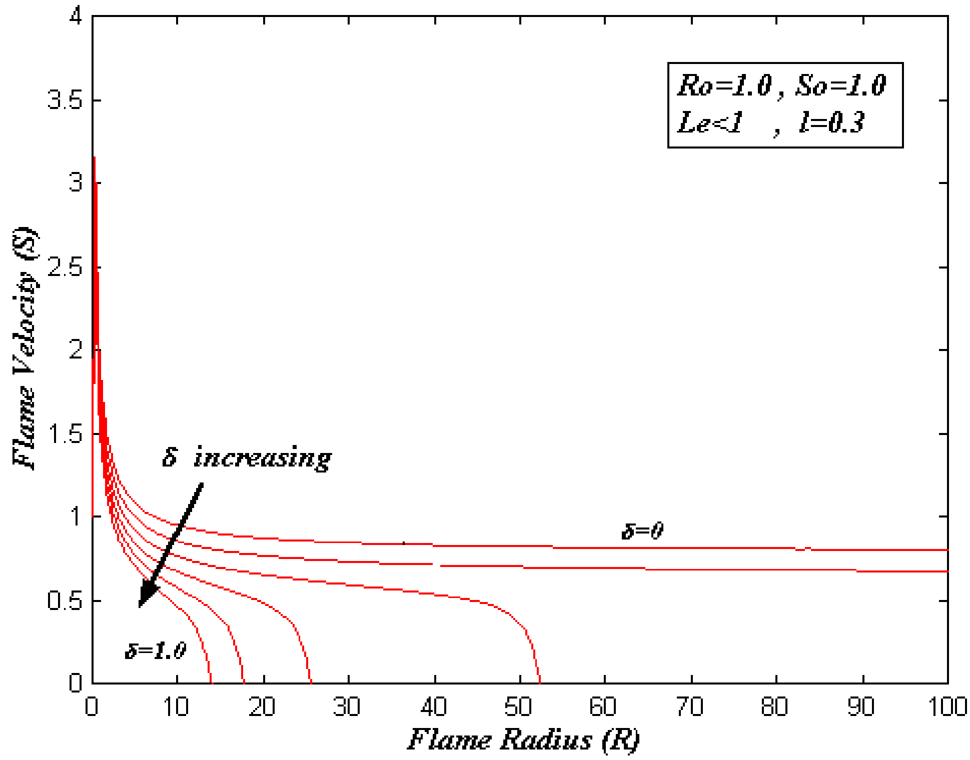

Figure 2: Effect of droplet loading on propagation and extinction of spherical spray flames.

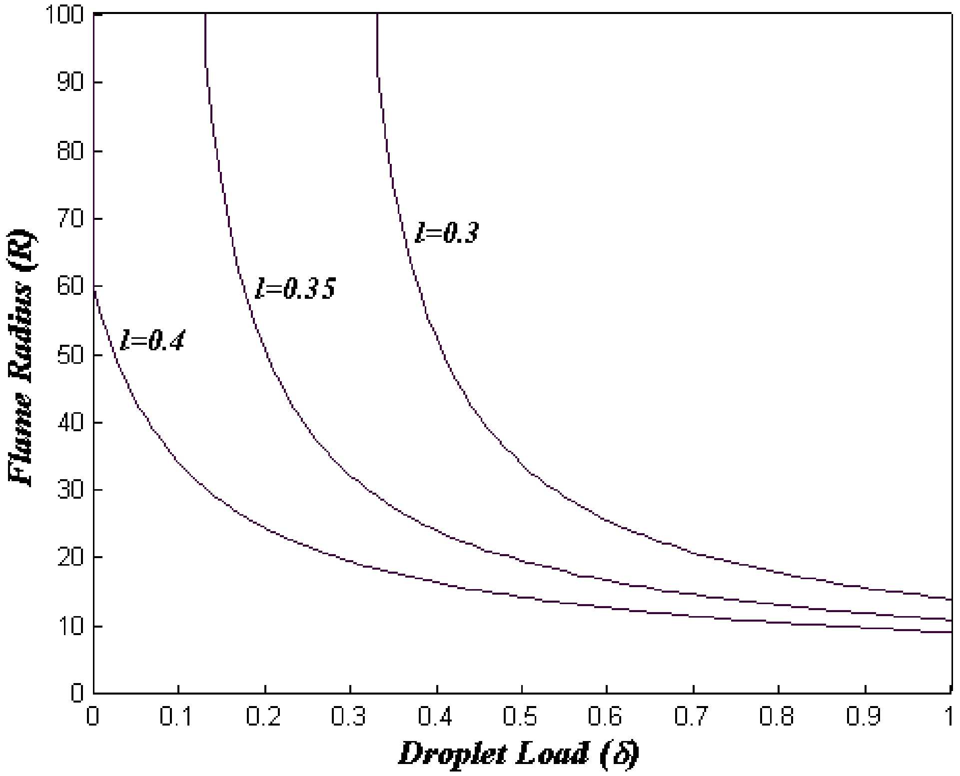

Figure 3: Influence of fuel droplet loading on critical extinction radius.