Concept of the Limit of Existence of 2-D Steady-State Structure of Fuel Liquid Film under Flame Propagation

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Two-dimensional steady-state structure of the flow in fuel liquid film on heat conductive substrate under combustion wave propagation is theoretically studied in the framework of hydrodynamic approach. Physical mechanisms of the structure forming are analyzed. It is shown that the important role belongs to thermocapillary effect. The conclusion that two-dimensional regime is possible only when the value of temperature gradient at the film surface is low enough is substantiated. The critical condition governing the transition to three-dimensional regime is derived. This condition means the balance between the velocity of the flow (induced for example by gravitation) and the velocity induced by thermocapillarity. If the temperature gradient exceeds certain critical value then the zone with reverse flow would appear according to 2-D model. In the previous works we suppose that such regime could not exist because of its instability relatively to 3-D perturbations. Indeed, the experiments with flowing liquid films upon immovable local heat source (without combustion wave) confirm the conclusion about the transition to 3-D regular flow structure when the temperature gradient is high enough. The first part of the paper is devoted to modeling 2-D film structure in critical regime. The second part of the paper deals with generalization of the problem to the case of heat source, moving with constant speed. This statement of the problem includes the flame propagation. Mathematical formulation of this problem allows us to conclude that existence of 2-D solution in this case is limited by the same condition. If the temperature gradient is more than critical then 2-D film structure would not exist. This concept substantiated at the present work explains the phenomena experimentally observed in liquid films under local heating.

Part I.

Experimental research [1, 2] shows that effects of thermocapillarity under certain conditions can significantly influence upon the character of film flow. Forming of horizontal "roller" of fluid is observed in the experiments near the place with high gradient of film surface temperature. The thickness of film decreases at heat source, the main quantity of fluid gathers in streams forming a periodical structure (with period $\lambda \approx (5 \div 8)$ mm). This spontaneous appearing of 3-D periodical flow structure is new physical phenomenon.

Let us analyze a film flow of viscous thermal-conductive incompressible liquid (with film thickness h) at the planar substrate with the angle of the inclination to the horizontal plane θ in gravitation field ($\mathbf{g} = \mathbf{x}|\mathbf{g}|\sin\theta - \mathbf{y}|\mathbf{g}|\cos\theta$ is the acceleration of free falling). Coordinate axes are directed as following: x-axis is directed along the plane in the direction of film flowing, z-axis is directed along the plane perpendicularly to the direction of film flowing, y-axis is directed normally to the plane in the side of liquid. We'll neglect the effects connected with fluxes of heat, mass and momentum through free surface of the liquid. The atmosphere pressure p^g is constant. The film flow allows one to define Reynolds number $\mathrm{Re} = Q/(\rho\nu)$, where Q is the flow rate of liquid, ρ is the density, ν is kinematic viscosity. Velocity profile doesn't depend on x and z when the temperature (T) field at the film surface is uniform, and the surface tension (σ) is constant. When a local heat source with constant power acts at the plane of substrate (the source has infinite size along z) then the thermal boundary layer is formed in the liquid, the non-uniformity (along x) of temperature field appears at film surface. This leads to the presence of surface tension gradient. At the region of remarkable value of surface tension gradient a capillary force (directed tangential to the free surface) interferes with the flow of liquid due to gravitation. The local slowing down of liquid flow near free surface results in film thickness increasing, the thickness turns out to be a function of surface tension gradient (and due to this – a function of coordinate x: $h=h(x)$). The transformation of free surface leads to establishment of a new stationary regime when thermocapillary forces are in equilibrium with gravitation. To find the distribution of thermocapillary force it needs to solve a heat problem. But in the case of non-uniform heat release and film flow the analytical solution of this problem can not be found, so we use the dependencies $T(x)$ and $\sigma(x)$ as the known from experiment functions (in [1, 2] the field of film surface

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temperature was measured). We have to note that in the experiments mentioned above the power of heat source is low enough so the liquid is thermally far from boiling regime.

To find the dependence $h(x)$ it is necessary to solve the Navier-Stokes system of equations with boundary conditions at the free surface ($y=h$), at the wall ($y=0$) and with the condition of constant liquid flow rate: $Q \equiv \rho \int_{y=0}^{y=h} u(x,y)dy = \text{const}$. As the thickness of liquid layer is small comparatively with characteristic length of free surface non-uniformity along x, then Navier-Stokes equations for stationary flow independent on z can be simplified and written in approximate form (1) introducing the following designations: the pressure $p(x,y)$, the velocity $\mathbf{u}(x,y)=\mathbf{x}u(x,y)+\mathbf{y}v(x,y)$, subscripts x and y denote derivatives, ν=const:

$$u_x + v_y = 0; \quad p_x = \rho\nu u_{yy} + \rho|\mathbf{g}|\sin\theta; \quad p_y = -\rho|\mathbf{g}|\cos\theta. \tag{1}$$

The boundary condition at the free surface in general form [3]:

$$\left[p - p^g - \sigma r^x\right] n_i = \left(\sigma'_{ik} - \sigma'^g_{ik}\right) n_k + \partial\sigma/\partial x_i; \tag{2}$$

where σ'_{ik} are the components of viscous tension tensor, r^x is the main radius of free surface curvature, n_i are components of vector $\mathbf{n}(x,y)=\mathbf{x}n_1(x,y)+\mathbf{y}n_2(x,y)$ normal to free surface, $n_1(x,y)\approx h_x$, $n_2(x,y)\approx -1$. Taking into account that, $\sigma'^g_{ik} = 0$ the condition (2) has the following form for long-wavelength approximation of stationary two-dimensional solution:

$$\begin{cases} \left[p - p^g + \sigma h_{xx}\right] n_1 = 2\rho\nu u_x n_1 + \rho\nu\left(u_y + v_x\right) n_2 + \sigma_x \;\Rightarrow\; \rho\nu u_y \approx \sigma_x, \; y = h; \\ \left[p - p^g + \sigma h_{xx}\right] n_2 = 2\rho\nu v_y n_2 + \rho\nu\left(v_x + u_y\right) n_1 \;\Rightarrow\; p = p^g - \sigma h_{xx}, \; y = h. \end{cases} \tag{3}$$

$$u(x,0) = 0 = v(x,0). \tag{4}$$

The solution of the problem (1), (3), (4) is:

$$p = p^g + (h-y)\rho|\mathbf{g}|\cos\theta - \sigma h_{xx}; \quad u = \frac{y(y-2h)}{2\nu}\{...\} + \frac{y\sigma_x}{\rho\nu}; \quad v = \frac{y^2 h_x}{2\nu}\{...\} + \frac{y^2(h-y/3)}{2\nu}\frac{d}{dx}\{...\} - \frac{y^2\sigma_{xx}}{2\rho\nu},$$

here $\{...\} \equiv \left\{h_x|\mathbf{g}|\cos\theta - |\mathbf{g}|\sin\theta - \rho^{-1}\left(\sigma h_{xx}\right)_x\right\}$. The condition Q=const allows to find the connection between h and σ which is in the other words the dependence $h(x)$ in parametric form:

$$h^3\left\{-h_x|\mathbf{g}|\cos\theta + |\mathbf{g}|\sin\theta + \rho^{-1}\sigma h_{xxx}\right\} + 3h^2\sigma_x(2\rho)^{-1} = h_\infty^3|\mathbf{g}|\sin\theta. \tag{5}$$

Here we used: $|hh_{xx}| \ll 1$, subscript ∞ denotes the condition $x\to-\infty$, where h_x, σ_x are equal to zero.

In the case of small relative deviation of $h(x)$ from h_∞ we can derive from (5) neglecting the dependence of pressure on surface tension: $h^+\sin\theta - 3^{-1}h^+_\xi\cos\theta + 2^{-1}\bar{\sigma}_\xi \approx 0$ for $h^+ \equiv (h-h_\infty)/h_\infty$, $|h^+| \ll 1$, $\xi \equiv x/h_\infty$, $\bar{\sigma}_\xi = \sigma_x/\left(\rho|\mathbf{g}|h_\infty\right)$. The last equation gives: $h^+ \approx 3(\sigma-\sigma_\infty)/\left(2\rho|\mathbf{g}|h_\infty\right)$, as $\theta\to0$, and $h^+ \approx -\sigma_x/\left(2\rho|\mathbf{g}|h_\infty\right)$, as $\theta\to\pi/2$. At the case $\theta\to0$ the form of surface looks like a "stage", but as $\theta\to\pi/2$ it is similar to a bell-like rise with maximum altitude at the point of maximum surface tension gradient in its absolute value.

If we neglect in (5) the dependence of pressure on surface tension $\left|(\sigma h_{xx})_x\left(\rho|\mathbf{g}|\right)^{-1}\right| \ll 1$ then in the case of vertical substrate plane (θ=0) we could show analytically that the condition $\left(u(x,h)\right)_{\min} = 0$ is equivalent to the following: $\left|\bar{\sigma}_\xi\right|_{\max} = 2^{-1/3}$ and $h_{\max} = 2^{2/3}h_\infty$. As one could see below (figure 2), in the critical regime equation (5) has the solution $h(x)$ with $h_{\max} < 2^{2/3}h_\infty$.

Equation (5) was solved numerically with the help of iterations using implicit finite-difference scheme and (three-point) sweep method. Physical quantities correspond to the experiments [1, 2] with 25% solution of C_2H_5OH in water, Re=2, T_∞=300 K, $\theta=\pi/2$. In this case we have: σ_∞=0,034 kg/s^2, $\partial\sigma/\partial T \approx\sim -1{,}1\cdot10^{-4}$ kg/(s^{2}K), ρ=956 kg/m^3, $\nu=1{,}8\cdot10^{-6}$ m^2/s, $h_\infty=1{,}3\cdot10^{-4}$ m. The distribution of surface tension is approximated by the formula: $\sigma_x = -|\sigma_x|_{\max}\cdot\exp\left(-(x/L)^2\right)$, $L=4h_\infty$ as $x<0$, and $L=12h_\infty$ as $x>0$ (see figure 1). The critical condition $\left(u(x,h)\right)_{\min} = 0$ means $\left|\bar{\sigma}_\xi\right|_{\max} = 0{,}92$, $(T_x)_{\max} \approx 10^4$ K/m. The results of numerical solution shown at the figures 2-4 were calculated for the critical regime, when $\left(u(x,h)\right)_{\min} = u(0,h) = 0$. If we put $\left|\bar{\sigma}_{\xi}\right|_{\max} > 0{,}92$ then calculations would give 2-D stationary solutions with $\left(u(x,h)\right)_{\min} < 0$, i.e. the 2-D model formally doesn't contain limitations on stationary solution existence. This contradicts to experimental data. So we are forced to suppose that the critical regime means the limit of stability of 2-D stationary solution. Weak 3-D perturbations begin to grow forming periodic stream-like flow structure. The instability has to be local because of the local character of heat release. One could expect the instability in the neighborhood of x=0, where the thermocapillary effect is significant. In the region, where $\sigma_x \to 0$, 3-D perturbations have to be damped due to energy dissipation by viscosity and thermal conductivity. The instability could appear only in critical regime, when x-component of liquid velocity locally tends to zero. In other case if $\left(u(x,h)\right)_{\min} > 0$ infinitesimal perturbations would not have enough time to grow because the flow would transfer them down stream to the zone with $\sigma_x \to 0$, where the amplitude of perturbations would decrease.

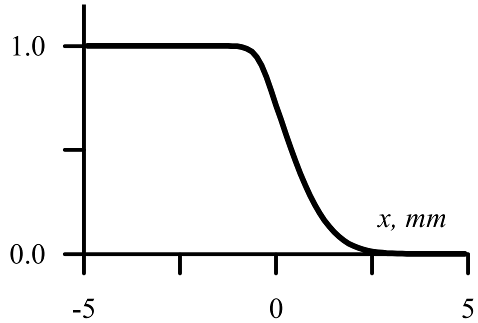

Figure 1. Distribution of the surface tension $\left(\sigma(x)-\sigma(\infty)\right)/\left(\sigma(-\infty)-\sigma(\infty)\right)$.

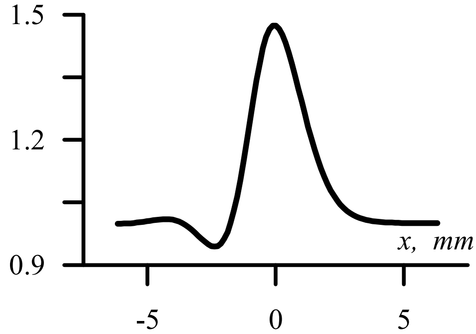

Figure 2. The coordinate of the free surface $h(x)/h_\infty$.

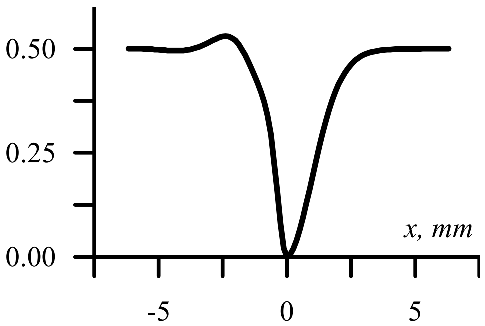

Figure 3. The x-component of velocity of liquid at the free surface y=h $u(x,h)\cdot\nu\left(|\mathbf{g}|h_\infty^2\right)^{-1}$.

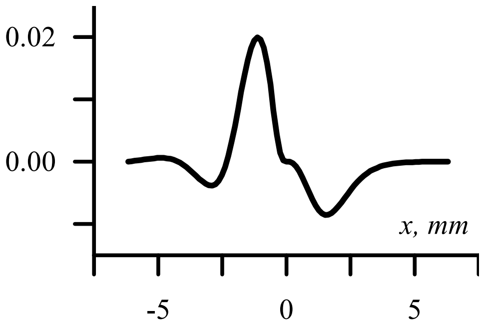

Figure 4. The y-component of velocity of liquid at the free surface y=h $v(x,h)\cdot\nu\left(|\mathbf{g}|h_\infty^2\right)^{-1}$.

The results of numerical solution are in good quantitative agreement with measurements [1, 2, 4]. For example the relative difference between curve $h(x)$ (figure 2) and measured film thickness [4] is less than 10%. The better correspondence can be obtained as the temperature dependence of viscosity $\nu(T)$ is taken into account [4]. But in that case we can use only numerical methods and lose the possibilities to make general conclusions on the base of analytical results.

Part II.

The similar situation takes place when a heat source is moving. If its speed doesn't depend on film thickness then it is possible to derive analytically the critical condition of existence of 2-D steady-state regime.

It was communicated in the recent work [5] about observation of a new phenomenon of spontaneous forming of regular 3-D flow structure in burning liquid film. Thin ($h \le 10^{-5}$ m) film of fuel liquid was placed on metal substrate possessing high thermal conductivity and inclined with different angles θ to horizontal plane.

After initiation of reaction the combustion wave was formed. This wave was propagating in direction of x-axis from the upper end of the substrate in steady-state regime (with the speed $c \geq 10^{-2}$ m/s approximately independent on θ). In experiments it was observed a local increasing of film thickness before the flame front, i.e. the horizontal roller of liquid was formed. The zone with length $\Delta x \approx 5\cdot 10^{-2}$ m was appearing at the front of the roller with periodical in direction of z-axis changing of film thickness (period was equal to $\lambda \approx (5\div 7)\cdot 10^{-3}$ m) [5, 6]. This means that 2-D structure of film flow is locally unstable. Let us analyze the physical mechanism governing the limits of 2-D regime existence.

We denote the laboratory frame of reference by (x, y, z, t) and the accompanying frame of reference (where the combustion wave doesn't move and regime is stationary) by (χ, ψ, η, τ), i.e. $\chi = x - ct$, $\psi = y$, $\eta = z$, $\tau = t$. Then $\partial/\partial y = \partial/\partial\psi$, $\partial/\partial z = \partial/\partial\eta$, $\partial/\partial x = \partial/\partial\chi$, $\partial/\partial t = \partial/\partial\tau - c\,\partial/\partial\chi$. Let $\chi \approx 0$ corresponds to flame front position. Combustion zone is placed at $\chi < 0$, and horizontal roller of liquid – at $\chi > 0$. Heat released due to chemical reaction near the film surface (at $\chi < 0$) is rapidly transferred through the thin liquid layer into substrate. The substrate due to high thermal conductivity rapidly transfers the heat to the area $\chi > 0$ heating the cold fuel liquid. This causes intensive evaporation. After the achievement of necessary concentration and temperature in the atmosphere above film surface the mixture of fuel vapor and oxidant is initiated. That is apparently the mechanism of propagation of self-sustained 2-D combustion wave [5 – 7].

As we are analyzing the physical situation in very thick films, so one can notes that the characteristic velocity of flow induced by gravitation is insignificant comparatively with the wave propagation velocity: $c \;?\; u_g \sim h^2\nu^{-1}|\mathbf{g}|\sin\theta$. Then in moving frame of reference we can observe 1-D flow of cold liquid coming with velocity $u=-c$ from $\chi \to +\infty$ to the warmed part of substrate. The temperature at the surface of this liquid at $\chi \to +0$ is increasing, and surface tension (σ) decreases. The induced thermocapillary force acts in the direction opposite to the flow and causes flow slowing down. Conservation of the flow rate (Q) of incompressible liquid leads to local increasing of the film thickness in area where $d\sigma/d\chi > 0$. So the thermocapillary effect results in horizontal liquid roller forming. But what is the reason of limits of this 2-D structure existence? Let us study this problem in frame of hydrodynamic approach, without account of such details as heat and mass transfer between phases and also chemical kinetics. So we shall take into account in our mathematical model only the flow of liquid and the non-uniform temperature field at the film surface, which is the consequence of all effects mentioned above.

The governing equations for 2-D steady-state uniform in η-direction long wavelength $\left(|dh/d\chi| = 1\right)$ solution have the following form:

$$p_\chi = \rho\nu u_{\psi\psi} + \rho|\mathbf{g}|\sin\theta - \rho c v_\psi;\quad p_\psi = -\rho|\mathbf{g}|\cos\theta + \rho c v_\chi;\quad u_\chi + v_\psi = 0. \tag{6}$$

The equations (6) are complemented by the boundary conditions:

$$u = -c,\quad v = 0,\quad \text{as}\quad \psi = 0;\quad v = (u-c)h_\chi,\quad \rho\nu u_\psi = \sigma_\chi,\quad p = p^g - \sigma h_{\chi\chi},\quad \text{as}\quad \psi = h. \tag{7}$$

If $|\mathbf{g}|\cos\theta \;?\; |cv_\chi| \sim \Delta h c^2(\Delta\chi)^{-2}$ ($\Delta h \sim h_\infty$ is characteristic change of film thickness at the distance $\Delta\chi$), then $p \approx p^g + \rho(h-\psi)|\mathbf{g}|\sin\theta - \sigma h_{\chi\chi}$. After integration of the first equation from (6) we obtain: $\nu u_\psi \approx cv + A(\chi) + \psi\left\{h_\chi|\mathbf{g}|\cos\theta - |\mathbf{g}|\sin\theta - \rho^{-1}\left(h_{\chi\chi}\sigma\right)_\chi\right\}$. We can neglect here the term cv comparatively with νu_ψ because of $|\nu u_\psi| \sim \nu c/h \;?\; |cv| \sim c^2|h_\chi|$. After the next integration using the boundary conditions we have: $\nu u \approx -\nu c + \psi\sigma_\chi/\rho + \left(\psi^2/2 - h\psi\right)\left\{h_\chi|\mathbf{g}|\cos\theta - |\mathbf{g}|\sin\theta - \rho^{-1}\left(h_{\chi\chi}\sigma\right)_\chi\right\}$. The third condition in (7) is equivalent to conservation of flow rate $Q = \rho\int_0^h u\,d\psi = -\rho c h_\infty$. This results in the equation for coordinate of free surface:

$$h^3\left\{-h_\chi|\mathbf{g}|\cos\theta + \rho^{-1}\sigma h_{\chi\chi\chi}\right\} + 3h^2\sigma_\chi(2\rho)^{-1} - 3\nu c\left(h - h_\infty\right) = 0.$$

Here we used the relations assumed and substantiated above: $c \;?\; u_g$, $|\sigma_\chi h_{\chi\chi}| = |\sigma h_{\chi\chi\chi}|$. This equation would coincide with equation (5) if $c = u_g$. Because of the assumption of long wavelength structure of solution we also have: $c\nu$? $h^2 h_\chi |\mathbf{g}|\cos\theta$ and $|\sigma_\chi|$? $|\sigma h h_{\chi\chi\chi}|$. Then the last equation can be simplified and written as the following:

$$2\rho\nu c\left(h - h_\infty\right) = h^2\sigma_\chi, \tag{8}$$

and also the solutions for p, u, and v are: $p \approx p^g$, $u \approx -c + \psi\sigma_\chi/(\rho\nu)$, $v \approx \psi^2\sigma_{\chi\chi}/(2\rho\nu)$. To verify the used assumptions one can take the following characteristic scales of physical values: $h \sim 10^{-5}$ m, $c \sim 10^{-2}$ m/s, $|\mathbf{g}| \sim 10$ m/s^2, $\sigma \sim 10^{-2}$ kg/s^2, $\Delta\sigma \sim 10^{-3}$ kg/s^2, $\nu \sim 10^{-6}$ m^2/s, $|d\sigma/dT| \sim 10^{-4}$ kg/(s^{2}K), $\rho \sim 10^3$ kg/m^3. It is easy to find that the derived equation (8) has a real solution only under the condition:

$$0 \geq 2h_\infty\sigma_\chi\left(\rho\nu\right)^{-1} - c = 2\left(u\left(\chi, h\right) + c\right)h_\infty/h - c. \tag{9}$$

This condition expresses the limit of existence of 2-D steady-state regime. It can be find from (8) that the critical condition (9) corresponds to $h = 2h_\infty$, and $u(\chi, h) = 0$. So one can expect that 2-D regime becomes locally unstable when the speed of liquid at the film surface equals to zero, i.e. the speed induced by thermocapillary effect achieves the value of flow speed (c). It is possible to estimate the value of local temperature gradient needed for stop of liquid flow at the free surface: $|T_\chi| \approx |dT/d\sigma|\rho\nu c/(2h_\infty) \sim 10^4$ K/m. This estimation is in good quantitative agreement with experimental data [5, 6, 1, 2]. We could suppose that the further increasing of heat flux and temperature gradient have to result in qualitatively new flow structure connected with the transition to a 3-D regime.

The analytical and numerical results described in Part I and Part II of the paper confirm the conclusion that critical regime of film flow with non-uniform temperature field at the surface is actually limiting the existence of 2-D stationary solution. If the liquid is locally slowed down and stopped due to thermocapillary effect then small 3-D perturbations can increase creating a stream-like periodic flow structure. This concept provides the explanation for experimentally observed new phenomena [1, 2, 5, 6]. It is important that our conclusions are based on 2-D model, which is in good agreement with experimental measurements. This allows us to be sure that the concept corresponds to reality.

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