Analysis of Acoustic Wave Transmission through Turbulent, Premixed Flames

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Introduction

This extended abstract describes an analysis of the characteristics of acoustic waves transmitted through turbulent flames. It is part of ongoing work to better understand the interactions of acoustic waves and turbulent flames. Such interactions play an important role in the characteristic unsteadiness of turbulent combustion processes that occur in a wide range of processing, power generating and propulsion applications. They arise because acoustic waves incident from other parts of the flame or from external sources impinge upon the flame and are scattered because of the significant change in sound speed and density at the flame front. The characteristics of the scattered waves are complex, due to the fact that they are interacting with a dynamic flame surface that is convoluted over a broad range of length and time scales; e.g., flames have been found to radiate acoustic waves between approximately 100 Hz – 25 kHz [1].

The work reported here is an extension of previously reported analyses of acoustic wave scattering by turbulent flames [3, 4]. These prior studies considered the idealized geometry shown in Fig. 1, consisting of a time varying, wrinkled flame surface whose average position is flat. These studies solved an integral equation formulation of the acoustic wave equation, that is similar in approach to studies of wave scattering from rough surfaces such as the ocean or arctic ice [2]. Reference [3] describes an analysis of the coherent, or average, wave field reflected by the flame (i.e., the field in Region 1, see Fig. 1). The diffuse field characteristics of the reflected field are discussed in Ref. [4]. These analyses show that the temporal and spatial characteristics of the reflected field depend upon the frequency of the incident wave, angle of incidence, statistical characteristics of the flame position and its spatial derivative, the temperature jump across the flame, and the temporal spectral characteristics of the flame position. These analyses are incomplete, however, in that they only analyze the characteristics of the acoustic field on one side of the flame and do not consider the characteristics of the transmitted wave field (i.e, the field in Region 2, see Fig. 1). As such, this paper describes an analysis of the coherent transmitted wave field.

Problem Statement and Basic Assumptions

The geometry under consideration is shown in Fig. 1. It consists of a time varying, wrinkled flame surface whose position is described by the equation $f(\vec{x}_s, t) = 0$ and whose average position is flat. The following basic assumptions are made in the analysis: (1) the flame has a thickness, δ, which is much smaller than an acoustic wavelength and, thus, can be treated as a surface of discontinuity, (2) outside of the flame itself, the flow field is isothermal, has a low Mach number, M, and is composed of a perfect gas; as such, mean flow affects (such as wave scattering by turbulent flow fluctuations) on acoustic wave propagation are neglected, (3) the acoustic field can be approximately described with the single scattering Kirchoff approximation (explained below), (4) the time scales over which flame surface properties and movement occurs is long relative to that of the acoustic period (see Ref. [3]), and (5) flame displacement due to the incident wave disturbance is small relative to that in the absence of the incident wave (see Ref. [3]).

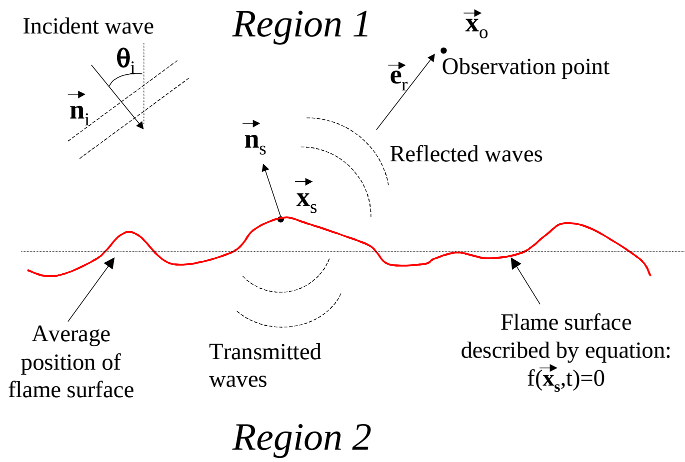

Figure 1. Schematic illustrating flame surface and acoustic field quantities.

With these basic assumptions, the acoustic field up and downstream of the flame can be described by the Kirchoff – Helmholtz integral (KHI) equation [5]:

Region 1: $$p'(\vec{x}_o,t)=p_i'(\vec{x}_o,t)+\frac{1}{4\pi}\iint_{S_s}\left(\bar{\rho}_1\frac{\frac{\partial}{\partial t}(\vec{n}_{s1}\cdot\vec{u}_1'(\vec{x}_s,t-R/\bar{c}_1))}{R}+\frac{\vec{e}_r\cdot\vec{n}_{s1}}{R}\left(\frac{1}{\bar{c}_1}\frac{\partial}{\partial t}+\frac{1}{R}\right)p_1'(\vec{x}_s,t-R/\bar{c}_1)\right)dS_s \quad [1]$$

Region 2: $$p'(\vec{x}_o,t)=\frac{1}{4\pi}\iint_{S_s}\left(\bar{\rho}_2\frac{\frac{\partial}{\partial t}(\vec{n}_{s2}\cdot\vec{u}_2'(\vec{x}_s,t-R/\bar{c}_2))}{R}+\frac{\vec{e}_r\cdot\vec{n}_{s2}}{R}\left(\frac{1}{\bar{c}_2}\frac{\partial}{\partial t}+\frac{1}{R}\right)p_2'(\vec{x}_s,t-R/\bar{c}_2)\right)dS_s \quad [2]$$

where p_i', $\vec{n}_s$, $\vec{x}_o$, $\vec{x}_s$, and $\bar{\rho}$ denote the incident wave, instantaneous flame surface normal vector, observation point, flame surface point, and the average gas density, respectively. In addition, the vector $\vec{e}_r$ points from the surface point, $\vec{x}_s$, to the observation point, $\vec{x}_o$, and R denotes the distance between the source and observation point, $R=|\vec{x}_s-\vec{x}_o|$. The subscripts 1 and 2 denote the value of the quantity in Regions 1 and 2. The surface integral is carried out over the surface of integration. In this case, this is over the flame surface, $\vec{x}_s$, although it could include other boundaries, such as combustor walls.

Equations (1-2) shows that the scattered acoustic field is determined by the value of the acoustic velocity and pressure at the surface of the flame. These quantities are not independent of each other; thus solving Eqs. (1-2) for the scattered pressure requires first solving for the pressure and velocity on the flame surface. This requires solving an integral equation which, in general, must be done numerically. In certain limiting cases, approximate analytical solutions to the Kirchhoff-Helmholtz integral equation can be obtained [5]. In this paper, we use the single scattering Kirchoff approximation to provide an approximate relationship between the acoustic pressure and velocity at the flame, see Assumption (3).

The basic Kirchoff approximation assumes that the acoustic field on the scattering surface is approximately equal to its value when the surface is flat. The single scattering approximation assumes that the scattered waves do not re-interact with the flame and, thus, neglects multiple scattering. The theory is exact for an infinitely long, smooth, plane scattering surface. As discussed in the literature [2], it is approximately true when the radii of curvature of the surface, Ra_s, is significantly larger than the wavelength of the disturbance, λ; i.e., the theory applies to disturbances with short wavelengths and, thus, at high frequencies. In this case, the scattering surface "looks" plane to the incident acoustic wave.

Denoting the surface reflection coefficient of the smooth scattering surface as $V(\vec{x}_s,t)$, the surface pressure and velocity are given in the single scattering Kirchoff approximation as:

$$p'(\vec{x}_s,t)=p_i'(\vec{x}_s,t)(1+V(\vec{x}_s,t))$$
$$\vec{n}_s\cdot\vec{u}'(\vec{x}_s,t)=\vec{n}_s\cdot\vec{u}_i'(\vec{x}_s,t)(1-V(\vec{x}_s,t)) \qquad [3]$$

It is shown in Ref. [3] that within the approximations of this analysis, the reflection coefficient is given by:

$$V(\vec{x}_s,t)=\frac{\dfrac{\bar{\rho}_2\bar{c}_2}{\bar{\rho}_1\bar{c}_1}(\vec{n}_i\cdot\vec{n}_{s1})-\sqrt{1-(\dfrac{\bar{c}_2}{\bar{c}_1})^2(1-(\vec{n}_i\cdot\vec{n}_{s1})^2)}}{\dfrac{\bar{\rho}_2\bar{c}_2}{\bar{\rho}_1\bar{c}_1}(\vec{n}_i\cdot\vec{n}_{s1})+\sqrt{1-(\dfrac{\bar{c}_2}{\bar{c}_1})^2(1-(\vec{n}_i\cdot\vec{n}_{s1})^2)}} \qquad [4]$$

Neglecting changes in molecular weight, specific heats ratio, and pressure of the gases across the flame, the density and sound speed ratios can be written in terms of the temperature ratio across the flame. Defining $\Lambda=T_2/T_1$, then $\dfrac{\bar{\rho}_2\bar{c}_2}{\bar{\rho}_1\bar{c}_1}=\sqrt{\dfrac{1}{\Lambda}}$ $\quad \dfrac{\bar{c}_2}{\bar{c}_1}=\sqrt{\Lambda}$.

Equations (1,3-4) were used in Ref. [3] to analyze the characteristics of the wave field in Region 1. Given the values of the acoustic pressure and velocity at the surface of the flame on the side on which the incident wave originates, the acoustic field on the transmission side of the flame (i.e., in Region 2, see Fig. 1) can be determined from matching conditions across the flame surface [6]. Neglecting terms of the order of the flame speed Mach number and higher, these matching conditions take the form [6]:

Momentum: $$p_1'(\vec{x}_s,t)=p_2'(\vec{x}_s,t) \qquad [5]$$

Energy: $$u'_{2,n}(\vec{x}_s,t)=u'_{1,n}(\vec{x}_s,t) \qquad [6]$$

It should be noted that more generally, the energy matching condition takes the form: $u'_{2,n}(\vec{x}_s,t)-u'_{1,n}(\vec{x}_s,t)=(T_2/T_1-1)S_1'$, where S_1' denotes the fluctuating flame speed. These flame speed fluctuations occur through chemical kinetic and flame strain sensitivities to the fluctuating pressure, temperature, and strain rate. As shown in Ref. [6], the fluctuations due to kinetic processes are of the order of the flame speed Mach number and, thus, are neglected to the order of approximation of this study. The flame strain terms can be shown to be of the order of $\mathbf{L}/\lambda$, where $\mathbf{L}$ is the Markstein length [7] and λ is the acoustic wavelength. Assuming that the Markstein length is of the order of the flame thickness, this term is also neglected to the order of approximation of this study, see assumption 1.

Analysis

Given the assumptions in the prior section, the scattered field can be explicitly written as an integral over the rough surface, rather than an integral equation in the more general case. The procedure we use to obtain the transmitted field is entirely analogous to that used in Ref. [3] to find the reflected field, except Eq. (2), rather than Eq. (1) is analyzed. After substituting Eqs. (3-6) into Eq. (2) and some algebra we obtain the following result for the coherent transmitted field:

$$p_2'(x_o,t)=p_{2,sm}'(x_o,t)\frac{<T>}{T_{sm}}F_2 \qquad [7]$$

where $p_{2,sm}'$ is the value of the pressure at the observation point if the flame surface were smooth and:

$$<T>=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{B(\varsigma_x,\varsigma_y)}{\sqrt{\Lambda(1-\Lambda\sin^2\theta_i)}}\frac{(\varsigma_x\sin\theta_i+\cos\theta_i)(\sqrt{(1+|\nabla\varsigma|^2)-\Lambda(1+|\nabla\varsigma|^2-(\varsigma_x\sin\theta_i+\cos\theta_i)^2)}+\sqrt{\Lambda}\varsigma_x\sin\theta_i+\sqrt{1-\Lambda\sin^2\theta_i})}{\frac{1}{\sqrt{\Lambda}}(\varsigma_x\sin\theta_i+\cos\theta_i)+(\sqrt{(1+|\nabla\varsigma|^2)-\Lambda(1+|\nabla\varsigma|^2-(\varsigma_x\sin\theta_i+\cos\theta_i)^2)}}d\varsigma_x d\varsigma_y$$

[8]

$$T_{sm} = \frac{2\cos\theta_i}{\cos\theta_i + \sqrt{\Lambda(1-\Lambda\sin^2\theta_{i)}}} \quad [9]$$

$$F_2 = \int_{-\infty}^{\infty} B(\varsigma)e^{-ik_1(\cos\theta_i - \sqrt{1/\Lambda-\sin^2\theta_i})\varsigma}d\varsigma \quad [10]$$

where the flame surface is assumed to be a single valued function of x_s and y_s and can be written as:

$$f(\vec{x}_s, t) = z - \varsigma(x_s, y_s, t) = 0 \quad [11]$$

<T> denotes the "average" transmission coefficient while T_{sm} denotes its values for a smooth flame surface. In writing Eqs. (7-10), it has been assumed that the statistical characteristics of the surface position are stationary. As such, it can be shown that the wave field vanishes in non-specular directions in the limit as the integration surface becomes very large relative to a wavelength.

Equations (7-10) are general results describing the dependence of the coherent scattered acoustic field in terms of the statistical characteristics of the flame position (ζ) and its spatial gradient (ζ_x and ζ_y), the angle of the incident wave, the wave number of the disturbance (k), and the temperature ratio across the flame (Λ). Note that the statistical characteristics of F_2 and <T> only depend upon the statistics of the surface height, ζ, and its gradient, ζ_x, respectively. We next present typical results explicitly illustrating these dependencies for one-dimensional flame surfaces with Gaussian characteristics. The following PDF's were used in these results:

$$B(\varsigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\varsigma^2/2\sigma^2} \Rightarrow F_2 = e^{-(k_1\sigma)^2(\cos\theta_i - \sqrt{1/\Lambda-\sin^2\theta_i})^2/2} \quad [12]$$

$$\zeta_x < 5\sigma_\Delta: \quad B(\varsigma_x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-\varsigma_x^2/2\sigma_x^2} \quad [13]$$

$$\zeta_x < 5\sigma_x: \quad B(\zeta_x) = 0$$

where σ^2 and $\sigma_x^{\,2}$ denote the variance of the surface position and gradient. A clipped Gaussian PDF is used for the gradient PDF because of the otherwise finite probability of very large gradients where the developed theory is not applicable. The results for <T> were obtained with numerical quadrature of Eq. (8).

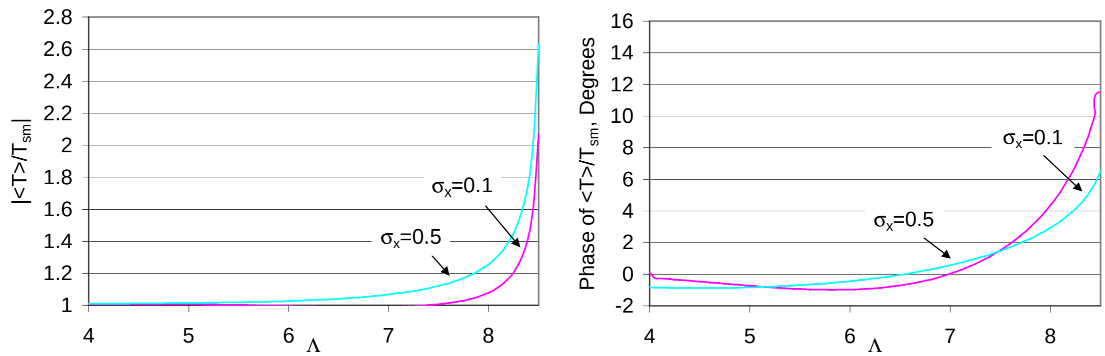

Figure 2 Dependence of <T>/T_{sm} amplitude and phase upon Λ for several values of the spatial slope variance of the flame surface. Wave incident from upstream at an angle of 20 degrees.

Figure 2 plots the dependence of <T>/T_{sm} upon the temperature ratio across the flame for several different roughness values for a case where the incident wave originates upstream of the flame (i.e., on the unreacted side) at an angle of θ_i=20. In interpreting the results shown in the figure, it is important to note the presence of a "cutoff" angle; i.e., a disturbance with an angle of incidence that is greater than $\theta_i = \sin^{-1}(c_1/c_2) = \sin^{-1}(1/\Lambda)^{0.5}$ does not transmit acoustic energy through the flame if it is completely smooth; e.g., for θ_i=20 degrees, acoustic energy is not transmitted through a smooth surface for Λ values above 8.55 (i.e., $p_{2,sm}$=0 in the farfield). This cutoff phenomenon is well known in acoustics and optics and occurs when a disturbance propagates from a medium of lower to higher sound speed. There is a discontinuous change in the characteristics of the acoustic field when θ_i is just above and below this angle. As such, only values of the temperature ratio up to this Λ=8.55 cutoff are shown in Fig. 2. The figure shows that the magnitude and phase of $\langle T\rangle/T_{sm}$ differs from unity and zero degrees only near Λ values in the vicinity of cutoff, which for the incident angle of θ_i=20 degrees in this figure is about Λ=8.5. This implies that for most of the temperature ratio's shown in the figure, the effect of flame roughness upon the "average" transmission coefficient is negligible.

In contrast, the term F_2 exhibits sensitivity to flame surface roughness over a wide temperature range, as can be seen in Fig. 3. It physically reflects the fact that the phase of the transmitted wave along the flame surface differs from its value when the surface is smooth. These phase differences arise because of the differences in surface height and, thus, distance the incident wave travels before impinging upon the flame.

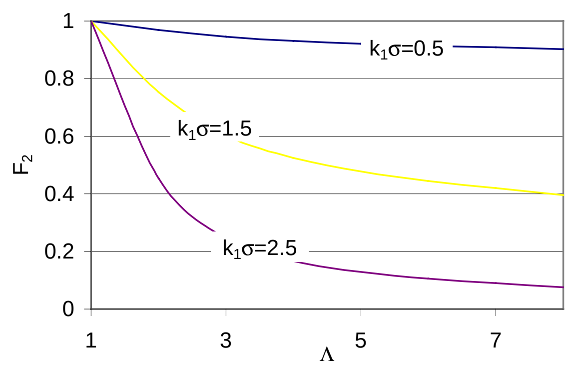

Figure 3 Dependence of F_2 upon Λ for several values of $k_1\sigma$. Wave incident from upstream at an angle of 20 degrees.

Figure 3 shows that the magnitude of F_2 decreases with increasing roughness ($k_1\sigma$) and temperature ratio (Λ). In contrast, the results presented in Ref. [3] show that the amplitude of the analogous term for the field in Region 1 (i.e., the reflected field) does not depend upon the temperature ratio across the flame but exhibits a similar dependence upon surface roughness. This additional temperature ratio dependence is due to the fact the time required for a disturbance to reach the observation point (and, hence, its phase) depends upon the speed of sound in both regions. Increasing differences in temperature ratio cause the phase of disturbances originating at different points along the flame surface to differ more from each other at the observation point if the flame surface is not smooth.

References

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