Modeling and Control of Mixing in a Transverse Jet

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The mixing properties of the transverse jet—a jet issuing normally into a uniform crossflow—are important to a variety of engineering applications. Transverse jets may function as sources of fuel in industrial furnaces, or as diluent jets for blade cooling or exhaust gas cooling in industrial or airborne gas turbines. Other industrial applications include pipe tee mixers and jets of oil and gas entering the flow in oil wells. Transverse jets have also been studied extensively for their relevance to V/STOL aerodynamics and to environmental problems such as pollutant dispersion from chimneys or the discharge of effluents into the ocean.

The mixing rate between jet and crossflow is a key design constraint in many of the systems mentioned above. Enhancement of this mixing rate can lead to significant improvements in many performance aspects. In gas turbines, for instance, mixing enhancement is essential to achieving a wider range of operability, lower emissions, smaller size, and lower noise output. The ultimate objective of this work is to develop control strategies for the transverse jet with the goal of enhancing or reducing the mixing rate between the jet fluid and the crossflow. To this end, we plan to develop models that capture the fundamental processes responsible for entrainment and subsequent mixing of fluid from the crossflow into the jet and the response of these processes to actuation.

Prospects for control are supported by numerous experimental studies which report the presence of a small number of large-scale coherent structures in the flow field. Experimental observations by Fric and Roshko [1] identify four such structures in the transverse jet: jet shear layer vortices; "wake vortices" arising from interaction between the jet and the channel wall boundary layer; horseshoe vortices that wrap around the jet exit; and a counter-rotating vortex pair that forms as the jet bends into the crossflow, persisting far downstream. The first two structures are inherently unsteady, while the last two are present in the mean flow, though with significant unsteady components. The overall structure of the flow field is governed by the jet-to-crossflow momentum ratio:

$$r = \left(\frac{\rho_j V_j^2}{\rho_\infty U_\infty^2} \right)^{1/2} \tag{1}$$

written here as an effective velocity ratio. Penetration of the jet into the crossflow increases with r. For relatively strong jets (e.g., $r > 4$), typical of engineered systems, interactions with the wall downstream of the jet nozzle thus have a diminished role in mixing [2]. Our modeling efforts therefore focus on the first and last of the coherent vortical structures listed above: the jet shear layer and the counter-rotating vortex pair. We plan to capture the dynamics of these structures at high Reynolds number, to understand their formation mechanisms and follow them downstream as they mature. For simplicity, we focus on incompressible flow, and for relevance to mixing in engineered systems we consider r in the range of 5–15.

We construct a computational model of a spatially evolving transverse jet in which actuations are imposed as time-dependent boundary conditions. Arbitrary pulsation of the jet velocity is allowed, and the jet velocity profile may itself be altered in a time-dependent fashion. Indeed, the computational model must accommodate arbitrary locations for actuation, as it is yet unclear whether the best mixing enhancement is achieved through actuation in the jet or elsewhere. In this framework, computation functions both as a testbed for proposed actuation mechanisms and as an investigative tool for the fundamental physics; before control can be intelligently designed, we must understand the dynamic response of flow structures to each mode of actuation, and the resulting impact on mixing. Yet the model also must balance physical fidelity with computational efficiency. Actuation acts on the large scales of the flow, and effective actuation will modify the large scale vortical structures described above. These are the dynamics that must be resolved; calculation of all the small scales, or of events in the wall boundary layer downstream of the jet, seems unnecessary in this context.

Despite these simplifications, the transverse jet presents several subtle physical issues of relevance to mixing and dynamic response to actuation. Chief among these is the origin of the counter-rotating vortex pair (CVP). Differing accounts of the mechanism by which the counter-rotating vortices form still persist. Recent experimental work [3] suggests that the CVP is initiated just above the jet exit as jet shear layer vorticity folds onto itself and Kelvin-Helmholtz instability leads to a simultaneous roll-up. The resulting flow pattern can be interpreted as the tilting and folding of vortex rings as they are ejected from the nozzle, where the downstream side of each vortex ring is approximately aligned with the jet trajectory. Various other studies support this view [4–7]. A different, though not contradictory, mechanism [8] points to quasi-steady "hanging vortices" formed in the skewed mixing layers on lateral edges of the jet. An adverse pressure gradient causes these vortices to break down into a weak CVP.

Flow visualization studies suggest that the CVP also contains vorticity generated in the channel wall boundary layer [3]. Though the relative magnitude of this contribution must decrease with higher r, it is not clear whether jet shear layer vorticity alone is sufficient to characterize the dynamics of the CVP in our parameter range of interest. Finally, with regard to the design of actuation strategies, many of the dynamic characteristics of the transverse jet are unknown. For free jets or co-flowing jets, for instance, the jet natural modes are well-known, and actuation typically consists of exciting the jet at a corresponding frequency. Analogous modes for the transverse jet and their relation to jet properties have yet to be determined.

The specifics of our computational approach are as follows. We employ a three-dimensional vortex filament method, similar in outline to that developed by Knio and Ghoniem [9]. Filaments representing both the azimuthal (produced in the pipe boundary layer) and axial (produced in the channel wall boundary layer) components of vorticity are introduced at each timestep. Image filaments of opposite circulation are used to model behavior in the semi-infinite domain (i.e., to enforce no-flow through the wall). A potential flow solution is superimposed on the vortical velocity field in order to model the outflow of the circular jet and the crossflow. Vortical velocity is obtained by integrating the discrete, de-singularized Biot-Savart law along the filaments.

$$\boldsymbol{u}_{\omega}(\boldsymbol{x}) = -\frac{1}{4\pi}\sum_{i=1}^{N}\Gamma_i \frac{(\boldsymbol{x}-\boldsymbol{\chi}_i)\times\boldsymbol{\delta\chi}_i}{|\boldsymbol{x}-\boldsymbol{\chi}_i|^3}\kappa_{\sigma}(\boldsymbol{x}-\boldsymbol{\chi}_i) \tag{2}$$

where
$$\kappa_{\sigma}(r) \equiv \kappa(|r|/\sigma); \quad \kappa(r) = 4\pi\int_0^r \xi^2 f(\xi)d\xi \tag{3}$$

and f is the smoothing kernel of the vorticity field. Each filament is represented by a finite number of nodes $\boldsymbol{\chi}_i$, but cubic splines are used to reconstruct continuous filaments at each timestep for stability and spatial accuracy. Nodes on the filament are simply advected by the velocity field:

$$\frac{d\boldsymbol{\chi}_i}{dt} = \boldsymbol{u}(\boldsymbol{\chi}_i) \tag{4}$$

A second order predictor-corrector method is used for time-integration. Lagrangian tracking of these nodes incorporates vorticity stretching and tilting without evaluating velocity gradients, since vortex lines are material lines in this limit of inviscid flow.

The present implementation incorporates a mesh refinement scheme analogous to that in [9]. Core overlap is maintained between neighboring segments of a filament by inserting new nodes as the filament is stretched. The location of each new node is determined by the cubic spline description of the filament. Also, to suppress the development of small scales, we introduce dissipation via nonlinear core expansion, after Leonard and Chua [10].

This Lagrangian vortex method provides an attractive model of the transverse jet, first of all for its explicit link to the formation and dynamics of vortical structures in the flow. Vorticity introduced at the boundary is tracked through the flow field, providing a clear, mechanistic view of its evolution. Also, inherent in the grid-free nature of the method is a dynamic clustering of computational points only where they are needed, i.e., over the compact support of the vorticity field. Filament methods in particular have other advantages. They allow efficient evaluation of vorticity stretch, as described above, and they preserve the fundamental invariants of three-dimensional inviscid flow, conserving circulation, impulse, and helicity, and maintaining the solenoidal nature of the vorticity field.

Preliminary computational results are shown in Figure 1. In this case, $r = 7$ and the vortex filaments are injected as rings of azimuthal vorticity from the jet nozzle. Coordinates are normalized by the jet diameter, and the non-dimensional time (normalized by d/U_{∞}) is 3.20 at this snapshot. Several key flow features are visible: Clustering of vortex rings in the near field of the jet indicates rollup of the jet shear layer. As they are advected further downstream, the vortex rings tilt and bend. By tracking the time evolution of a single vortex ring isolated from the full simulation, Figure 2 reveals the tilting/bending process more clearly. Here, a single vortex filament injected into the flow at a non-dimensional time of $\tilde{t} = 1.00$ is followed until $\tilde{t} = 4.55$. As in Figure 1, $r = 7$ and each vortex filament is initially a ring of azimuthal vorticity at the nozzle. The filament's position is projected onto the x-y plane at each timestep, and successive projections are superimposed on a single plot. Consistent with earlier observations [3–5], the upstream side of the vortex ring remains in a plane essentially normal to the jet trajectory, while the downstream side tilts into the crossflow. Most of the stretching and folding is confined to the downstream side of the ring. The downstream side of the ring thus provides vorticity aligned with the jet trajectory and is the source of the incipient counter-rotating vortex pair.

These results show that the model is indeed capable of capturing the vortical structures essential to entrainment and mixing. Work is underway to examine the precise mechanisms of CVP formation, the role of axial vorticity at large *r*, and the response of vortical structures to actuation. To further elucidate the role of vortical structures in mixing processes, we will take advantage of recent methods [11] employing dynamical systems theory to identify material surfaces bounding coherent structures in turbulent flows.

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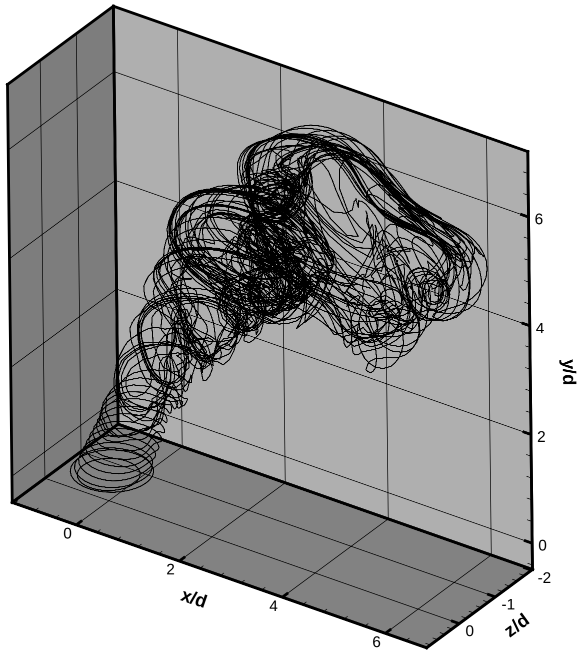

Figure 1: Perspective view of vortex filaments in the initial stages of a spatially evolving transverse jet; $r = 7$, $\tilde{t} = 3.20$. Coordinates are normalized by the jet diameter. Crossflow is in the x-direction.

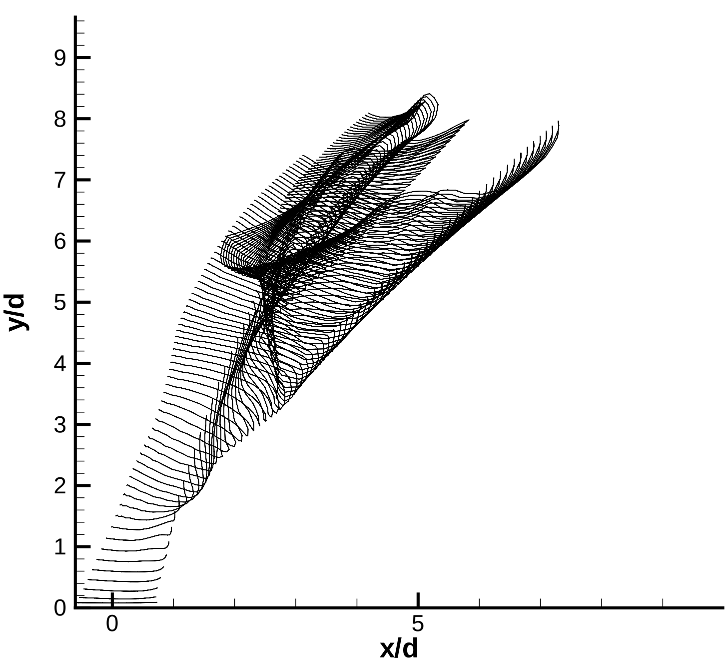

Figure 2: Time evolution of a single vortex filament in the transverse jet flow field, injected at $\tilde{t} = 1.00$ and followed until $\tilde{t} = 4.55$. Flow configuration is the same as in Figure 1.