

# Analysis of the Critical Conditions of Ignition of Gas-suspended Solid Fuel with a Heated Surface

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The present paper reports the results of the theoretical analysis of the process of ignition of gaseous suspensions with a heated body at the pulse energy supply.

The following model process is considered. At the moment  $t = 0$ , a surface heated up to  $T_{ig}$  ( $T_{ig} > T_0$ ,  $T_0$  is the initial temperature) is brought to contact with a homogeneous gaseous suspension containing  $N$  particles per volume unit. At  $t = t_{imp}$ , the surface is drawn away so that at  $t > t_{imp}$  the gas suspension is considered to be heat-insulated. At  $0 < t < t_{imp}$ , the temperature gradient causes conductive heating of the gas, which in turn warms up the solid particles. The particle temperature growth entails a sharp increase in the rate of heterogeneous interaction of the particles and the oxidant contained in the gas, which can result in the gas suspension ignition. It is supposed that at the initial temperature the reaction rate is equal to zero, the inter-particle heat exchange obeys the Newtonian law and is characterized by the coefficient of heat exchange  $\alpha$ , the heat loss to the walls round the suspension is negligibly low, the reaction is described by the first-order kinetics, there is no thermic interaction among the particles, and the oxidant transport towards the surface occurs via a diffusion mechanism.

The process is described by a one-dimensional mathematical model suggesting the difference between the particle and gas temperatures. The study was aimed at determination of such  $t_{imp} = t_{imp,cr}$  value as a function of the governing parameters that at  $t_{imp} > t_{imp,cr}$  the gas suspension is ignited, *i.e.* the combustion wave propagation is initiated, while at  $t_{imp} < t_{imp,cr}$  the process of chemical conversion could be stagnated.

The process is described by the following non-dimensional set of equations:

$$\frac{\partial \theta_g}{\partial \tau} = \frac{\partial^2 \theta_g}{\partial \xi^2} - B Z (\theta_g - \theta_p); \quad (1)$$

$$\frac{\partial \theta_p}{\partial \tau} = Z(\theta_g - \theta_p) + \frac{1}{\gamma} \frac{\partial \eta}{\partial \tau}; \quad (2)$$

$$\frac{\partial \eta}{\partial \tau} = W = \begin{cases} \gamma c_s \exp\left(\frac{\theta_p}{1 + \beta \theta_p}\right), & \text{if } \eta < 1; \\ 0, & \text{if } \eta \geq 1 \end{cases} \quad (3)$$

$$c_s = \left(1 + \gamma_1 / z \times \exp\left(\frac{\theta_p}{1 + \beta \theta_p}\right)\right)^{-1} \quad (4)$$

**Initial Conditions:**

$$\tau = 0 : \theta_p = \theta_g = -\theta_{in}, \quad \eta = 0, \quad c_s = 1, \quad (5)$$

**Boundary Conditions**

$$\begin{aligned} 0 < \tau \leq \tau_{imp}, \quad \xi = 0 : \theta_g = 0; \quad \xi \rightarrow \infty : \frac{\partial \theta_g}{\partial \xi} = 0.. \\ \tau > \tau_{imp}, \quad \xi = 0 : \frac{\partial \theta_g}{\partial \xi} = 0.; \quad \xi \rightarrow \infty : \frac{\partial \theta_g}{\partial \xi} = 0. \end{aligned}$$

The non-dimensional variables and parameters used were:

$$\begin{aligned} \theta_p &= \frac{E(T_p - T_{ig})}{RT_{ig}^2}; \quad \theta_g = \frac{E(T_g - T_{ig})}{RT_{ig}^2}; \quad \tau = \frac{t}{t_*}; \quad t_* = t_{ch} = \frac{RT_{ig}^2 c_p \rho_p r_0}{EQ \rho_g c_g k_0} \exp\left(\frac{E}{RT_{ig}}\right). \\ \xi &= \frac{x}{x_*}; \quad x_* = \sqrt{at_*}; \quad a = \frac{\lambda}{c_g \rho_g}; \quad \eta = \frac{r_0 - r}{r_0}; \quad \theta_{in} = \frac{E(T_{ig} - T_{in})}{RT_{ig}^2}; \\ B &= \frac{Nm c_p}{c_g \rho_g}, \quad \gamma = \frac{RT_{ig}^2 c_p}{EQ}, \quad \gamma_1 = \frac{RT_{ig}^2 c_g}{EQ c_0}, \quad Z = \frac{t_{ch}}{t_z} = \frac{\alpha s t_*}{m c_p}, \end{aligned}$$

Here:  $T_p, T_g$  are the particle and gas temperatures;  $x$  - is the space coordinate;  $t$  is the time;  $\rho_g$  is the gas density;  $c_0$  is the oxidant concentration in the gas;  $r$  - is the size of the non-reacted portion of the particle;  $Q$  is the reaction thermal effect per mass unit of the particle;  $c_g$  is the gas heat capacity;  $c_p, \rho_p$  are the particle heat capacity and density, respectively;  $\lambda$  is the coefficient of the gas thermal conductivity;  $a$  is the coefficient of heat diffusivity;  $N$  is the number of particles contained in the unit of the gaseous suspension volume;  $m$  is the particle weight;  $t_{ch} = t_*$  is the specific time of the chemical reaction;  $t_z$  is the specific time of the particle-gas heat exchange;  $E$  and  $k_0$  are the activation energy and the pre-exponential multiplier of the reaction;  $R$  is the universal gas constant;  $\theta, \xi, \tau$  - are non-dimensional temperature, coordinate, and time;  $\eta$  is the depth of the initial substance conversion;  $W$  is the non-dimensional reaction rate;  $Z$  - is the main parameter equal to the ratio of the specific time of the gas-particle heat exchange to that of the chemical reaction (an analog of the Semenov criterion);  $\theta_{in}$  is the non-

dimensional gradient of the initial temperatures;  $B$  – is the ratio of the volume heat capacity of the condensed to gaseous phases;  $\beta, \gamma, \gamma_1$  are the minor parameters of the combustion theory. It is assumed that the ignition temperature  $T_{ig}$  is much lower than the adiabatic temperature.

An earlier performed criterial analysis of the starting set of equations for  $t_{imp} \rightarrow \infty$  showed that dependent on the ratio of the characteristic time of the heat exchange between the particles and the gas  $t_z$  to that of the chemical reaction  $t_{ch}$  there could occur two different regimes of ignition [1]. If  $t_z$  is  $e$  ( $e = 2.72$ ) or more times is higher than  $t_{ch}$  (high ignition temperatures, low values of the parameter  $Z = t_{ch}/t_z$ ) the process of ignition proceeds via the diffusion mechanism, *i.e.* the rate of the particle interaction with the gas is determined by the diffusion of the oxidant to the particle surface and the particle temperature in the reaction zone is much higher than that of the heater even at the stage of ignition.

At  $t_{ch} > t_z$  (the duration of the heat exchange is lower than the reaction characteristic time), the kinetic regime of ignition is observed, *i.e.* the suspension is ignited at the same particle and gas temperatures and the process can be described with a one-temperature model. The specific features of both regimes were analyzed with the use of the wave theory of ignition (WTI) [2]. According to WTI, the time characteristics of ignition are presented via the parameters of the intermediate combustion wave (ICW), which is a steady-state combustion wave with a maximum gas temperature equal to that of ignition. The dependences of the time characteristics of ignition on the major parameters of the process (the particle mass concentration in the gas, the particle size, the kinetic characteristics of the reaction, the ignition temperature) were described by simple expressions.

**The duration of the ignition stage  $\tau_0$  in the kinetic (one-temperature) regime of ignition (high values of the  $Z$  parameter)[1]**

$$\tau_0 = \frac{(1+B)}{B} \left( \frac{\theta_{in}^2}{2\pi} + \theta_{in} \right)$$

$\tau_0$  is the time of the zero gradient establishment on the suspension-heater interface at  $t_{imp} \rightarrow \infty$ .

**The duration of the ignition stage in the diffusion (two-temperature) regime of ignition (low values of the  $Z$  parameter)[1]**

$$\tau_0 = \tau_{in} + \frac{\gamma_1 (1+B)}{Z B} \left( \frac{\theta_{in}^2}{2\pi} + \theta_{in} \right),$$

where the time of inert heating  $\tau_{in}$  is equal to the time of the particle transition to the diffusion regime of interaction  $\tau_d$  in the cross-section  $\xi = 0$ ,

$$\tau_d = \tau_h + \tau_{sh},$$

$$\text{where } \tau_h = -\frac{1}{Z} \ln \left( -\frac{\theta_{p,*}}{\theta_{in}} \right) \quad \tau_{sh} \approx \exp(-\theta_{p,*}).$$

The temperature  $\theta_{p,*}$  is found from the equality  $W1 = W2$ , where  $W1$  is the rate of the particle heating with the hot gas and  $W2$  is the rate of the particle self-heating:

$$W1 = \frac{d\theta_p}{d\tau} = -z\theta_p \quad W2 = \frac{d\theta_p}{d\tau} = c_s \exp\left(\frac{\theta_p}{1 + \beta\theta_p}\right)$$

According to the wave theory of ignition [2],  $\tau_{imp,cr} \sim \tau_0$  ( $\tau_{imp,cr}$  is the critical time of the igniting pulse action), since by this moment the necessary and sufficient conditions of the combustion wave propagation along the gaseous suspension have been already developed.

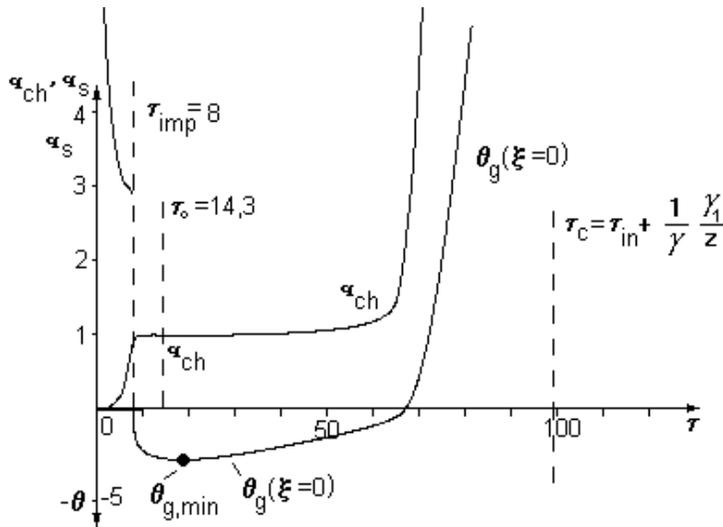
As shown by the results of numeric calculations,  $\tau_{imp,cr} = \tau_0$  for  $Z > 3$  (the kinetic one-temperature regime of ignition), while in the diffusion regime of ignition ( $Z < 1$ ) could be expected at  $\tau_{imp,cr} < \tau_0$ .

The result is the consequence of the absence of the positive reverse relation between the heat release rate and the particle temperature in the diffusion regime of interaction and the hysteresis of the ignition and stagnation conditions characteristic of the particle interaction in the oxidant medium. Figure 1 shows the development of the process of ignition of the gaseous suspension in the diffusion regime ( $\tau_{imp,cr} < \tau_{imp} < \tau_0$  and  $z = 0,75$  ; )

**Fig.1 The time-dependent changes in the surface temperature  $\theta_g(\xi = 0)$  and the integral**

**chemical heat release**  $q_{ch} = B \int_0^{\infty} c_s \exp\left(\frac{\theta_p}{1 + \beta\theta_p}\right) d\xi$  for  $\tau_{imp} = \delta > \tau_{imp,cr}$

$\tau_{imp,cr} < \tau_{imp} < \tau_{imp,cr} + o(\tau_{imp,cr})$ ,  $\theta_{in} = 10$ ;  $\gamma = 0,0012$ ;  $\beta = 0,03$ ;  $B = 0,5$ ;  $\gamma_1 = 0,075$



As seen from the Figure, after heating has been stopped ( $\tau > \tau_{imp}$ ) the gas temperature within the plane  $\xi = 0$  lowers down, achieves its minimum value  $\theta_{g,min}$  and starts growing again. Integral heat release

$q_{ch}$  is not practically changed, though the particle temperature  $\theta_p(\xi = 0)$  changes within a wide range following the changes in the gas temperature in the same cross-section ( $\theta_p(\xi = 0) - \theta_g(\xi = 0) \sim 1/\gamma_1$ ). This result is the consequence of the absence of the positive reverse relationship between the particle temperature and the reaction rate in the diffusion regime.

The numerical calculations show that the  $\tau_{imp,cr}$  value is dependent on the ratio of  $\theta_{g,min}, \theta_{g,exp}, \theta_{g,ext}$ , where  $\theta_{g,exp}$  and  $\theta_{g,ext}$  are the gas temperatures on the Semenov diagram corresponding to the conditions of the particle ignition and extinction [3]. The more detailed calculations showed that the minimum ignition time can be achieved at  $\tau_{imp,cr} = \tau_0$ . The formulas for the ignition delay time and the critical conditions of ignition at  $\tau_{imp,cr} < \tau_0$  were derived.

### Conclusion

1. Two mechanisms of the critical phenomena arising at the pulse energy supply have been observed.
2. The formulas that allow determination of the necessary and sufficient conditions of the gaseous suspension ignition with the heated body at the pulse energy supply have been obtained.
3. At high values of the heat exchange coefficient  $Z$  the critical time of the pulse is equal to the time of establishment of the zero gradient on the heater-gaseous suspension interface  $\tau_0$ .
4. At low values of the heat exchange coefficient  $Z$ , the critical time of the pulse is less than  $\tau_0$  and depends on the particle weight concentration  $B$  in the gaseous suspension and the non-dimensional thermal effect  $1/\gamma$  of the particle interaction with the oxidant contained in the gas.
5. The results of numeric calculations have shown the applicability of the approximate formulas to determination of the minimum pulse duration required for the gaseous suspension ignition.

### References.

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