# On the Exit Boundary Condition for One-dimensional Calculations of Pulsed Detonation Engine Performance

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## Abstract

In one-dimensional calculations of pulsed detonation engine (PDE) performance, the exit boundary condition is frequently taken to be a constant static pressure. In reality, for an isolated detonation tube, after the detonation wave arrives at the exit plane, there will be a region of high pressure, which will gradually return to ambient pressure as an almost spherical shock wave expands away from the exit, and weakens. Initially, the flow is supersonic, unaffected by external pressure, but later becomes subsonic. Previous authors have accounted for this situation either by assuming the subsonic decay to be a relaxation phenomenon, or by running a two-dimensional calculation first, including a domain external to the tube, and using the resulting exit pressure temporal distribution at the tube exit as the boundary condition for one-dimensional calculations. These calculations show that the increased pressure does affect the PDE performance.

In the present work, a simple model of the exit process is described. The planar shock wave emerging from the tube is assumed to transform into a spherical shock wave. The initial strength of the spherical shock wave is determined from comparison with experimental results. Its subsequent propagation, and resulting pressure at the tube exit, is given by a numerical blast wave calculation. The model agrees reasonably well with other, limited results.

Finally, the model was used as the exit boundary condition for a one-dimensional calculation of PDE performance to obtain the thrust wall pressure for a hydrogen-air detonation in tubes of length to diameter ratio ( L/D ) of 4, and 10, as well as for the original, constant pressure boundary condition. The modified boundary condition had no performance impact for values of L/D > 10, and moderate impact for L/D = 4.

## Introduction

At some point in the cycle of a pulsed detonation engine, a strong compression wave will arrive at the exit of the tube, and propagate into the region beyond the detonation tube. This wave will either be the detonation wave itself, if the tube is completely filled with combustible mixture, or the transmitted shock from the interaction of the detonation wave with the combustible gas-air interface. In calculating the cycle, it is necessary to know how this wave reflects at the tube exit. For weak waves, it is well known that a shock wave reflects at the exit of a tube as an expansion wave, with the exit pressure approximately constant[1]. This result is so ingrained that it is tempting to use it even for strong shocks such as those found in the pulsed detonation engine. However, this is not true for strong shocks. Rudinger[1] concludes that in this case, the outflow will be supersonic, and since pressure waves can not travel upstream in supersonic flow, the pressure can not immediately return to ambient conditions. He states that "final expansion to the exterior pressure must then take place outside the duct, and is of no concern here". For the calculation of a PDE cycle however, it is of concern, as it can affect the result. Considered here will be the case of an isolated detonation tube, which though not representative of an actual engine, is typical of many experiments.

Kailasanath[2] treated this problem by assuming that, after the flow became subsonic, the exit pressure decayed as a relaxation process, and found that the higher pressure at the tube exit increased the PDE performance. However, it is not clear what relaxation time should be used. Ebrahimi et al.[3] performed two-dimensional calculations, including a region external to the tube, to establish a pressure-time relationship at tube exit to use in one dimensional calculations. These two-dimensional calculations must be repeated for calculating a different geometry, so this destroys the easy use of one-dimensional calculations. What is needed is a simple way of applying a two-dimensional ( or more ) result to a one-dimensional calculation. This is the objective of the present work, and is achieved by using a model of the external flow.

**Model Description**

The planar wave leaving the exit of the tube will be the transmitted wave from the interaction of the detonation wave on the mixture/air interface (assuming a fuel-air reaction). The solution to the problem of a shock (or detonation) wave incident on an interface is given by Rudinger[1]. Taking the conditions behind a hydrogen-air detonation from Borland and Ragland[4], the transmitted shock in air is found to have a Mach number of 3.385, with a pressure ratio of 13.2. This is the initial pressure at the exit after the shock emerges. Although this is a one-dimensional result, it will hold until expansion waves from the edges have reduced the pressure. The development of the shock wave is envisioned in fig. 1. On emergence, the wave is still planar, except at the edges - fig. 1a. As the edge waves grow, the wave becomes more spherical, although at the beginning, there will still be a planar portion near the axis – fig. 1b. At later time, the wave will become essentially spherical – fig. 1c. After this it will grow as a spherical wave. That this actually occurs can be seen from photographs of a similar situation, namely the precursor blast wave from a gun[5], showing an almost spherical shock wave propagating ahead of the bullet. For calculating pressures, it is necessary to ascertain the strength of the spherical shock. For this, recourse is made to the experiments of Ungut et al.[6]. Ungut et al. measured the centerline trajectory of the transmitted wave from detonations in various mixtures in a tube of diameter D, expanding into a larger region, also containing the combustible gas mixture. In cases in which detonation was not re-initiated in the larger region, the transmitted wave stays constant in velocity until about 0.7D, decelerates rapidly to 1.3D, and then decelerates more slowly. From this, it will be assumed that the wave can be considered essentially spherical at a radius of 1.3D, which will be defined as the initial radius $R_0$ of the spherical wave. At this point, the experiments indicate that the wave velocity is very close to half the detonation velocity. Thus the spherical wave pressure jump at $R_0$, $\Delta P_0$, will be that corresponding to a shock wave travelling at half the detonation velocity. This defines the spherical blast wave. For hydrogen-air with a detonation velocity of 1971 m/sec, $\Delta P_0 = 8.58$ ats. From this point on, the wave will propagate as a spherical blast wave. The propagation of a spherical blast wave has been calculated by Brode[6]. Initially, if the wave is sufficiently strong ($\Delta P > 9$), the pressure at the centre of the wave system P(R= 0) is 0.375 of the pressure behind the shock front. Later the value of P(R=0) is a more complicated function of the shock pressure, and drops below atmospheric for a while, returning to atmospheric pressure when the shock pressure ratio is 1.14. A given shock pressure jump $\Delta P$ will occur at a dimensionless shock radius $\lambda = R_s/\varepsilon$, according to the relation:

$$\Delta P = 0.137/\lambda^3 + 0.119/\lambda^2 + 0.269/\lambda - 0.019 \quad \text{(ats)}$$

in which $\varepsilon = (E/P_0)^{1/3}$ is a length determined by the energy E which produced the shock wave. For the detonation case, since $\Delta P_0$ is known when the shock is at a radius $R_s$ = 1.5 D , the value of $\lambda$ follows from the above equation , and hence the value of $\varepsilon$ can be determined. For hydrogen-air detonations, with $\Delta P_0 = 8.58$, $\lambda = 0.282$, and $\varepsilon = 4.61$ D. The pressure at the wave center, P(R = 0) is given by Brode in terms of a dimensionless time $\tau = t\, c_0 / \varepsilon$ , in which t = real time, and $c_0$ is the speed of sound in the ambient air, and is shown in fig 2. With $\varepsilon$ known, this can be converted into real time. This pressure is the pressure at the detonation tube exit, which is the desired exit boundary condition.

**Comparison with other results**

Moen et al.[8] have reported measurements of the overpressure at three distances from the exit of the Norwegian large explosion experiment, and also report a calculation of overpressure versus distance by Hjertager for a methane-air detonation. For methane-air, The detonation velocity is 1801 m/sec, and hence $\Delta P_0 = 6.96$. For this experiment, the exit diameter is 2.5 m. With these values, the overpressure versus distance can be evaluated using the above model, and gives overpressures of 94% at 10m, and 70% at 40m, of the values found by Hjertager. This is quite good agreement. Unfortunately, there do not appear to be data for the detonation case.

Ebrahimi et al. performed calculations of the pressure distribution at the end of a 20 mm high detonation tube assuming a cylindrical blast wave. Since a cylindrical blast wave will decay at a slower rate than a spherical one, there is not a direct comparison here, but there should be qualitative agreement. A comparison of the prediction of the present model with that of Ebrahimi et al. is given in fig. 3.

**Application to PDE calculation**

A one-dimensional, time accurate, CFD code for analysis of PDE cycles has been developed at the NASA Glenn Research Center by Paxson[9]. Using a high-resolution scheme, the code numerically solves the governing equations for a reacting, two species (single progress variable), calorically perfect gas with specified boundary conditions. In the code, a non-dimensional time is used, defined as

$$\tau_{PDE} = c_{ref}\, t / L$$

where $c_{ref}$ is some appropriate speed of sound, and L is the length of the device. For hydrogen-air detonations, $\varepsilon = 4.61$ D, from which;

$$\tau = \tau_{PDE} ( L/D ) (c_0 / c_{ref}) / 4.61$$

in which $\tau$ must be measured from the arrival of the detonation wave, or transmitted wave, at the tube exit. From this, the exit pressure can be found from fig. 2. Calculations for tube L/D ratios of 4, and 10, as well as a calculation with the constant pressure boundary condition, have been made, and the results are shown in fig. 4. Plotted in fig. 4 is the evolution with time of the pressure on the front, i.e. thrust, wall. The pressure distribution with the constant pressure boundary condition was identical to the L/D = 10 result. In these cases, the outflow from the tube is sonic, or greater, until after the external pressure has returned to atmospheric, and so the external pressure distribution has no effect. For the case of L/D = 4, the below atmospheric portion of the external pressure does influence the pressure in the detonation tube, lowering it, and hence the pressure behind the detonation, giving less total thrust.

## Conclusions

A blast wave model appears to give realistic values for the externally imposed pressure distribution with time for an isolated detonation tube. However, it is only for tubes with L/D ratios less than 10 that any effect on the thrust is noticed, when the effect is to reduce the thrust. The pressure decay is even faster than the fastest decay used by Kailasanath[2].

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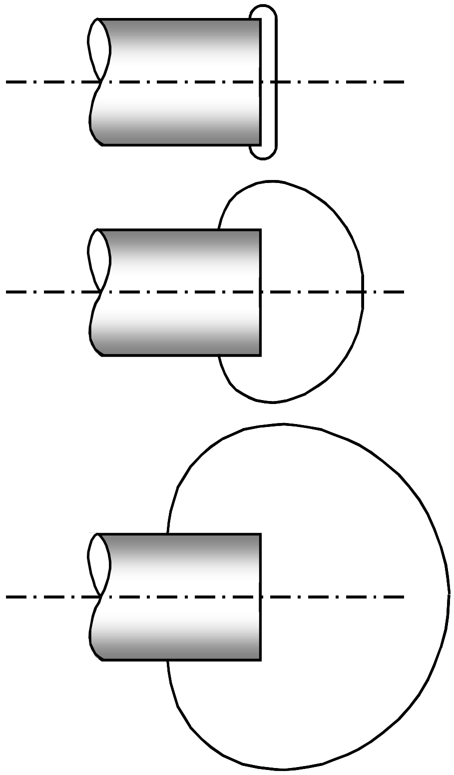

Fig. 1. (a) Shock wave emerging from tube is mostly planar, (b) at a distance slightly less than 0.7 D, only small planar portion remains, (c) at distance greater than 1.3 D, wave is almost spherical.

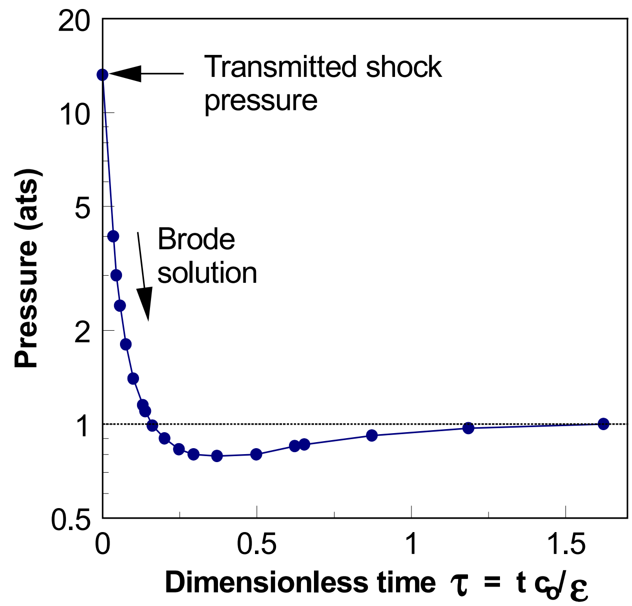


Fig. 2. Pressure at wave center versus dimensionless time, which is the desired boundary condition.

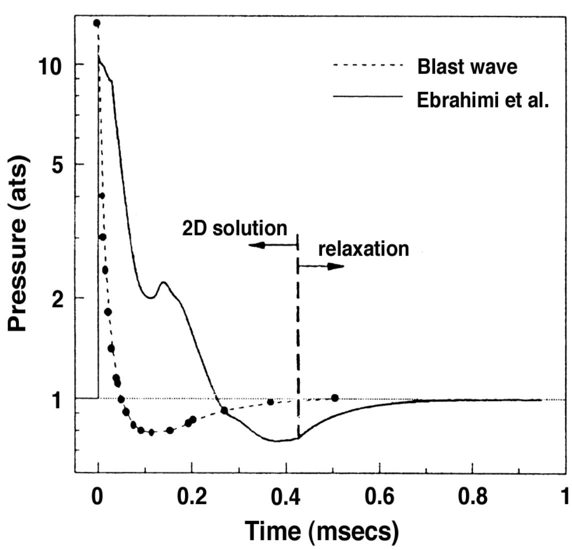


Fig.3. Exit boundary condition with blast wave model compared with boundary condition of Ebrahimi[3].

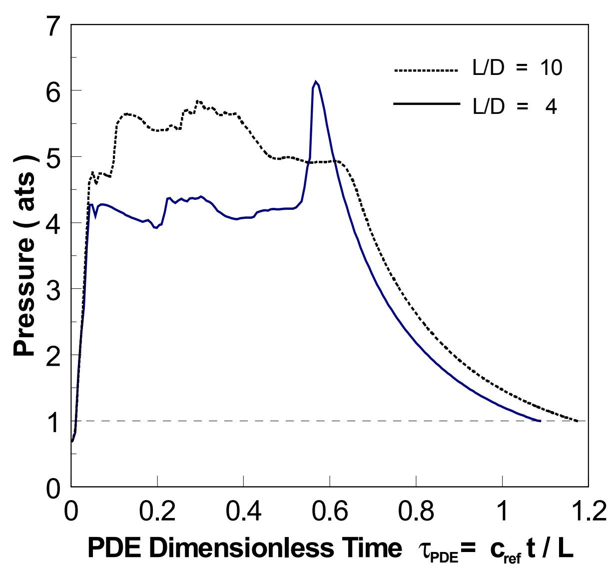


Fig. 4. Pressure on the thrust wall of the PDE versus time, for two values of tube L/D.