Surface Instability and Droplet Pinch-Off for Liquid Films and Filaments

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1 Motivation

Combustion instabilities still remain of major concern in liquid-fuelled rocket propulsion, since their occurrence keeps leading to unexpected mission failures. A predictive model of the combustion chamber processes would therefore be highly welcome since it could provide an a-priori indication of how to avoid those dangerous operating conditions where small perturbations of pressure and gas velocity are strongly amplified. If they are to be reliably predictive, combustion models obviously need to be based on sound physical understanding.

Both combustion and the acoustic vibrations of the chamber volume are distributed in space and time, and combustion instabilities result if their interaction is resonant. Injection of liquid propellants has a strong influence on this interaction, since it controls the spatial distribution and the mixing of the propellants prior to combustion. It is therefore unfortunate that a well-founded physical description of liquid decomposition into a spray does not exist. This deficiency is one of the reasons why existing models of combustion in liquid-fuelled rockets are still of limited predictive capacity.

Liquid decomposition is by way of three stages. In the first stage, thermodynamic or hydrodynamic instability of the injected liquid bodies causes surface deformations to be strongly amplified. This stage is describable in terms of the hydrodynamics of curvature-driven free surface flows. It is followed by the second stage where surface deformation is strong enough to bring about close contact and thus strong interaction for certain surface portions which were initially well separated so that initially their interaction was negligible. For this type of interaction, ordinary hydrodynamics ceases to be applicable. As it turns out, a description derived from an asymptotic limit of continuum theory is still possible. In the third stage, the liquid parent body and the drop are fully separated so that their evolution can again be described by the hydrodynamics of curvature-driven free surface flow, as for the first stage. The overall process therefore consists of two hydrodynamic stages, one before and one after the pinch-off. The details of pinch-off determine how to join the initial stage with the final one. This connection of the two stages is obtained by asymptotic matching.

2 Weber-Number Hydrodynamics

Free-surface flows in thin liquid membranes and filaments are characterized in terms of the Weber number *We*:

$$We = \frac{\sigma}{\rho h \upsilon^2}$$

Here σ = surface tension, h = thickness of the liquid membrane or filament, ρ = liquid density, $\upsilon = |\upsilon_l - \upsilon_g|$, with υ_l = velocity of the liquid flow, υ_g = velocity of the ambient gas phase.

The flow of the liquid is described by a set of three equations, namely the Navier-Stokes equation, the equation of mass continuity, and the equation for the surface kinematics. Strongly deformed thin liquid ligaments can be uncoupled from the dynamics of the ambient atmosphere, which gives:

$$\upsilon \simeq \upsilon_l$$

In this case, the velocity of the liquid determines the Weber number as defined above. The Reynolds number Re and the Capillary number Ca are given as:

$$Re = \frac{\upsilon h}{\nu} \quad , \quad Ca = \frac{\rho \nu \upsilon}{\sigma}$$

Re and Ca express the influence of internal friction which is given by the kinematic viscosity ν. In terms of non-dimensional variables the Navier-Stokes equation is obtained as:

$$Re\left(\frac{\partial \vec{\upsilon}}{\partial t} + (\vec{\upsilon} \cdot \nabla)\vec{\upsilon}\right) = \frac{1}{Ca}\nabla^3 h + \nabla^2 \vec{\upsilon}$$

For ligaments which are thin enough the pressure in the Navier-Stokes equation is essentially a functional of the thickness h, with $p[h] \simeq \sigma \nabla^2 h$.

The Weber number We may be expressed in terms of Re and Ca as follows:

$$\frac{1}{We} = Re\, Ca$$

Strong deformation leads to pinch-off if $h \to 0$. Therefore pinch-off occurs asymptoticly in the following limit:

$$We \to \infty$$

As with other examples of asymptotic limiting cases of a continuum-theoretic description, the transition by which the limit is approached is restricted by supplementary conditions. In the case of droplet pinch-off, these supplementary conditions express the fact that the strong capillary forces, due to strong curvature, can only be counterbalanced by friction forces which are expressed by Re and Ca. The actual form of the supplementary conditions depends on the particular boundary conditions. Two examples are discussed to illustrate the role of the boundary conditions, namely surface films and unsupported films.

For surface films and filaments, the no-slip boundary condition on a supporting solid leads to the following requirement:

$$Re \sim \varepsilon^2 \quad , \quad Ca \sim finite, \neq 0$$

The asymptotic limit of $We \to \infty$ is then characterized by:

$$\frac{1}{We} = Re\, Ca \sim \varepsilon^2.$$

Here, ε represents a small parameter with $\varepsilon \to 0$.

For unsupported liquid membranes, the limit $We \to \infty$ is characterized by:

$$Re \sim \varepsilon^2 \quad , \quad \frac{1}{Ca} \sim \varepsilon.$$

The limit of $We \to \infty$ is then given as follows:

$$\frac{1}{We} = Re\, Ca \sim \varepsilon.$$

Since the asymptotic limit of $We \to \infty$ is of second order in $1/\varepsilon$ for surface films, it leads to simplifications in this case which allow to reduce the equation set to a single equation, the so-called *film equation*. For unsupported films, where the limit is of first order in $1/\varepsilon$, such a reduction is impossible, and a set of two equations remains. It turns out, though, that the case of an unsupported liquid membrane is more suited for an understanding of droplet pinch-off. This is connected with the fact, that for planar liquid membranes, be they supported or unsupported, curvature-driven flow does not lead to droplet pinch-off. It can in fact be demonstrated that pinch-off does eventually occur for thin cylindrical filaments. These are seen to be related to unsupported planar films by the following argument. If a free planar film is wrapped around so as to give a cylindrical filament, an additional contribution to surface curvature arises. This curvature is of the opposite sign to that of the neck which would occur if a droplet were to separate from a planar membrane. It is due to this curvature contribution that a given neck is not smoothed out but is rather driven towards pinch-off so that eventually a droplet is fully severed from the parent body.

Comparison with a range of experiments on liquid decomposition may be used to confirm the result of model calculations, namely that liquid pinch-off is primarily observed for thin 1-d filaments rather than for membranes of 2-d extension.