Non-Adiabatic Strained Premixed Flames – the Effect of Sudden Transient Cooling by a Pressure Drop

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1 Introduction

It has been shown that a sufficiently sharp pressure drop can lead to the extinction of a premixed flame with the peak rate vanishing and the mass burning rate tending to zero, while a relative small or moderate pressure drop leads to recovery to a second steady state (Johnston et al, 1995). The earlier work (Johnston et al, 1995) shows that there exists a critical value of sharp pressure drop beyond which extinction occurs. This critical value rises somewhat with activation energy and decreasing Lewis number, but still represents a considerable drop for flat flames in a straight flow, and thus is not generally very accessible for a flame with a near unity Lewis number.

Strained flames have also been investigated for different ranges of parameter space (Sivashinsky et al, 1982; Matalon, 1988; Buckmaster, 1997). It is well-known that for a non-strained premixed flame, radiative heat loss defines the intrinsic flammability limits. In a simplified asymptotic analysis with radiative heat loss, Buckmaster (1997) found that there exists a weak flame solution at mixture strengths for which the non-strained flame is not possible. The typical response of flame-location or flame-temperature versus strain rate is a curve with a radiation-defined quenching limit at small strain rates, and a stretch-defined quenching limit at large strain rates.

When sharp pressure drops are now put through strained flames near the quenching limit, it is found that not only does the critical value of pressure drop depend on the strain rate for these flames (McIntosh et al., 2001), but that the character of the extinction curve is substantially altered as well.

2 Model Equations

Based on the earlier work (Buckmaster and Mikolaitis,1982; Buckmaster, 1997; McIntosh et al, 2001), we can write the nondimensional equations in the mass-weighted co-ordinates, and we have

\[
p = \frac{\rho T}{T_{0i}}, \tag{1}
\]

\[
\frac{\partial C}{\partial t} + (m_0 - \alpha x) \frac{\partial C}{\partial x} - p \frac{\partial^2 C}{\partial x^2} = -\mathcal{R}, \quad \mathcal{R} = \Lambda C e^{\theta(1-\lambda)}, \tag{2}
\]

1
\[
\frac{\partial T}{\partial t} + (m_0 - \alpha x) \frac{\partial T}{\partial x} - \frac{p}{Le} \frac{\partial^2 T}{\partial x^2} = -q(T - T_\infty) + QR + (1 - \gamma^{-1}) \frac{T}{p} \frac{dp}{dt}.
\]

where \( T \) and \( C \) are temperature and fuel concentration. \( p \) is pressure, \( \theta \) is activation energy, \( m_0 \) is the mass flux and \( \theta \) is the non-dimensional activation energy. All quantities are non-dimensionalised with respect to upstream values, except for temperature which is non-dimensionalised with respect to the unstrained steady flame temperature \( T_{b1} \) and \( Q \) which is non-dimensionalised as

\[
Q \equiv \frac{Q'}{C_p T_{b1}}
\]

The coefficient of heat loss \( q \) is defined as:

\[
q \equiv \frac{\theta \lambda' c_\infty}{(\rho' u'_{ad})^2 c_p'(c_\infty)},
\]

where \( \rho' u'_{ad} \) is the dimensional adiabatic mass flux. The Lewis number is defined as \( Le \equiv \frac{p'_{B}}{u_0 \rho'_{ad}} \). In this work, in keeping with earlier work, we regard the strained flow as fixed and prescribed, viz. \( u = \alpha(-x, y) \) with \( \alpha = \alpha' D_0'/u_0'^2 \). Thus we ignore the small amplitude change in the velocity field by the pressure disturbance as a higher order effect. Due to the symmetrical nature of the flow field, we need only consider the half-space \( x < 0 \) (Figure 1). The related boundary conditions are

\[
C = C_\infty, \quad T = T_\infty, \quad \text{for} \quad x \to -\infty,
\]

\[
C = 0, \quad T = T_a \quad \text{(or} \quad T_b \quad \text{for} \quad \alpha = 0), \quad \text{for} \quad x \to x^*.
\]

This defines a moving boundary problem as the flame moves in response to the input perturbation of pressure \( p(t) \), which is a prescribed function of \( t \). We introduce the equivalence ratio as

\[
\phi \equiv \frac{c_\infty}{c_{\infty, stoich}},
\]

and define a modified heat release variable \( \tilde{Q} \) as

\[
\tilde{Q} = QC_{\infty, stoich},
\]

so that

\[
T_b = T_u + \tilde{Q} \phi.
\]

In this work we have used \( \tilde{Q} = 0.8 \). Following Buckmaster (1997), typical values for \( c_{\infty, stoich} \) and \( Q \) would be: \( c_{\infty, stoich} \approx 0.045 \) with \( Q \approx 18 \).

For a sudden lowering of pressure to \( p_0 (= p_{\text{final}}/p_{\text{ambient}}) \), the adiabatic decompression effect gives an intermediate profile of temperature which in non-dimensional terms is given by \( T = T_s(x)p_0^{(1-\gamma^{-1})} \), where \( \gamma \) is the ratio of specific heats \( (c'_p/c'_v) \) and \( T_{01} \) is the ratio of initial unburnt to initial burnt temperature. The final temperature is then eventually given by \( T_B = T_{01} p_0^{(1-\gamma^{-1})} + \tilde{Q} \phi \) and it is found that this raises the threshold for the pulsating instability to a value closer to accessible Lewis numbers near unity.
3 Numerical Simulations

Our main purpose in this work is to find numerical solutions of equations (2) and (3) subject to large initial pressure drops. This is in keeping with the earlier zero-strain analysis (Johnson et al, 1995). We use a finite-difference scheme to solve the full unsteady equations (2) and (3) in a similar way to that used in our earlier work (Johnson et al, 1995; McIntosh et al., 2001). However, a more dense adaptive grid scheme is used near the flame to follow the evolving temperature profile more accurately.

Abrupt Pressure Drops and Effect of Heat Loss

From the features of extinction and gradual recovery of a premixed flame, there exists a critical value of pressure drop $p_{min}$ at which the extinction only just occurs. Numerical simulations show that this critical value of minimum pressure (Figure 2) varies with heat loss for a given strain rate and fixed concentration ($\phi = 1$). The value of heat loss has a great effect on the pressure drop $p_{min}$ required for extinction. We can see that a large heat loss makes the flame less stable, such that only a very small sharp pressure drop will extinguish it. But for very small heat loss, the flame is relatively stable such that a large pressure drop is required to extinguish the flame. We note also that for significant heat loss, moderate positive strain initially stabilises the flame to pressure drops, but larger strain has a destabilising effect.

The Quenching Curves: Effect of Sharp Pressure Drops

It is well known from earlier work (Dixon-Lewis,1996; Buckmaster 1997) that there exists a quenching curve of extinction strain rates versus equivalence ratio $\phi$ for a strained premixed flame. For a given heat loss and a fixed equivalence ratio, there exist two quenching limits. Refererring to the dotted curve in Fig.3, at small strain rates,
there is a radiation-defined quenching limit such that the effect of strain is relatively small compared with that of heat loss. There is also a large strain limit where heat loss is not the major effect. As the equivalence ratio reduces from unity, these two limits converge to an absolute limit of $\phi \approx 0.46$.

**Figure 2** Effect of heat loss on the critical pressure drops at different strain rates for $\theta = 10$, $\phi = 1$, and $Le = 1$. Above each curve there is recovery, and a pressure drop below the critical pressure yields extinction.

**Figure 3** Effect of pressure drops on the quenching curves for $\theta = 10$, $q = 0.1$, $Le = 1$
For a sharp pressure change, this quenching curve changes markedly. Figure 3 shows the effect of a pressure drop on the shape of the quenching curve for different final pressures. Thus, for a given strain rate and a fixed equivalence ratio, there exists a critical pressure drop that can just extinguish the premixed flame, and the different curves correspond to the contours of these different levels of critical minimum pressure $p_{\text{min}}$ which is now a function of strain rate $\alpha$ and equivalence ratio $\phi$. The stable flame region shrinks as the magnitude of the pressure drop increases. In short, increasing the size of the pressure drop has a similar effect to increasing the heat loss. This is an important finding for flames near extinction limits experiencing rapid pressure variations.

4 Concluding Remarks

This paper has extended the earlier work on extinction limits of non-adiabatic strained premixed flames, to include the effect of sharp pressure drops. The investigations consider the response of a flat flame in a prescribed flow field $\mathbf{u} = (-\alpha x, \alpha y)$ where the flow is regarded as essentially incompressible and uniformly strained. For a given heat loss, strain and equivalence ratio, the quenching pressure drop has been evaluated, and it has been found that for an equivalence ratio well below unity, only a small pressure drop is required for extinction.

References


