Critical Condition for Stabilized Chapman-Jouguet Oblique Detonation Waves Around Hypersonic Bodies

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ABSTRACT
Five-mm-diameter projectiles, whose speed was beyond the Chapman-Jouguet (C-J) detonation speed, were fired into mixtures containing stoichiometric hydrogen-oxygen gases plus argon diluents. The flowfield around the projectile was visualized using a CCD camera. We measured the minimum normal velocity components of detonation waves around the projectiles and the effective curvature radius of a bow detonation wave near the projectile tip. From these experimental results we concluded that the steady-state C-J detonation wave is stabilized around a projectile when the effective curvature radius of the bow detonation wave is larger than 8.6 times of the detonation cell size.

NOMENCLATURE
\(d\) : projectile diameter, mm
\(d^*\) : critical projectile diameter, mm
\(p_0\) : initial pressure of a mixture gas, atm
\(r\) : effective curvature radius, mm
\(r^*\) : critical effective curvature radius, mm
\(T_0\) : initial temperature of a mixture gas, K
\(V_{CJ}\) : C-J detonation velocity, km/s
\(V_{nm}\) : minimum normal velocity component of a shock or a detonation wave around a projectile, km/s
\(V_p\) : projectile velocity, km/s
\(X_{Ar}\) : argon mole fraction
\(\beta_{CJ}\) : C-J detonation wave angle, deg
\(\lambda\) : detonation cell size, mm

INTRODUCTION
The propulsion systems using stabilized oblique detonation waves are Oblique Detonation Wave Engine (ODWE) \(^1\) \(^2\) \(^3\) and Ram Accelerator (superdetonative mode) \(^4\). Although a lot of research were done about the systems, there are a few experimental study associated with the stabilized oblique detonation wave \(^3\) \(^5\). Particularly, the critical condition of initiation for the stable Chapman-Jouguet (C-J) oblique detonation wave is not clear. In the present experimental study, we investigated the critical condition by using hypersonic free projectiles injecting static detonative mixtures, \(2\text{H}_2+\text{O}_2\) plus argon diluents. Projectile diameter was 5 mm. Projectile velocity was beyond the C-J velocity of the mixtures.

EXPERIMENTAL APPARATUS AND CONDITIONS
The Schlieren system was used for visualizing the flowfield around the projectile launched from a two-stage light-gas gun. The light source of the system had spark duration of 20 ns. A high-resolution CCD camera (1024×768 pixel) was used. The position of the projectile is determined by a He-Ne laser-cutting signal. The camera and light source were triggered by the signal. We performed the experiments in a stoichiometric...
hydrogen-oxygen gas mixture with argon diluents (diluents' mole fractions were varied from 0.2 to 0.6), at initial fill pressures from 0.5 to 1.1 atm, and room temperature of $291 \pm 4$ K. The projectiles had a conical nose of 120-degree open angle, 5 mm in diameter, and were made of polyethylene. The projectile velocity was ranged from 1.97 km/s to 2.81 km/s, but it was always faster than the C-J velocity, to ensure an overdriven stabilized detonation condition on the projectile nose.

DETONATION CELL SIZE AND C-J DETONATION PROPAGATION VELOCITY
Strehlow and Engel\textsuperscript{7)} investigated the detonation cell sizes of stoichiometric hydrogen-oxygen mixtures plus argon diluents. We extrapolated their cell size data by using the equation (1) to which least-squares fitting was applied.

$$\lambda = p_0^{-1.398} \sum_{n=0}^{9} a_n X_{Ar}^n$$ \hspace{1cm} (1)

$$0.0 \leq X_{Ar} \leq 0.8$$

$$a_0 = 0.60242151, a_1 = -0.10444428, a_2 = 1.4350771, a_3 = 8.5126829, a_4 = -80.207640, a_5 = -17.323234, a_6 = 1221.4973, a_7 = -3153.1402, a_8 = 3098.7268, a_9 = -1062.6686.$$  

We measured the C-J detonation wave velocities\textsuperscript{4, 5)}. Assuming the phenomena were steady state for the projectile, the normal velocity component of the detonation wave, $V_{CJ}$ was calculated by $V_{CJ} = V_p \sin \beta_{CJ}$, where $\beta_{CJ}$ is C-J detonation wave angle. If the C-J detonation waves were not caught in the Schlieren picture, we interpolated our C-J detonation velocity data by using the equation (2).

$$V_{CJ} = 2.6266 - 0.0161 X_{Ar}$$ \hspace{1cm} (2)

$$0.2 \leq X_{Ar} \leq 0.6$$

RESULTS AND DISCUSSIONS
We classified these phenomena in five cases of Figs.1 to 5. In these figures, Schlieren picture (right) and schematic diagram (left) were shown. Since we used negative photographs for clarity, the projectile color is white in the pictures. The projectiles were flying leftward in the pictures.

In Figs.1 to 5, we defined the effective curvature radius, $r$, as the sphere-approximation curvature radius of the bow detonation wave whose normal velocity component was attenuated to C-J velocity, $V_{CJ}$ (detonation wave angle is $\beta_{CJ}$). We also defined $V_{nm}$ the minimum normal velocity component of the shock or detonation wave within 25-mm area behind the normal detonation wave. The rate of the velocities, $V_{nm}/V_{CJ}$, was called attenuation rate.

Figure 1 is the case of the smallest attenuation rate ($V_{nm}/V_{CJ} = 0.299$). In the Fig.1, the decoupled shock induced combustion wave was observed. In the case of Fig.1 the bow-shock envelope was oscillating caused by the interaction between the compression (shock) wave and shock wave\textsuperscript{8)}. Figure 2 is the case of the larger attenuation rate ($V_{nm}/V_{CJ} = 0.443$) than Fig.1. In the Fig.2, the decoupled shock induced combustion wave was also observed. The interaction of shock and heat release was stronger and induction length from the shock wave to the heat release zone was shorter than Fig.1. In Fig.2 C-J detonation wave was observed as a straight line, but in the previous paper\textsuperscript{9}) we confirmed that the C-J detonation wave in such a case of Fig.4 was unsteady-state moving away from the projectile. Figure 3 is the case of $V_{nm}/V_{CJ} = 0.756$. In the Fig.3, interaction of shock and heat release was stronger, and the shock wave was coupled with the heat release zone. The C-J detonation wave will exist outside of the Fig.3 frame. Figure 4 is the case of $V_{nm}/V_{CJ} = 0.912$. In Fig.4, the interaction of shock and heat release became stronger than Fig.3, and stabilized steady-state C-J detonation wave was generated around the projectile. But, quasi C-J detonation wave being attenuated by the rarefaction waves from the projectile shoulder was generated. Figure 5 is the case of the largest attenuation rate ($V_{nm}/V_{CJ} = 0.989$). In Fig.5, the stabilized steady-state C-J detonation wave was generated perfectly around the projectile.

The attenuation rate ($V_{nm}/V_{CJ}$) is plotted against the effective curvature radius divided by the detonation cell size ($r/\lambda$) in Fig.6. In Fig.6 the dash-dotted line means $V_{nm}/V_{CJ} = 1.000$, and broken line is an asymptote where the bow shock wave is attenuated to Mach wave ($V_{nm}/V_{CJ} = 0.19$, no argon dilution). Argon's mole fraction, initial pressure, and projectile velocity were varied in the range shown in "CONDITIONS". From Fig.6 we found that
the attenuation rate, $V_{nm}/V_{CJ}$, only depends on the effective curvature radius divided by the detonation cell size ($r/\lambda$). From previous results\(^4\), the steady-state C-J detonation waves were generated under the condition $V_{nm}/V_{CJ} \geq 0.8$. Recently we obtain the steady-state numerical solution of $V_{nm}/V_{CJ} = 1.000$ case as shown in Fig.7 (axisymmetric flow, $2\text{H}_2+\text{O}_2$, $P_0=0.506$ atm, $T_0=302.1$ K, $V_p=3.71$ km/s, $d=10$ mm). From these results, we determined the critical attenuation rate is 0.8 ($V_{nm}/V_{CJ} = 0.8$) and defined $r^*$ as critical effective curvature radius. From Fig.6 we obtain,

$$r^* = 8.6\lambda$$

That is, the steady-state C-J detonation wave is stabilized around a projectile when the effective curvature radius of a bow detonation wave near the projectile is larger than 8.6 times of the detonation cell size. We thought the dispersion of the data in Fig.6 was due to cell-size dispersion, asymmetry of the wave front and unsteady-state wave propagation.

The initial pressure is plotted against the mole fraction of in Fig.8. The open-circle symbol and cross symbol are above the critical attenuation rate ($V_{nm}/V_{CJ} \geq 0.8$) and, below the rate ($V_{nm}/V_{CJ} <0.8$), respectively. From Fig.8, critical cell size is about 1.2 mm at 5-mm projectile diameter. That is, the steady-state C-J detonation wave is stabilized around a projectile when the projectile diameter is larger than roughly 4 times of the detonation cell size.

$$d^* = 4\lambda$$

But equation (4) does not hold for $r$ decrease when the projectile speed is much higher than C-J velocity.

**CONCLUSIONS**

Five-mm-diameter projectiles, whose speed was beyond the Chapman-Jouguet (C-J) detonation speed, were fired into mixtures containing stoichiometric hydrogen-oxygen gases plus argon diluents. The steady-state C-J detonation wave is stabilized around a projectile when the effective curvature radius of a bow detonation wave near the projectile is larger than 8.6 times of the detonation cell size.

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Fig. 1 The case of a decoupled reactive shock wave. \( p_0 = 0.50 \text{ atm}, \ T_0 = 293.7 \text{ K}, \ X_{Ar} = 0.33, \ V_p = 2.10 \text{ km/s}, \ V_{nm}/V_{CJ} = 0.299. \)

Fig. 2 The case of a decoupled reactive shock wave with unsteady-state (not stabilized) C-J detonation wave (Straw-Hat Type). \( p_0 = 0.55 \text{ atm}, \ T_0 = 291.9 \text{ K}, \ X_{Ar} = 0.20, \ V_p = 2.59 \text{ km/s}, \ V_{nm}/V_{CJ} = 0.443. \)

Fig. 3 The case of a coupled reactive shock wave. \( p_0 = 0.80 \text{ atm}, \ T_0 = 293.8 \text{ K}, \ X_{Ar} = 0.50, \ V_p = 2.43 \text{ km/s}, \ V_{nm}/V_{CJ} = 0.756. \)
Fig. 4 The case of a steady-state (stabilized) quasi C-J detonation wave. 
$p_0=0.85$ atm, $T_0=294.7$ K, $X_{Ar}=0.50$, $V_p=2.23$ km/s, $V_{nm}/V_{CJ}=0.912$.

Fig. 5 The case of a steady-state (stabilized) C-J detonation wave. 
$p_0=0.90$ atm, $T_0=294.2$ K, $X_{Ar}=0.50$, $V_p=1.97$ km/s, $V_{nm}/V_{CJ}=0.989$.

Fig. 6 The critical condition for initiation of steady-state C-J detonation waves around hypersonic projectiles.

Fig. 7 Numerical solution of steady-state C-J detonation wave around the projectile.

Fig. 8 Dependence of the critical initiation pressure on the detonation cell size.