Pulsating Cellular Flames

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Abstract

Laminar premixed flames often exhibit cellular patterns envolving flickering, i.e. periodic time evolution. The model considered here, describes the pattern formation of flames stabilized by a porous plug burner. We derive a Kuramoto-Sivashinsky-equation coupled to a heat equation for the burner surface temperature. Even though the variations of the burner surface temperature are small, they induce an observable effect. Namely, in a transition of the primary bifurcation from stationary to oscillatory, which may explain experimental observations.

Introduction

In many instances the transition from laminar to turbulent combustion is associated with the development of increasingly complex spatiotemporal patterns in the flame front. In premixed gaseous combustion flame instability is often manifested in the formation of an array of cells along the flame front. Due to the elevated temperature of the cells, they appear as bright troughs in the flame front pointing in the direction of the fresh mixture. As time evolves the cells may pulsate.

Experimental observations of cellular flames date back to the 1950s [1]. In recent experiments in [2] [3], cellular flames were established above a circular porous plug burner, and their dynamics were described in detail. A common feature of all the reported patterns is the time-dependence of the patterns. Some patterns exhibit periodic dynamics but many also show a very complex time evolution. The spatial structure can be characterized as either ordered or disordered, where the ordered patterns are often organized in the form of a single ring or several concentric rings of cells. In some patterns a whole ring of cells rotates around a fixed point. In other patterns the cells flicker, i.e. the cells periodically expand and contract and also become brighter and dimmer.

The objective of this paper is to give a possible explanation for the observed rotations and pulsations in the flame front.

The cellular instability mechanism for freely propagating premixed flames was described theoretically by Sivashinsky [4], who considered a weakly nonlinear theory based on the diffusional thermal model [5] resulting in a Kuramoto-Sivashinsky equation (KS) .

The stability of premixed flames stabilized by a porous plug burner was studied for one-dimensional perturbations in [6] and for two-dimensional perturbations in [7]. It was shown that, if the flame has a stand-off distance within a range of several preheat lengths, the pulsating instability, which for freely propagating flames exists only for high Lewis numbers, may be shifted toward lower Lewis numbers more commonly encountered in gaseous mixtures. The Lewis number is defined as the ratio of thermal to mass diffusivity. In [8], based on a linear stability analysis, a modified Kuramoto-Sivashinsky equation (MKS) was proposed to describe cellular porous plug burner flames. This MKS equation was derived by systematic asymptotic methods in [9] where a logarithmically large stand-off distance was considered, allowing a balance between the stabilizing effects of the burner with the terms responsible for the instability. Numerical solutions in 2D [9] of the MKS equation describing combustion in three dimensions demonstrate that flickering or pulsating patterns may exist in the limit of large stand-off distances. However, patterns such as a rotating single cell are not described by the MKS. It seems that in the experiments there is a greater tendency to rotation and flickering than in the numerics. The experimental observations [2] [3], suggest that the primary bifurcation always is to a flickering or rotating state even when the flame is located at large stand-off distances from the burner. This is in contrast to MKS results since the primary bifurcation to cellular flames as described by the MKS-equation is to a stationary state.

Here we present a coupled system consisting of a heat equation coupled to a KS equation, where the heat equation models small perturbations of the burner surface temperature about its equilibrium value.

Relaxing the assumption of a constant surface temperature alters the primary bifurcation from stationary to pulsating, thereby explaining the experimental observation that cellular flames flicker or rotate even at large stand-off distances.

Model

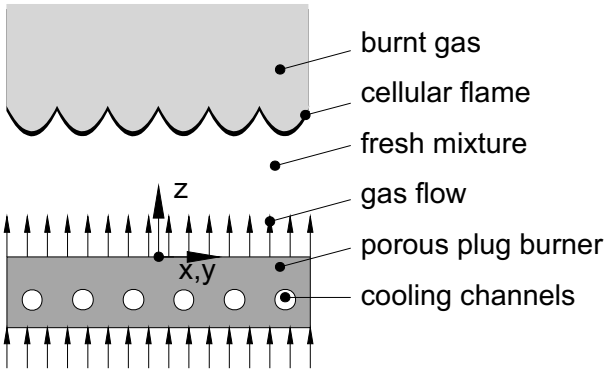

Figure 1: Burner geometry

Figure 1 shows the burner geometry considered here, which is related to the experimental configuration employed in [2] where a premixed cellular flame is established above a circular porous plug burner. A mixture of fuel and oxidizer emerges from a cooled porous plug and reacts in a thin laminar flame to form the burnt products.

The main tasks of the burner are to uniformly deliver the fresh mixture and to stabilize the flame at a fixed location. The gas velocity at the burner is below the propagation speed of a freely-propagating adiabatic planar flame. Therefore, a flame that is ignited sufficiently far away from the burner, will initially propagate toward the burner. As the flame approaches the burner, the heat losses to the burner are growing and therefore the reaction temperature, and also the flame speed is lowered. Eventually the flame is stabilized at a fixed stand-off distance.

The burner consists of a porous plate which is cooled by an embedded cooling channel. We use a one temperature model, i.e. we do not distinguish between the temperature of the solid matrix of the porous plate, and the gas temperature in the pores of the matrix. This is valid as long as the heat transfer between the gas and the porous matrix is sufficiently high. In contrast to the models [7] or [6] where the burner surface temperature is assumed to be constant, we assume that the temperature of the coolant is fixed. Therefore, small temperature variations along the burner surface may arise. This assumption is consistent with the observation that the heat capacity of the burner, and therefore its inertia, is typically large as stated in [7]. The effect of the large inertia is to restrict temperature variations within the porous plug to a thin layer at the surface, and to allow for slow time evolution only. By assuming a high heat capacity, and very small temperature variations of the burner surface temperature, we find an observeable effect; the transition of the primary instability from stationary to oscillatory.

In suitable nondimensional variables, the equations for the stand-off distance ϕ and the burner surface temperature B are,

$$\begin{aligned} \phi_t + \nabla^4\phi + 2\nabla^2\phi + (\nabla\phi)^2 + B &= 0, \\ a\,B_t - b\nabla^2 B + B - c\,\phi &= 0, \end{aligned} \tag{1}$$

where a is proportional to the inverse of the Fourier-number, b is proportional to the heat conductivity of the burner and c is a measure of the stand-off distance of the flame. Here $c = 0$ corresponds to a freely propagating flame which is not influenced by the burner, and $c > 0$ stands for a flame which is stabilized by heatlosses to the burner. For a burner with constant surface temperature, $B = 0$, equation

(1) reduces to a damped Kuramoto-Sivashinsky equation, $\phi_t + \nabla^4\phi + 2\nabla^2\phi + (\nabla\phi)^2 + c\phi = 0$, which exhibits cellular patterns for $c < 1$.

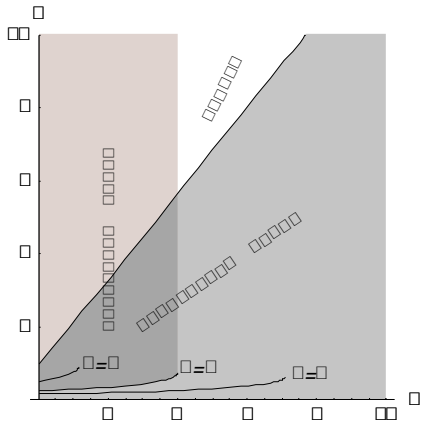

Figure 2: Stability map

In figure 2 the results from a linear stability analysis of (1) are shown.

For $a < 1$ we find the cellular instability, which is associated with a real eigenvalue which crosses the imaginary axis as c is lowered below a critical value $c_{\text{crit}}(b) > 1$. I.e. the stand-off distance of the flame has to be sufficiently large.

For $a > 1$, and simultaneously $b < b_{\text{crit}}$ and $c > c_{\text{crit}}$, we find an oscillatory instability, which is associated with a pair of conjugate complex eigenvalues crossing the imaginary axis as either b or c cross their critical values.

The nature of the new solution which originate from the instability is studied by numerical simulations of equation (1).

Figure 3 shows a rotating two cell solution which was computed for $a = .6$, $b = 0$ and $c = .96$.

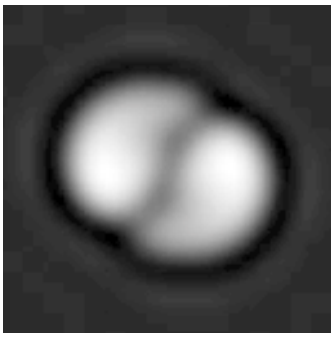

Figure 3: Rotating 2 cell solution

The grey scale annotates the depth ϕ of the cells where white corresponds to small ϕ and black to large ϕ. The pattern rotates counter-clock-wise. The shape and depth of the cells do not change in a rotating frame of reference.

Other patterns such as rotating one, two and four cell solutions and also target patterns are computed.

Summary

Premixed flames stabilized by heat losses to a porous plug burner often exhibit pulsating cellular patterns. We suggest that a possible origin for the pulsations is the large heat capacity of the burner.

In the present paper we present a model consisting of a Kuramoto-Sivashinsky equation describing the evolution of the flame front coupled to a heat equation which describes the burner surface temperature. It is assumed that the burner has a large inertia and a high transverse heat conductivity which applies for typical burners.

With an increasing stand-off distance from the burner, the stationary cellular instability is promoted. The inertia, i.e. the heat capacity of the burner has no effect on the onset of the stationary cellular instability. However, the transverse heat conductivity of the burner is promoting stationary cellular patterns.

Pulsating cellular patterns are observed for small stand-off distances and a large inertia of the burner. The transverse heat conductivity of the burner retards the onset of the pulsating instability. As the stand-off distance of pulsating flames is increased, the frequency of the pulsations is lowered. For sufficiently large stand-off distances, bifurcation to pulsating cellular flames is through a infinite period bifurcation.

We speculate that accounting for the inertia of the burner in more sophisticated flame models that include effects such as, large amplitude patterns, or strong thermal expansion will lead to qualitative similar results.

References

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