The multi-dimensional stability of weak heat release detonations

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Abstract

We examine the multi-dimensional linear stability of an overdriven detonation wave in the limit of a small, shock-temperature-scaled, heat release β and with a ratio of specific heats γ such that $(\gamma - 1) = O(1)$. Under these assumptions, the steady structure can be evaluated explicitly, allowing us to derive a wholly analytical representation of the dispersion relation governing the detonation stability behaviour. The dispersion relation predicts that for finite detonation overdrives and $\beta \ll 1$, the detonation is always stable to two-dimensional disturbances. For large overdrives f, the boundaries between regimes of stability or instability are found to depend on a choice of distinguished limits between β and f.

Introduction

The problem of detonation stability and the formation of detonation cells has enjoyed much attention recently. Direct numerical simulations (Bourlioux & Majda 1992; Quirk 1994; Williams, Bauwens & Oran 1996; Quirk & Short 1998) reveal a range of cell sizes and regularity as the four fundamental detonation parameters, the heat release, activation energy, ratio of specific heats and detonation overdrive, are varied.

Recent theoretical attempts at explaining the mechanism underlying the cell formation include Buckmaster & Ludford (1986), Buckmaster (1989), Yao & Stewart (1996), Short & Stewart (1997) and Short (1997). These assume a large activation energy coupled with a pressure change through the main reaction layer of the order of the Von-Neumann shock pressure. In this regime, these studies reveal an intricate coupling between the shock state and the response of the main reaction layer in driving the cell formation. Other theoretical studies include that by Clavin, He & Williams (1997) who studied the multi-dimensional stability of overdriven waves in the limit of large detonation Mach numbers.

Having established the mechanisms behind detonation instability for large activation energies in a one-step Arrhenius reaction model, we now progress to the study of problems with different limits of equally important practical interest, namely those of a moderate activation energy and a low heat release measured on scales associated with the steady post-shock detonation temperature. The latter limit could, for example, account for the large amounts of inert diluent that are typically added to the chemical mixtures when conducting experiments on cellular detonation instabilities (Strehlow 1970), but as in Clavin, He & Williams (1997), also covers situations of large detonation overdrive. However, unlike Clavin, He & Williams (1997) who predict unconditional detonation instability, the limits we investigate uncovers the range of parameters where the neutral stability boundary occurs.

Model

We assume an ideal gas which undergoes a unimolecular, first-order, irreversible Arrhenius reaction with constant mole fraction and ratio of specific heats.

Parameter choices

The scaled heat release β and activation energy θ are defined by

$$\beta = \gamma \widetilde{Q}/\widetilde{T}_s^*, \quad \theta = \gamma \widetilde{E}/\widetilde{T}_s^*,$$

where $\widetilde{Q}$ and $\widetilde{E}$ are the dimensional heat release and activation energy respectively for the reaction mixture and $\widetilde{T}_s^*$ is the immediate post-shock temperature in the steady detonation wave. The ratio of specific heats is denoted by γ, while the detonation overdrive

$$f = (D^*/D^*_{CJ})^2,$$

where D^* is the steady detonation Mach number and D^*_{CJ} the steady Chapman-Jouguet detonation velocity.

We study the multi-dimensional stability of detonations in the parameter range

$$\beta \ll 1, \quad \theta = O(1), \quad (\gamma - 1) = O(1).$$

Our analysis thus applies to situations where $\widetilde{Q} \ll \widetilde{T}_s^*$, and since $\widetilde{T}_s^*$ is a function of the detonation Mach number D^*, does not restrict us to small values of $\widetilde{Q}$. We also assume that the detonation overdrive $f > 1$, to avoid the complex transonic flow problem that occurs when $f = 1$, $\beta \ll 1$ and $M_s^* = 1 + O(\sqrt{\beta})$, where M_s^* is the steady post-shock flow Mach number.

For the purpose of the perturbation analysis we define the product of $(\gamma - 1)$ and β as

$$(\gamma - 1)\beta = \epsilon, \quad \epsilon \ll 1.$$

Steady detonation wave structure

In the above parameter range, and in the shock-attached co-ordinate frame X, the pressure variation p through the steady detonation wave can be evaluated as,

$$p^* = p_b^* - \epsilon\alpha(Y_0^* - 1),$$

where p_b^* is the constant pressure at the burnt state ($X \to \infty$), α is a constant and Y_0^* is the leading-order reactant mass fraction variation through the detonation wave. This has the straightforward form,

$$Y_0^* = 1 - (1/2)^X.$$

Expressions for the velocity and density variations can be derived similarly.

Normal-mode stability analysis

A normal-mode linear stability analysis is possible by expanding pressure and the perturbation to the shock front, h, in the form

$$p = p^*(x) + p'(x)e^{\lambda t + iky}, \quad h = h'e^{\lambda t + iky},$$

where $\mathrm{Re}(\lambda)$ represents the disturbance growth rate, $\mathrm{Im}(\lambda)$ the disturbance frequency and k the disturbance wavenumber. The shock-attached coordinate is x, while the transverse coordinate is y. Similar expansions can be written down for the other variables. The perturbation eigenfunctions ($p'(x)$, etc) satisfy perturbed Rankine-Hugoniot relations at the shock front ($x = 0$) and a standard acoustic radiating condition at the equilibrium point $x \to \infty$.

Dispersion relation

Since the steady detonation structure is known analytically, it transpires that by expanding the perturbation eigenfunctions in the form $p' \sim p'_0 + \epsilon p'_1$, a dispersion relation governing the multi-dimensional stability of detonation can be derived in the truncated form,

$$F_0(\lambda; a_i^*(\epsilon)) + \epsilon F_1(\lambda; a_i^*(\epsilon)) = 0,$$

allowing an analytical representation for λ to be determined in the truncated form,

$$\lambda \sim \lambda_0 + \epsilon\lambda_1 + \epsilon^2\lambda_2.$$

An analysis of form of λ allows us to determine the ranges of parameters for which the detonation is either stable or unstable. We have identified three significant regimes as explained below.

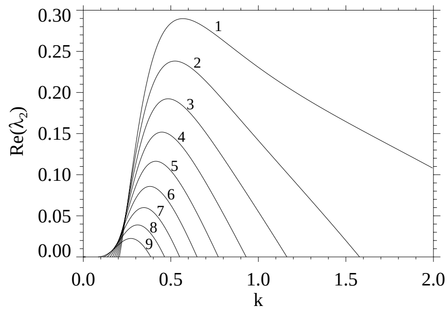

Figure 1: $\mathrm{Re}(\lambda_2)$ versus k for $\gamma = 1.4$ and $(\epsilon D^{*\,2})^{-1} = 0$ with (1) $\theta = 4.5$, (2) $\theta = 4.0$, (3) $\theta = 3.5$, (4) $\theta = 3.0$, (5) $\theta = 2.5$, (6) $\theta = 2.0$, (7) $\theta = 1.5$, (8) $\theta = 1.0$ and (9) $\theta = 0.5$.

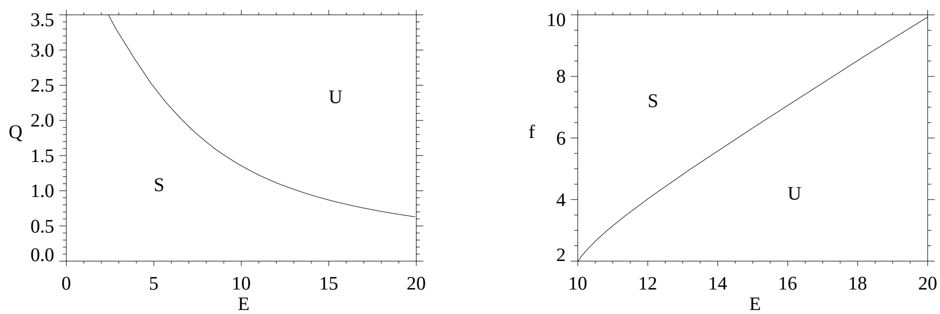

Figure 2: The neutral stability boundary in (a) the Q–E plane for $\gamma = 1.4$ and $f = 5$, and (b) the f–E plane for $\gamma = 1.4$ and $Q = 1$. The regions to the right of the curves are unstable.

Stability Results

Before proceeding, we define the traditional heat release and activation energy scalings

$$Q = \gamma \widetilde{Q}/\widetilde{T}_0^*, \quad E = \gamma \widetilde{E}/\widetilde{T}_0^*,$$

to present our results, where T_0^* is the pre-shock reactant temperature.

Unconditional detonation stability - $f = O(1)$, $Q \ll 1$.

In the parameter range of order one overdrives, and a small heat release $Q = O(\beta) \ll 1$, our dispersion relation predicts unconditional stability of the detonation wave to multi-dimensional disturbances.

Unconditional detonation instability - $f \gg 1$, $Q \gg 1$.

In the parameter range defined by large overdrives, $f \gg 1$, and a large heat release $Q \gg 1$, our dispersion relation predicts unconditional instability of the detonation wave to multi-dimensional disturbances. For each value of θ and γ, we find a single unstable mode consisting of a finite range of unstable wavenumbers. An example is shown in figure 1.

Location of neutral stability boundaries - $f \gg 1$, $Q = O(1)$.

In the parameter range defined by large overdrives, $f \gg 1$, but with an order one heat release $Q = O(1)$, our dispersion relation predicts the presence of a neutral stability boundary defined by a further relation between the four parameters Q, γ, f and E. Typical neutral stability boundaries in Q-E space and f-E space are shown in figure 2. Their behaviour are qualitatively similar to those calculated numerically. For example, for fixed E, f and γ, an increase in Q will render the detonation unstable. On the other hand, for fixed E, Q and γ, an increase in f generally will render the detonation stable. In addition to quantitative evaluation, the explicit analytical dispersion relation we have derived provides a mechanism for understanding the reasons behind detonation instability in the above parameter ranges.

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