Stability analysis of gaseous detonations

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Gaseous detonation waves adopt a wide range of unsteady propagation modes at near limit conditions. Among them, the galloping detonation has experimentally been shown to exhibit large velocity fluctuations over relatively long periods [1, 2]. Recent theoretical work has also been directed towards the analysis of this phenomenon [3, 4].

We recently performed the detailed characterization of unstable gaseous detonation waves in propane/oxygen mixtures, diluted or not with argon or helium, at near limit conditions, in a 25–m long circular tube using a coaxial Doppler interferometer [5]. In particular, it was found that the galloping mode could not be obtained at dilutions equal to or above 70% argon and 60% helium molar dilution. Figure 1 shows an example of a galloping detonation in an undiluted stoichiometric propane/oxygen mixture at an initial pressure of 0.8 kPa in a 38–mm diameter tube. Large velocity fluctuations, averaging about 110% of the theoretical Chapman-Jouguet velocity for this mixture, can readily be observed; the period Δt_p of these cyclic variations is measured at about 7.2 ms. It was found that, for a given dilution, the galloping wave is produced at higher initial pressures for helium as compared to argon dilution. Arguments based on computed induction times were used to explain the influence of dilution and diluent on the onset of the galloping regime. These arguments could not be used, however, to explain the properties of the galloping waves, in particular their specific amplitude and period. Since that study has shown that the features of the galloping wave were independent from the initiation conditions, they must therefore be intrinsic properties of the confined mixture.

Eckett *et al.* (1997) [6] proposed an analytical model for the direct initiation of gaseous detonation which identified unsteadiness in the induction zone as the primary physical mechanism by which a detonation might fail to initiate. Since the galloping propagation mode can be assimilated to successive direct initiations followed by failure, we use this approach to study the stability of detonation waves subject to velocity fluctuations. In particular, we use Eckett's model in the context of the unstable propagation regimes that were experimentally observed.

By applying the usual conservation equations in an unsteady 1D reaction zone behind a shock wave and assuming a first order Arrhenius reaction-rate law with large activation energy E_a, Eckett *et al.* (1997) [*op. cit.*] obtain an asymptotic time-evolution equation for the temperature T as:

$$(1-M_s^2)C_p\frac{DT}{Dt} = \frac{(1-\gamma M_s^2)qk}{\exp(2E_a/RT_s)}\exp(E_aT/RT_s^2) + 6\frac{(\gamma-1)}{(\gamma+1)^2}U\frac{dU}{dt}\ , \tag{1}$$

where the strong shock approximation has been used, and M_s, C_p, R, q, k, γ and U are, respectively, the shock Mach number, the constant pressure specific heat, the perfect gas constant, the heat of reaction,

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the pre-exponential rate multiplier, the ratio of specific heats and the shock velocity; subscript s indicates the conditions just after the shock wave.

In particular, it is found that thermal runaway, i.e., initiation (corresponding to $T \rightarrow \infty$), is achieved within finite time provided that the so-called initiation parameter α, which characterizes the unsteadiness of the leading shock, remains below 1. Again assuming strong shock conditions, the initiation parameter α is obtained as:

$$\alpha = -\frac{12(\gamma-1)\exp(E_a/RT_s)}{qk(3-\gamma)(\gamma+1)^2}\, U\frac{dU}{dt} \ . \tag{2}$$

In our first attempt to apply this model to unstable detonations, we prescribe, *a priori*, the motion of the leading shock to be purely sinusoidal with an amplitude $A/2$ and a period Δt_p, around a mean value U_m, as $U(t) = U_m\,(1 + A/2\cos(\omega t))$, where $\omega = 2\pi/\Delta t_p$.

For example, the parameters A and Δt_p can be obtained from our experimental velocity histories. The time evolution of the initiation parameter α for two particular cases is shown in Fig. 2. Figure 2a shows the results for the galloping detonation of Fig. 1. It can be seen that, if failure occurs for $\alpha > 1$, this model identifies the deceleration of the leading shock wave as the mechanism by which the galloping detonation fails. Furthermore, when $\alpha < 1$ this can lead to reinitiation. Using the data from Fig. 2a to interpret the velocity history of Fig. 1, we can see that failure takes place between the velocity peak and the velocity minimum and that reinitiation occurs just at the minimum.

The time evolution of the initiation parameter α for another propagation mode is presented in Fig. 2b: this applies to the stuttering mode, obtained in the same mixture at p_1=1.33 kPa. For the stuttering mode, which is characterized by rapid velocity fluctuations at a frequency around 5 kHz with a small amplitude of about 0.1 V_{CJ} around a mean value of 0.9 V_{CJ}, Fig. 2b shows that α always remains below 1 and therefore that the detonation is not supposed to fail in this regime.

For a given mixture, α varies with U_m, $A/2$ and Δt_p. To examine the stability of unsteady detonations subject to sinusoidal variations of leading shock velocity, we have computed the conditions, defined by the parameters U_m, $A/2$ and Δt_p, which produce the limiting behavior $\alpha = 1$. These results are shown in Fig. 3 for three values of average shock velocity U_m. A datum on any one of these curves defines the required amplitude and period of the velocity fluctuations of the leading shock wave which would produce extinction of the detonation at some point in the cycle. Any amplitude-period combination above the curve would produce extinction and any combination below could be qualified as "stable." It is seen that increasing the average velocity effectively increases the area of the $A - \Delta t_p$ region for which the detonation is stable. It can also be seen that, according to this model, a detonation can stably sustain velocity perturbations as long as their amplitude is sufficiently small or that their period is sufficiently long. The two cases presented in Fig. 2 are also shown in Fig. 3. The galloping case (square), corresponding to an average velocity $U_m = 0.75V_{CJ}$, is seen to lie above the corresponding curve while the stuttering case (circle), corresponding to an average velocity $U_m = 0.90V_{CJ}$, lies below the corresponding curve.

Our efforts are now directed towards acquiring a truly predictive capability by coupling the motion of the shock to that of the reaction zone.

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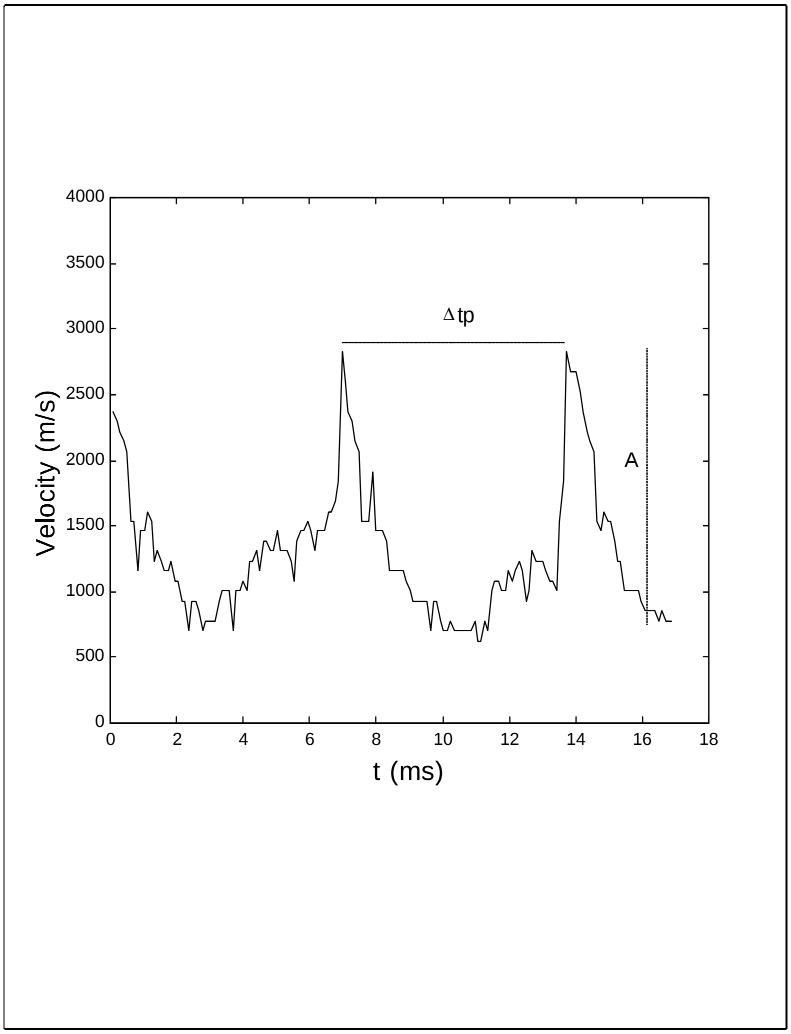

Figure 1: Velocity versus time for $C_3H_8/5O_2$ at $p_1 = 0.8$ kPa.

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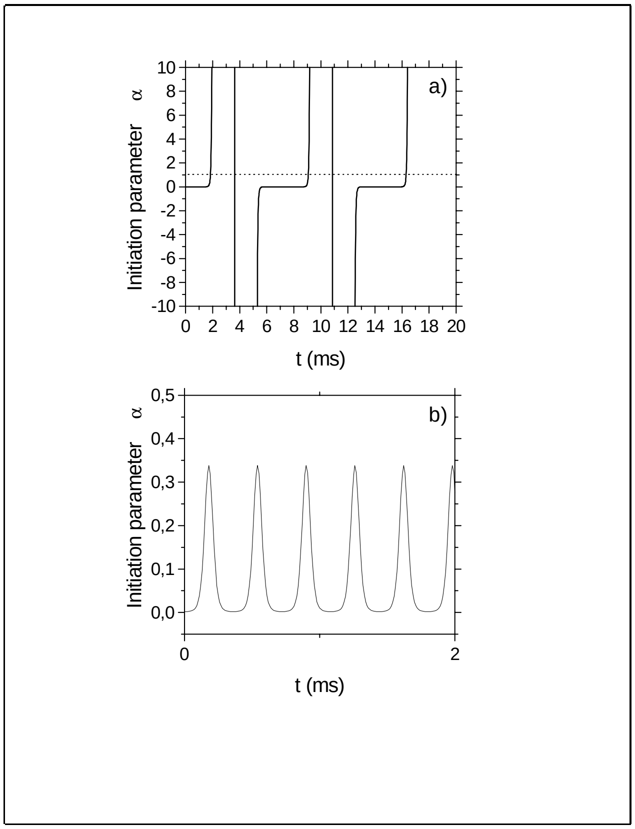

Figure 2: Initiation parameter α versus time for $C_3H_8/5O_2$ at: a) $p_1 = 0.8$ kPa — galloping detonation, and b) $p_1 = 1.33$ kPa — stuttering detonation.

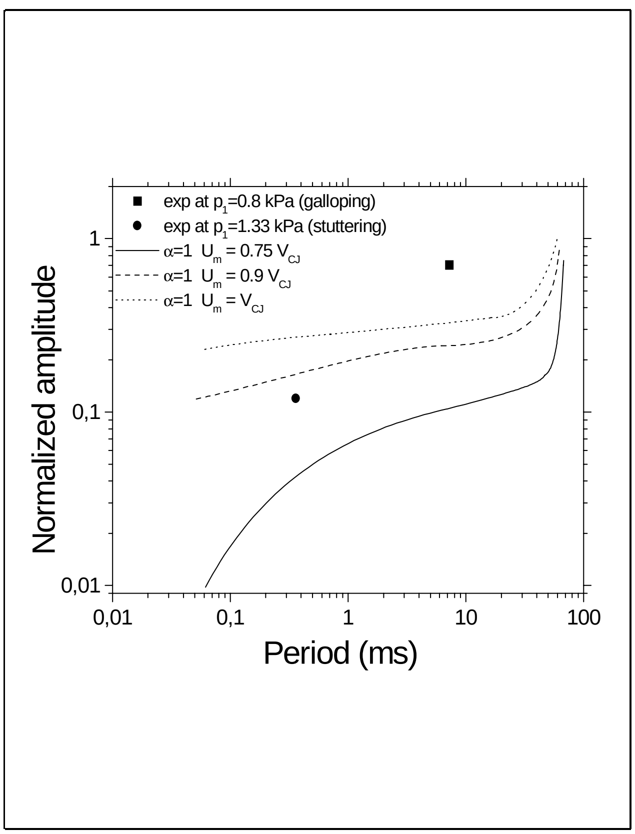

Figure 3: Conditions, in amplitude–period space, which produce the limiting behavior $\alpha = 1$ for the $C_3H_8/5O_2$ mixture.