Dynamics of Curved Flames in Closed Tubes

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Abstract

Recent analytical and numerical results on interaction of curved flames with acoustic and shock waves are reported, as well as numerical simulations of flame dynamics in closed tubes. Analytical scalings for the problem of flame-acoustic interaction are obtained including both stabilization of the Darrieus-Landau instability by sound waves of small amplitudes and excitation of the parametric flame instability by sound waves of sufficiently large amplitudes. Investigation of flame-shock collisions shows that it may result in both stabilization and destabilization of the flame front depending on the shock intensity and on the expansion coefficient of burning matter. It is found that flame dynamics in a closed tube depends strongly on the reaction order. The flame-acoustic coupling in a closed tube may be so strong that it may lead to detonation triggering.

Introduction

Flame dynamics in closed tubes is a very important subject of combustion science since it models the burning process in a spark ignition engine. Still many problems of flame dynamics in closed tubes remain unsolved such as interaction of a flame with acoustic waves under confinement [1, 2]. Even detonation triggering and knock in engines is described up to now in a rather empiric way despite of its industrial importance. The other process that requires better understanding is spontaneous development of a curved shape of a flame front in enclosure called the "tulip flame" phenomenon [3, 4]. It is a general belief that "tulip flames" are directly related to the Darrieus-Landau (DL) instability, which makes a planar flame front corrugated [5]. When a flame propagates in a closed burning chamber, then many additional effects influence flame dynamics and development of the DL instability in comparison with the cofiguration of a flame in an unbounded fuel [4-7]. One of these effects is generation of pressure waves by a moving flame and interaction of flames with these waves. The generated pressure waves may be described as weak shocks if the typical time needed for sound to traverse the burning chamber is longer than the characteristic time of flame dynamics. In the opposite case the pressure waves should be considered as acoustic waves. When a flame front acquires a curved shape because of the DL instability, the interaction of acoustic waves with the flame front becomes more effective, which makes easier the spontaneous reaction in the fresh fuel ahead of the flame front with possible detonation ignition.

In the present paper we report recent analytical and numerical results on interaction of curved flames with acoustic and shock waves, as well as numerical simulations of flame dynamics in closed tubes. Analytical scalings for the problem of flame-acoustic interaction are obtained including both stabilization of the DL instability by sound waves of small amplitudes and excitation of the parametric flame instability by sound waves of sufficiently large amplitudes. The obtained stability limits are in excellent agreement with experimental results on propane flames. Investigation of flame-shock collisions shows that it may result in both stabilization and destabilization of the flame front depending on the shock intensity and on the expansion coefficient of burning matter. It is found that flame dynamics in a closed tube depends strongly on the reaction order: for flames with a first order reaction the flame front is typically flattened compared to the case of an open tube, while for flames with a third order reaction flame-acoustic and flame-shock interaction leads to additional bending of the flames front in the second half of the tube. The flame-acoustic coupling may be so strong that it may lead to detonation triggering at the end of the tube.

Analytical scalings for flame-acoustic interaction

Interaction of acoustic and shock waves with a slightly curved almost planar flame front may be described on the basis of equation for linear perturbations $F(\mathbf{x},t) = F(t)\exp(i\mathbf{k}\cdot\mathbf{x})$ at a flame front in a gravitational field derived in [8]

$$A\frac{d^2F}{dt^2}+BU_f k\frac{dF}{dt}-CU_f^2k^2F-C_1gkF=0, \qquad (1)$$

where U_f is the planar flame velocity and the gravitational field $\mathbf{g}$ is directed normally to the initially planar flame. The effective gravitational field produced by pressure waves depends on time as $g=-\omega U_a\cos(\omega t)$ for acoustic waves of frequency ω and amplitude U_a. The numerical factors in Eq. (1) depend on the thermal-chemical fuel properties, such as the expansion coefficient of the fuel Θ defined as the ratio of the fuel density to the density of the burnt matter. For example, in the case of unit Lewis number and a constant coefficient of thermal conduction the numerical factors take the form

$$A=1,\quad B=\frac{2\Theta}{\Theta+1}\left(1+\frac{\Theta\ln\Theta}{\Theta-1}kL_f\right),\quad C=\Theta\frac{\Theta-1}{\Theta+1}\left(1-\frac{k\lambda_c}{2\pi}\right),\quad C_1=\frac{\Theta-1}{\Theta+1}, \qquad (2)$$

where L_f is the flame thickness and λ_c is the cut-off wavelength for which the DL instability is suppressed by thermal conduction. The cut-off wavelength is proportional to the flame thickness with a large coefficient exceeding 20 for realistic flames with $\Theta=5\text{-}10$: $\lambda_c/2\pi L_f=1+\Theta\ln\Theta(\Theta+1)(\Theta-1)^{-2}$. Expressions for the coefficients A, B, C, C_1 in a more general case of an arbitrary Lewis number and temperature dependent thermal conduction may be found in [2]. In absence of gravity, either real or effective, flame front perturbations grow as $F(t)=F\exp(\sigma_0 t)$ with the DL instability growth rate $\sigma_0=\Gamma U_f k(1-k\lambda_c/2\pi)$ and the numerical factor $\Gamma=\Theta\left(\sqrt{\Theta+1-1/\Theta}-1\right)/(\Theta+1)$.

Flame-acoustic interaction involves two effects [2, 9]: suppression of the DL instability by high frequency sound waves of moderate amplitudes and the parametric instability of a flame front induced by sound waves of a sufficiently large amplitude. Stabilization of a flame front by acoustic waves is similar to the stabilization of a pendulum in a gravitational field with oscillating acceleration. In that case the approximate solution of Eq. (1) consists of a slow component $F_1(t)$ that varies on a time scale of the DL instability and a fast component oscillating with the acoustic frequency: $F=F_1(t)+F_2\cos(\omega t)$. It follows from Eq. (1) that the DL instability is suppressed by the sound waves, if the amplitude of the sound waves exceeds the critical amplitude $U_{a1}=U_f\sqrt{2\Theta(\Theta+1)/(\Theta-1)}$. An interesting point is that the critical acoustic amplitude is determined by the fuel expansion coefficient only and it is independent of the internal flame structure. For flames with realistic expansion coefficients the critical acoustic amplitude is $U_{a1}/U_f=4\text{-}5$.

The parametric instability of a flame front in an acoustic wave implies oscillations of the perturbation amplitude with a period twice larger than the acoustic period, so that the perturbations may be described approximately as $F(t)=F\exp(\sigma t+i\omega t/2)$. At the stability limits one has $\sigma=0$ and the second critical acoustic amplitude follows from Eq. (1):

$$U_{a2}^2/U_f^2=\left(\frac{B}{C_1}\right)^2+\left(\frac{A\omega^2+4CU_f^2k^2}{2C_1\omega U_f k}\right)^2. \qquad (3)$$

The parametric instability develops when the acoustic amplitude is larger than the second critical value $U_a>U_{a2}$, which is about $U_{a2}\approx 5U_f$ for rather small acoustic frequencies $\omega<U_f/L_f$ and may increase with increase of frequency up to $U_{a2}\approx 20U_f$ for $\omega=10U_f/L_f$. Close to the stability limits development of the parametric instability results in a cellular structure at a flame front with a wavelength $\lambda=(20\text{-}40)L_f$ depending on the internal flame structure. For example, for a propane flame in an acoustic wave of frequency $\omega=U_f/L_f$ the characteristic wavelength of the parametric instability is $\lambda\approx 41L_f$. The obtained analytical scalings are in excellent agreement with the experimental results on propane flames in an acoustic wave [2].

Flame interaction with weak shocks

In the case of a head-on collision of a curved flame with a weak shock the acceleration in Eq. (1) takes the form $g=\Delta U\delta(t)$, where ΔU is the velocity jump in the shock behind the flame front after the collision. The velocity jump is related to the intensity of the original shock ΔU_0 as $\Delta U=2\sqrt{\Theta}\Delta U_0/\left(\sqrt{\Theta}+1\right)$. The first attempt of theoretical analysis of flame-shock interaction has been undertaken in [9], where linear response of a slightly perturbed almost planar flame to a passing shock has been considered. However, unrealistic initial conditions at the flame front adopted in [9] led to incorrect conclusions about the results of the flame-shock collision. Accurate

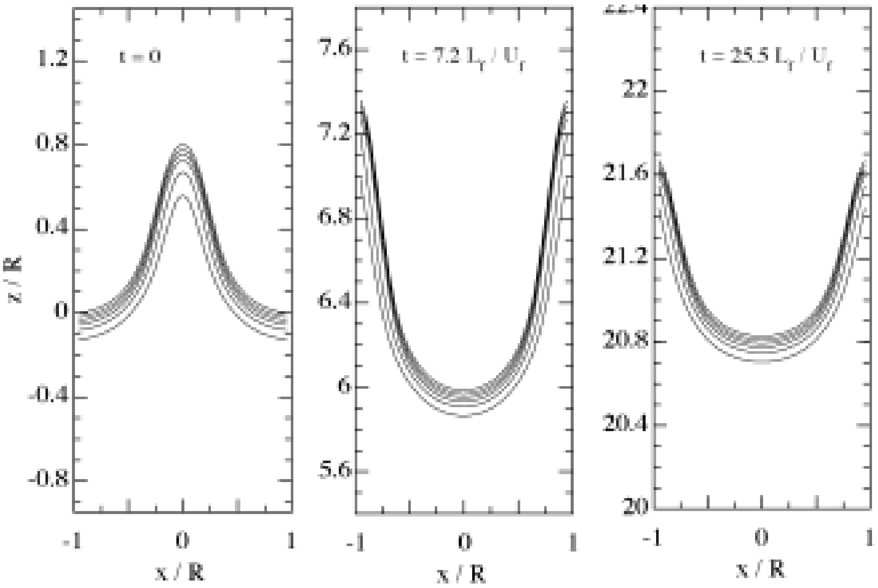

Fig. 1. Flame isotherms for $\Theta = 8$ and $R = \lambda_c$ at different time instants after the flame collision with a shock of intensity $\Delta U_0 = 9U_f$. The flame in the figures propagates downwards.

solution of the problem of the linear flame response to a passing weak shock has been obtained in [10] on the basis of Eq. (1). It was obtained that a shock may invert the flame shape with convex parts of the flame front becoming concave and vice versa, if the shock intensity is larger than the critical value ΔU_c proportional to the flame velocity U_f:

$$\Delta U_c = \frac{U_f}{2\sqrt{\Theta}} \frac{\Theta + 1}{\sqrt{\Theta} - 1} \left(\frac{2\Gamma}{\Theta} \frac{\Theta + 1}{\Theta - 1} C + B \right). \tag{4}$$

It was shown that a head-on collision of a shock and a perturbed flame may cause temporary stabilization or additional destabilization of the flame front. According to the linear theory, shocks of relatively small intensities lead to temporary stabilization of the flame. The stabilization is accompanied by inversion of the flame shape if $\Delta U_c < \Delta U_0 < 2\Delta U_c$, and it happens without flame inversion if $\Delta U_0 < \Delta U_c$. Collision of a shock of a large intensity $\Delta U_0 > 2\Delta U_c$ and a perturbed flame front results in inversion of the flame shape with additional destabilization of the flame. The solution of the linear problem of a flame-shock collision provides scalings for the nonlinear problem of interaction of a curved flame and a weak shock.

2D numerical simulations of a flame-shock collision in tubes with ideally slip and adiabatical walls on the basis of the complete set of hydrodynamical equations including thermal conduction, viscosity, fuel diffusion and chemical kinetics confirm predictions of the linear theory. Simulations allow one to study nonlinear flame-shock interaction for the case of a considerably curved flame front. In that case the basic configuration corresponds to a curved stationary flame, so that stabilization and destabilization of the flame front by a shock imply increase or decrease of the flame curvature in comparison with the stationary one. In agreement with the linear theory [10] we have obtained three possible outcomes of a head-on collision of a curved stationary flame and a weak shock depending on the shock intensity and the flame parameters: 1) temporary flame stabilization with some flattening of the flame front by weak shocks; 2) the stabilization is accompanied by inversion of the flame shape for stronger shocks; 3) inversion of the flame shape happens together with flame destabilization for very strong shocks as presented in Fig. 1. Particularly, according to the numerical simulations of a flame-shock collision in the case of $\Theta = 8$ and the tube width $R = \lambda_c$ (corresponding to $k = \pi / \lambda_c$ in Eq. (1)) the critical shock intensity needed to invert the flame shape is $\Delta U_c = 3.7U_f$, and the linear theory predicts practically the same value $\Delta U_c = 3.6U_f$. An interesting point is that the citical shock intensity is almost independent of the expansion coefficient being $\Delta U_c / U_f = 3.6\text{-}3.9$ in the domain of flames with realistic expansion coefficients $\Theta = 5\text{-}10$. In the case of flame destabilization by a shock agreement of the linear scalings with the numerical simulations is not so perfect. For example, for the above flame parameters the linear theory predicts flame destabilization for a shock intensity $\Delta U_0 > 7.2U_f$, while the numerical simulations demonstrate destabilization for $\Delta U_0 > 5.2U_f$. Finally, both the linear theory and the numerical simulations show strong destabilization of a curved flame with considerable increase of the flame curvature if the shock comes to the flame front from the burnt matter. In the last case the flame velocity after the collision may exceed considerably the velocity of a curved stationary flame. Still flame stabilization or destabilization by a single weak shock are always temporary with the typical time scale determined by the inverse DL instability growth rate $\Delta t \propto \sigma_0^{-1}$. Flame relaxation back to the stationary state after the flame-shock collision takes about $4\Delta t$.

Flame dynamics in a closed tube

The frequency of an acoustic wave in a closed tube is determined by the tube length L and by the sound speed c_s as $\omega = \pi c_s / L$. Thus the flame generated pressure waves with $\omega / \sigma_0 >> 1$ should be considered as acoustic waves in the case of short tubes $L << R / M$, where $M = U_f / c_s$ is the Mach number and R is the tube radius. In the opposite limit of long tubes $L >> R / M$ pressure waves take the form of weak shocks. Amplitudes of acoustic waves and weak shocks generated by a flame in a closed tube are not independent values, but instead they are determined by flame dynamics. Particularly, when a flame propagates from a closed wall it pushes a weak shock of intensity $\Delta U_0 = (\Theta - 1)U_f$. Taking the last value as an evaluation of the shock intensities in a long closed tube with $L >> R / M$, one can conclude that weak shocks destabilize a flame front in enclosure for the expansion coefficients $\Theta = 6 - 10$. For smaller expansion coefficients a flame front is stabilized with the strongest stabilization expected for $\Theta \approx 5$. In a similar way, the amplitude of the acoustic waves in short tubes may be evaluated as $U_a = (\Theta - 1)U_f$, so that according to the above scalings the acoustic waves should lead to strong stabilization of the DL instability for flames with realistic expansion coefficients $\Theta = 5\text{-}10$. At the same time the amplitude of the sound waves is not so strong to produce a noticeable parametric instability.

We studied dynamics of laminar flames in closed tubes by means of 2D numerical simulations taking into account thermal conduction, fuel diffusion, viscosity and chemical kinetics [7]. Development of the DL instability has been investigated for flames with chemical reactions of the first and third order. The basic difference between these two cases is that the normal flame velocity U_f increases with the pressure build-up cuased by burning under confinement for the third order reactions, and it increases for the first order reactions [5]. Other dimensionless flame parameters were taken $\Theta = 5$, $M = 10^{-2}$ with the tube width $R = 20L_f$ and different tube lengths $L / L_f = 100; 200; 500; 2000$. We found that the DL instability is strongly reduced or even suppressed for a flame with a first order reaction in closed tubes of all considered tube lengths in agreement with the above conclusions. The stabilization was especially strong for short tubes $L / L_f < 200$ when the acoustic frequency is larger. In longer tubes the pressure waves took the form of weak shocks and flame stabilization was somewhat weaker. Flame dynamics was more complicated in the case of a third order reaction. While in the first half of a tube flames with first and third order reactions behave in a similar way, close to the end of the tube strong nonlinear coupling of a flame of a third order reaction with acoustic waves has been observed. The coupling resulted in considerable destabilization of the flame front and even in detonation triggereing in the fresh fuel ahead of the flame.

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