Dynamics and Stability of Curved Flames and Development of Fractal Flame Structure

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Abstract

Analytical and numerical solutions of two basic problems of combustion theory are obtained, which are the problems of dynamics and stability of curved stationary premixed flames. A curved stationary shape develops at an initially planar flame front in tubes of a moderate width as a result of the Darrieus-Landau instability. It is found that the curved shape amplifies the flame velocity by the factor 1.25-1.35 for 2D flames and by the factor 1.6-1.8 for 3D flames. Stability analysis shows that curved stationary flames become unstable in wide tubes with respect to the secondary Darrieus-Landau instability. The critical tube width for the secondary instability is calculated. Because of the secondary instability extra cusps arise at a flame front leading to additional increase of the flame velocity. On the basis of the obtained results the fractal dimension of a flame is evaluated.

Introduction

Dynamics of curved premixed flames deserves much attention in combustion science since, as it was observed experimentally [1-3], a flame almost never propagates as a planar front. Quite often a curved shape of a flame develops because of the hydrodynamic Darrieus-Landau (DL) instability [1]. According to the linear theory of the DL instability [4] a planar flame in a gaseous fuel is unstable against all perturbations bending the flame front, if the perturbation wavelength exceeds the cut-off wavelength λ_c determined by thermal conduction and by the finite thickness of a flame front. The instability growth rate depends on the expansion coefficient Θ of the flame defined as the ratio of the fuel density to the density of the burnt matter, which takes the values $\Theta = 5-10$ for most laboratory flames. The cut-off wavelength is proportional to the flame thickness with a large numerical factor about 20 and larger, while perturbations of a shorter wavelength $\lambda < \lambda_c$ are suppressed by thermal conduction.

Outcome of the DL instability at the nonlinear stage has been a subject of long discussions starting from the original papers by Darrieus and Landau. First it was assumed that the DL instability leads to flame self-turbulization [1]. Then it was proposed that the instability results in a smooth curved stationary shape of a flame front instead of the self-turbulization [5]. The stationary shape may be described qualitatively as a large smooth hump directed towards the fresh fuel and a cusp pointing to the burnt matter as shown in the first part of Fig. 1. For a long time the theory of the nonlinear stage of the DL instability was restricted to qualitative estimates [5, 6]. The rigorous solutions were obtained only in the peculiar limit of a small expansion coefficient $\Theta - 1 << 1$ [7, 8], which is quite far from the case of realistic laboratory flames. Therefore the theory of curved flames with small expansion coefficients provided only qualitative, but not quantitative description of the nonlinear stage of the DL instability. Besides, the theory [7, 8] predicted that curved stationary flames are always linearly stable independent of the radius of curvature of the flame, which obviously contradicts the basic physical understanding of flame dynamics and stability [6].

In the present paper we report recent results of the nonlinear theory of dynamics and stability of flames with realistic expansion coefficients $\Theta = 5-10$. The theoretical results on the velocity amplification caused by the curved shape of stationary flames are in a very good agreement with the results of direct numerical simulations of flame dynamics in tubes predicting the velocity increase by the factor 1.25-1.35 for two-dimensional (2D) flames and by the factor 1.5-1.7 for three-dimensional (3D) flames. Stability analysis of curved 2D stationary flames with realistic expansion coefficients shows that stationary flames do become unstable as soon as the size of the flame hump exceeds the cut-off wavelength of the DL instability by a factor about 3.9-4.2. On the basis of the obtained results we evaluate the fractal dimension of a flame as 1.18-1.22 for 2D fractal flames and as 2.3-2.35 for 3D flames. The estimates of the fractal dimension agree well with experimental observations of self-accelerating spherical flames.

Curved stationary flames

We have derived the nonlinear equation for curved stationary flames with realistic expansion coefficients [9], which allows quantitative investigation of properties of curved flames. In the reference frame of a curved stationary flame front $z = F(\mathbf{x}) - U_w t$ the nonlinear equation takes the form

$$1 - U_w / U_f + \frac{\Theta}{2}(\nabla F)^2 + \frac{(\Theta-1)^3}{16\Theta}\left[(\nabla F)^2 - \left(\hat{\Phi}F\right)^2\right] = \frac{\Theta-1}{2}\left[\hat{\Phi}F + \frac{\lambda_c}{2\pi}\nabla^2 F\right], \tag{1}$$

where U_f is the velocity of a planar flame, the operator $\hat{\Phi}$ is defined as

$$\hat{\Phi}F = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} |\mathbf{k}| F_k \exp(i\mathbf{k}\cdot\mathbf{x}) d\mathbf{k}, \tag{2}$$

and F_k is the Fourier transform of F. The nonlinear equation (1) is valid for any kind of fuel (both viscous and inviscid) with any temperature dependence of the transport coefficients and any Lewis number for which the thermal-diffusion instability of a flame does not happen. The transport properties of the fuel influence only the expression for the cut-off wavelength λ_c, while the nonlinear terms are not affected by the fuel properties and the mathematical structure of Eq. (1) remains the same independent of a particular premixed fuel.

The stationary nonlinear equation predicts quite well the velocity amplification of curved stationary flames both for the cases of 2D flames [9] and 3D flames in cylindrical tubes [10]. Particularly, it follows from Eq. (1) that a planar flame front in a tube with ideal walls acquires a smooth curved shape with the velocity of the curved flame U_w exceeding the respective planar flame velocity U_f, if the tube width is larger than some critical value $R > R_c$. In the configuration of 2D curved flames the critical tube width is $R_c = \lambda_c / 2$ because of the ideal boundary conditions at the tube walls that can be interpreted also as symmetry axes. The velocity amplification for 2D curved stationary flames depends on the tube width R as [9]

$$U_w / U_f - 1 = 4\beta_{2D}\frac{MR_c}{R}\left(1 - \frac{MR_c}{R}\right), \tag{3}$$

where $M = \mathrm{Int}\left[R/2R_c + 1/2\right]$ and the factor β_{2D} characterizing the maximal velocity increase $\max U_w / U_f = 1 + \beta_{2D}$ is a function of the expansion coefficient

$$\beta_{2D} = \frac{1}{2}\frac{\Theta(\Theta-1)^2}{\Theta^3 + \Theta^2 + 3\Theta - 1}. \tag{4}$$

The analytical formula for the velocity amplification of curved stationary flames Eqs. (3), (4) is in a very good agreement with results of direct numerical simulations of flame dynamics in tubes of moderate width with ideally slip and adiabatic walls [11] performed on the basis of the complete set of hydrodynamic equations including thermal conduction, viscosity, fuel diffusion and chemical kinetics.

Dynamics of 3D curved stationary axisymmetric flames in cylindrical tubes has been investigated as an eigenvalue problem Eq. (1) in [10] and by use of direct numerical simulations of the complete set of hydrodynamic equations in [12]. Dynamics of 3D curved flames involves many specific features such as existence of two principally different solutions corresponding to convex and concave flames, the nonlinear DL instability in narrow tubes with $R < R_c$, etc. Still basic properties of the principal solution for the 3D axisymmetric convex flame are similar to the properties of 2D curved stationary flames. At the same time the characteristic velocity amplification $\beta_{3D} = U_w / U_f - 1$ for the 3D flames is about twice larger than the respective velocity amplification for 2D flames and may be described well by the formula

$$\beta_{3D} = 2\beta_{2D} = \frac{\Theta(\Theta-1)^2}{\Theta^3 + \Theta^2 + 3\Theta - 1}. \tag{5}$$

Thus, development of a curved stationary shape at a flame front with a realistic expansion coefficient $\Theta = 5 - 10$ results in amplification of the flame velocity by the factor $U_w / U_f = 1.25 - 1.35$ in 2D configurations and by the factor $U_w / U_f = 1.5 - 1.7$ in 3D configurations.

Stability of curved stationary flames

In order to investigate stability of curved stationary flames a time dependent version of the nonlinear equation (1) has been derived and then linearized around the stationary solutions $F_s(\mathbf{x})$ with respect to small perturbations $\tilde{F}(\mathbf{x},t)=\tilde{F}(\mathbf{x})\exp(\sigma t)$. Solution of the eigenvalue stability problem for 2D curved flames in tubes with ideally slip and adiabatic walls on the basis of the linearized equation shows that curved stationary flames do become unstable as soon as the tube width exceeds some critical value R_w. Below we will call R_w the second critical

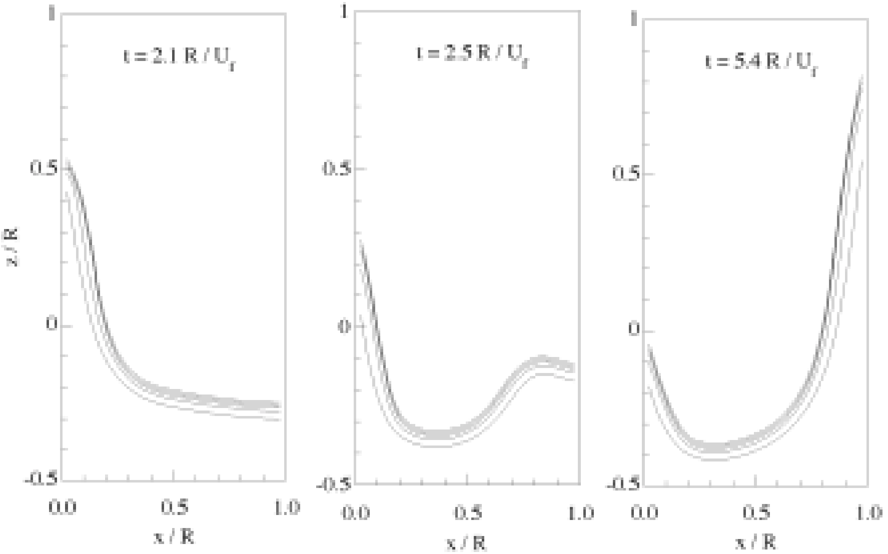

Fig. 1. The secondary DL instability at a flame front with $\Theta = 8$ in a tube of width $R = 4R_c$ at different time instants after the beginning of calculations. The flame in the figures propagates downwards.

tube width in order to distinguish it from the first critical tube width R_c. Though a priori the growth rate σ of the obtained secondary DL instability might be an arbitrary complex value, numerical solution of the eigenvalue stability problem shows that σ is real for all problem parameters with $\operatorname{Im}\sigma = 0$. Therefore, the condition $\operatorname{Re}\sigma = 0$ at the stability limits implies also $\sigma = 0$, so that the stability limits may be found on the basis of the linearized equation for curved stationary flames (1)

$$\Theta\nabla F_s\nabla\tilde{F}+\frac{(\Theta-1)^3}{8\Theta}\left(\nabla F_s\nabla\tilde{F}-\hat{\Phi}F_s\hat{\Phi}\tilde{F}\right)-\frac{\Theta-1}{2}\left(\hat{\Phi}\tilde{F}+\frac{R_c}{\pi R_w}\nabla^2\tilde{F}\right)=0 \quad (6)$$

with the ratio R_c / R_w playing the role of an eigenvalue of Eq. (6). Numerical solution of the linearized equation gives the stability limits $R_w / R_c \approx 4.2$ for curved stationary flames with realistic expansion coefficients $\Theta = 5-10$. The ratio of the second and first critical tube width somewhat increases with decrease of the expansion coefficient: e.g. $R_w / R_c = 4.35$ for $\Theta = 3$. Taking into account the results [8] for flames with small expansion coefficients $\Theta - 1 << 1$ one may conclude that the second critical tube width becomes infinite $R_w / R_c \to \infty$ as the expansion coefficient goes to unity $\Theta \to 1$. Direct numerical simulations of 2D curved flames in wide tubes with ideally slip and adiabatical walls on the basis of the complete set of hydrodynamical equations confirm the theoretical predictions. Particularly, according to the numerical simulations the second critical tube width is $R_w / R_c = 3.6;\ 3.7;\ 3.95$ for flames with the expansion coefficients $\Theta = 10;\ 8;\ 6$, which is only slightly smaller than the theoretical predictions.

In wider tubes $R > R_w$ the secondary DL instability develops at a curved stationary flame front. As illustrated in Fig. 1 development of the secondary instability close to the stability limits leads to an extra cusp arising at the initially smooth hump of the stationary flame. The second cusp may become even deeper than the primary one, though quite often the resulting new regime of flame propagation is not stationary with the depth of the cusps pulsating in time. Another important point is that the new shape of the flame front results in considerable increase of the flame velocity in comparison with the maximal velocity of curved stationary flames. For example, it is observed that the flame with the expansion coefficient $\Theta = 8$ propagates in a tube of width $R_w = 4R_c$ with velocity $U_w = 1.82U_f$ exceeding noticeably the respective maximal velocity of curved stationary flames $U_w = 1.33U_f$. Taking into account physical similarity between the primary and secondary DL instabilities one should expect that the secondary instability leads to amplification of the flame velocity by the factor $U_w / U_f = (1+\beta_{2D})^2$ for the configuration of 2D flames in ideally slip and adiabatic tubes of widths

$R_w < R < R_w^2 / R_c$. This estimate is in agreement with the results of direct numerical simulations of flames in wide tubes.

Discussion: fractal flame structure

For wider tubes $R >> R_w$ further development of the DL instability is expected leading to a fractal flame structure similar to that observed in the experiments [13]. A fractal structure of a flame front implies cascades of humps and cusps of different sizes imposed one on another. Let us assume that every step of the cascade increases the size of the humps and the flame velocity by the values b and $1+\beta$, respectively, which is supported by the observed velocity amplification in the secondary DL instability. The largest length scale of the fractal flame structure is limited by the tube width R. Then the velocity of the fractal flame with a large number of cascades $N = \ln(R / R_c) / \ln b >> 1$ depends on the tube width as [14]

$$U_{fractal} = U_f (1+\beta)^N = U_f (R / R_c)^d, \qquad (8)$$

where $d = \ln(1+\beta) / \ln b$ is the excess of the fractal flame dimension over the embedding dimension. Evaluating the factor β for 2D fractal flames with the help of Eq. (4) and the factor b as $b_{2D} = R_w / R_d$ we find the estimate for the fractal excess d_{2D} of 2D flames with realistic expansion coefficients $d_{2D} = 0.18 - 0.22$, so that the fractal flame dimension is 1.18-1.22. The fractal dimension depends on the expansion coefficient of the flame increasing with increase of the fuel expansion. In the case of 3D flames the DL instability is stronger at the nonlinear stage and a larger fractal excess over the embedding dimension is expected. The velocity amplification for 3D flames on every step of the fractal structure is about twice larger than in the 2D case, which determines the factor β_{3D} Eq. (5). There are no results on stability limits of 3D curved stationary flames yet, therefore the only estimate for the factor b_{3D} available so far comes from the theory of 2D flames. Adopting the estimate $b_{3D} \approx 4$ in the 3D case we obtain the evaluation for the fractal excess of a 3D flame $d_{3D} = 0.3 - 0.35$ with the respective fractal dimension 2.3-2.35 for flames with realistic expansion coefficients. Though the last estimates are rather approximate, they agree well with the experimentally measured values 2.33 of the fractal dimension for spherically expanding laboratory flames [13].

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