

The interaction of acoustics and compressible combustion fronts - application to flames in tubes.

A.C. McIntosh, Fuel and Energy Department, University of Leeds, LEEDS, LS2 9JT, U.K.

Paper for the 17th International Colloquium on the Dynamics of Explosions and reactive Systems, Heidelberg, Germany, July 26th-30th 1999.

1. Introduction

In this paper, we address the question of the amplification of long wavelength acoustic waves reflected from fast deflagration waves where the entropy change across such fronts is significant, but where the structure of the deflagration is not resolved. Effectively the premixed combustion front with its associated entropy change is treated as a discontinuity in the flow with standard Rankine-Hugoniot jump conditions applied across it. If such a combustion front is (by some means) travelling fast enough to produce significant compressibility, there is then the possibility of the combustion front sending its own compression waves (i.e. strong acoustic disturbances) through the combustible mixture. This work considers acoustic waves interacting with a fast (but still subsonic) combustion front using the significant work of Ni and Goel (1995) which has pioneered this type of analysis. The combustion front, though one-dimensional, could in fact be turbulent in its structure, and the unsteady pressure disturbances would then be characterised by a long length scale.

With entropy no longer approximately conserved across the combustion front due to the high speed, the perturbation in pressure just before and after is now not the same, but one can obtain a connection between the amplitude of acoustic disturbances before and behind the front and which has (as a parameter) the dependence of burning rate on pressure. Thus the whole theory can be applied to fast turbulent as well as conventional laminar structures. For fast turbulent flames it is recognised that the mean mass burning rate may itself vary due to small but significant baroclinic terms. However in this simple model this effect is not included - the theory depends on a given input of the sensitivity of mass burning rate with pressure rise. This is termed χ and for a laminar premixed flame with an overall reaction rate characterised by an activation energy E'_a and flame temperature T'_b , it will have the approximate form

$$\chi = \frac{1}{2}(\gamma - 1)M_{01} \frac{E'_a}{RT'_b} . \quad (1)$$

where M_{01} is the Mach number of the propagation of the flame into the cold upstream flow. Essentially this arises from the small heating effect due to the pressure work term, which then slightly increases the flame temperature. The mass burning rate is very sensitive to flame temperature rises, due to the activation energy in the exponential of the Arrhenius reaction rate term of the energy equation. In general for laminar low speed flames, the value of χ is too small, but as pointed out by Ni and Goel (1995), it can be significant for a combustion front which may no longer be laminar, but is driven possibly by turbulence. This work explores the effects on compressibility as one allows χ to increase.

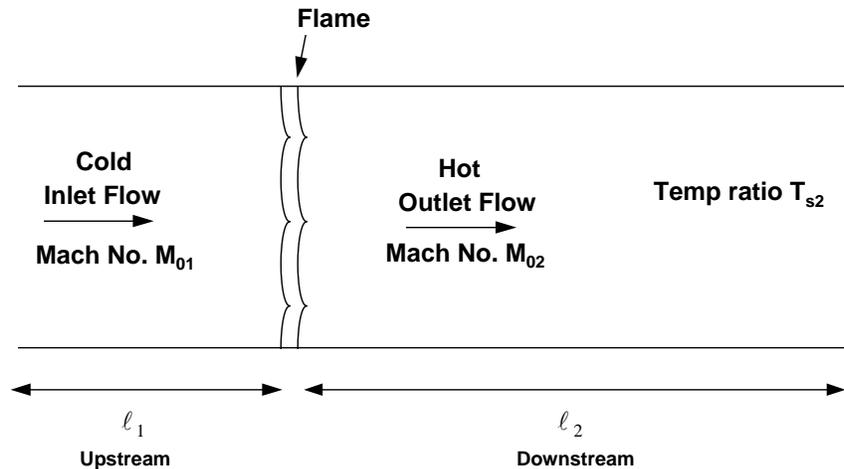


Fig. 1. Schematic of compressible combustion front within a tube which can undergo acoustic interactions.

In an earlier paper (McIntosh 1999) it has been shown, that by building on the conceptual advance of Ni and Goel (1995), one can construct frequency conditions which must be obeyed for acoustic oscillations interacting with such a compressible reaction front. These frequency conditions are typically of the form

$$\frac{\cosh(\omega \ell_1)}{\sinh(\omega \ell_1)} + \frac{M_{02}}{M_{01}} \frac{\cosh(\omega \ell_2 / \sqrt{T_{s2}})}{\sinh(\omega \ell_2 / \sqrt{T_{s2}})} = \chi \quad (2)$$

This is the form of the condition for a tube which is open at both ends. In this equation ω is complex frequency, ℓ_1 is the distance from one end of the tube to the flame, with ℓ_2 being the distance on the hot side of the flame (see Fig. 1). T_{s2} is the ratio of the temperature of the hot side of the flame to the cold and M_{01} , M_{02} are the Mach numbers of the flow in the upstream and downstream sides respectively. The condition for the tube closed at the cold end is

$$\frac{\sinh(\omega \ell_1)}{\cosh(\omega \ell_1)} + \frac{M_{02}}{M_{01}} \frac{\cosh(\omega \ell_2 / \sqrt{T_{s2}})}{\sinh(\omega \ell_2 / \sqrt{T_{s2}})} = \chi \quad (3)$$

and that for a tube closed at the hot end is

$$\frac{\cosh(\omega \ell_1)}{\sinh(\omega \ell_1)} + \frac{M_{02}}{M_{01}} \frac{\sinh(\omega \ell_2 / \sqrt{T_{s2}})}{\cosh(\omega \ell_2 / \sqrt{T_{s2}})} = \chi \quad (4)$$

If one knows how the mass burning rate of the front changes with pressure, then it is possible to predict the resonance or otherwise of fast compressible combustion waves within predefined geometries.

2. Results and discussion

Single waves from one side only.

Some initial results are indicated in the following figures. First of all we consider two simple cases:

(1) The transmission and reflection of an acoustic wave approaching from the hot side which has a length l_2 (where it should be noted that $\hat{\ell}_2 \equiv \ell_2 / \sqrt{T_{s2}}$ and T_{s2} denotes the ratio of the hot and cold zone temperatures.

Fig. 2 shows the plot of the growth rate (plotted as $\omega_r \hat{\ell}_2$) of the lowest modes, which are $\omega_i \hat{\ell}_2 = \pi/2$ and $\omega_i \hat{\ell}_2 = \pi$. This plot is for a given value of χ and $\hat{M} \equiv M_{02}/M_{01}$. The equation satisfied is the simpler relationship

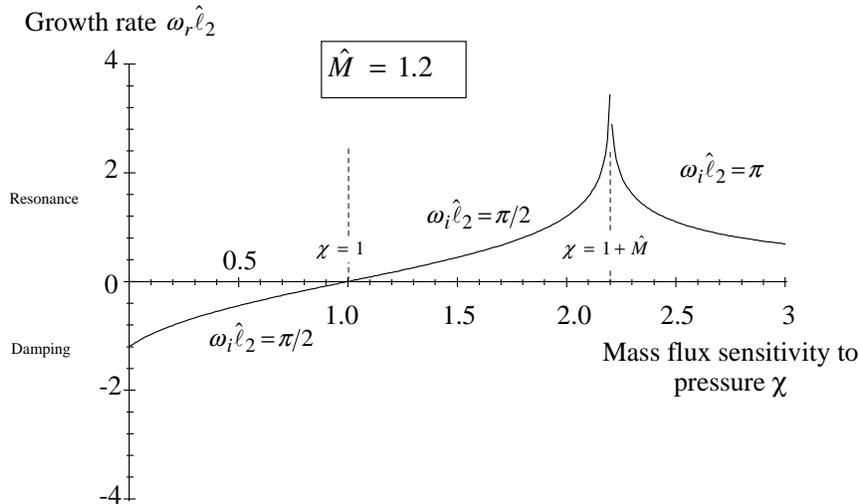


Fig. 2. Regions of resonance and damping for an acoustic signal approaching the fast combustion front from the hot side.

$$1 + \frac{M_{02}}{M_{01}} \frac{\cosh(\omega \ell_2 / \sqrt{T_{s2}})}{\sinh(\omega \ell_2 / \sqrt{T_{s2}})} = \chi \quad (5)$$

It is evident that only for $\chi < 1$ can one avoid resonance in this case.

(2) The transmission and reflection of an acoustic wave approaching from the cold side which has a length l_1 .

Fig. 3 shows a similar plot of the growth rate (but this time plotted as $\omega_i l_1$) again of the lowest modes, for the same value of χ and $\hat{M} \equiv M_{02}/M_{01}$. The simpler relationship is now

$$\frac{\cosh(\omega \ell_1)}{\sinh(\omega \ell_1)} + \frac{M_{02}}{M_{01}} = \chi \quad (6)$$

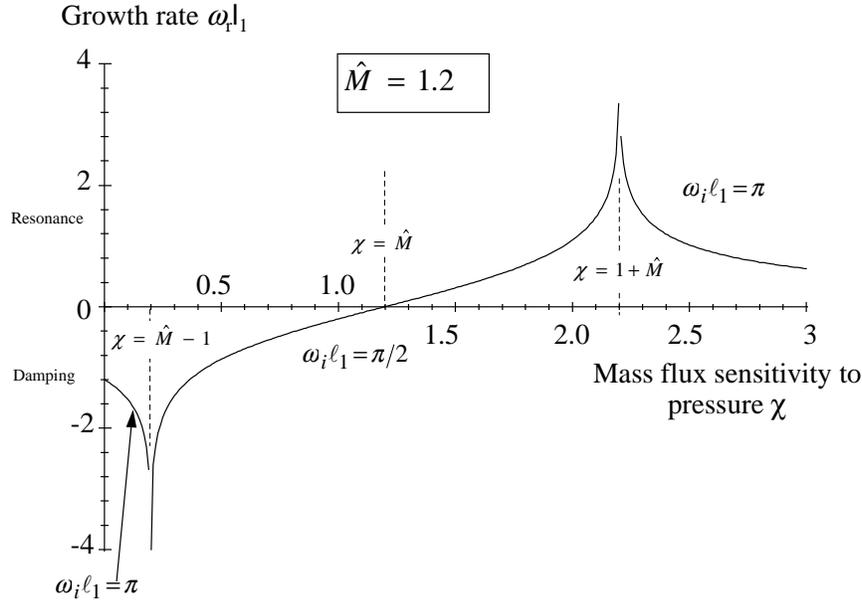


Fig. 3. Regions of resonance and damping for an acoustic signal approaching the fast combustion front from the cold side.

Resonance is now achieved when $\hat{M} < \chi < 1 + \hat{M}$ for the mode $\omega_i l_1 = \pi/2$ and when $\chi > 1 + \hat{M}$ for the mode $\omega_i l_1 = \pi$.

Multiple acoustic waves

If one now considers the solution to the problem of a front which is steadily progressing down a finite length tube, then one always obtains resonance, but with a distinct series of maxima in the growth rate as the combustion front progresses down the tube. Typical results are shown in Figures 4 and 5 for the case of an open tube (equation (2)). As the front reaches the extreme of the tube, the frequency increases (Fig. 4), and the amplitude of the growth rate cycles much more rapidly (Fig. 5) as the end is reached. It is also of interest to note that the *peak* growth rate increases as the flame comes to the end of the traverse. Both the plots are for a total tube length $l_1 + l_2 = 20$, mass burning rate sensitivity to pressure $\chi = 0.5$, $M_{02}/M_{01} = 1.2$. In these figures, the combustion front should be considered as travelling from right to left.

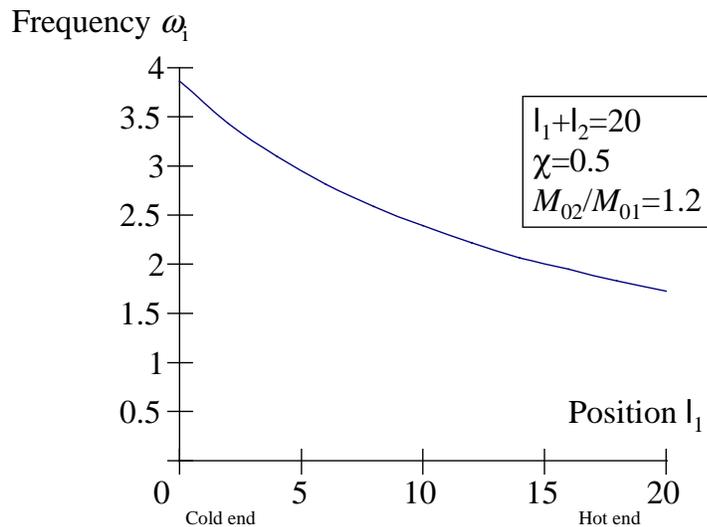


Fig. 4. Resonant frequency for a fast flame in a tube as a function of position.

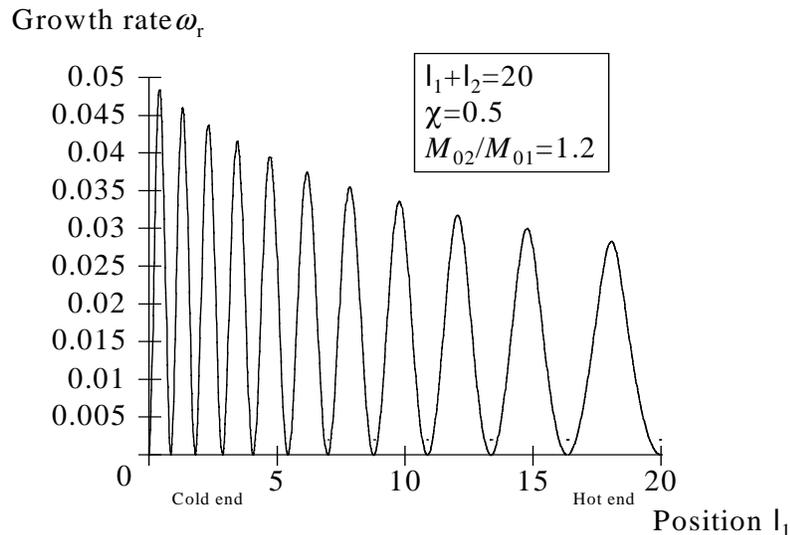


Fig. 5. Growth rate for resonance of a fast flame in a tube as a function of position.

These results are relevant to the propagation of flames in tubes where it has often been noted that there can be severe oscillations in the final stages of the propagation. Although this theory does not predict the propagation velocity (because the structure of the front has *not* been assigned), one can estimate the resonant behaviour of the wave given an estimate of the initial Mach Number M_{01} and the sensitivity χ of mass flux to pressure.

References

- McIntosh, A.C. (1999) "Deflagration Fronts and Compressibility", Philosophical Transactions of the Royal Society, Combustion Science Theme Issue (Proceedings of the conference entitled "Combustion Science at the end of the Millenium", D.A.M.T.P., University of Cambridge, 14-16 April 1997). *To appear*.
- Ni, A.L. and Goel, B. (1995) "Stability of a deflagration front in compressible media", Z. angew. Math. Phys. (ZAMP) **46**, 328-338.