Direct numerical simulation of the propagation of a flame front in a homogeneous turbulent flow field

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Abstract

In this work we study the interaction of premixed combustion and turbulence by means of a level set equation that describes the motion of the flame front. This approach reduces many of the complexities of the involved chemistry and enables us to analyse the arising phenomena over a wide range of parameters with relatively modest computational costs. We perform a direct numerical simulation of a homogeneous isotropic turbulent flow in a cubic box. Into that flow field we insert a scalar field with a mean gradient in the z-direction. An iso-surface of this scalar represents a flame front. It is transported and wrinkled by the flow field and propagates in a direction normal to itself with its laminar flame speed.

We present results both for the case of a passive flame front, where the effects of heat release are neglected, and the case where we model these effects by a volume source located on the flame front.

Introduction

The level set approach to turbulent premixed combustion was originally introduced by Williams [1]. First numerical calculations were made by Kerstein et al. [2] and Peters [3] presented a spectral closure for the corrugated flamelet regime. It has recently been extended to the thin reaction zone regime [4]. These two regimes differ in the aspect that in the former the smallest structures of the turbulence have to be larger than the flame thickness, whereas in the latter regime these structures may enter the preheat zone of the flame but not the much thinner reaction zone.

The level set equation that determines the position of the flame front is the G-equation:

$$\frac{\partial G}{\partial t} + \boldsymbol{v}_u \cdot \nabla G = s_L |\nabla G| - D\, \kappa |\nabla G|. \tag{1}$$

The first term on the right hand side of that equation comes from the propagation of the flame front into the direction normal to itself with the laminar flame speed s_L. It is the most important term in the corrugated flamelet regime. The second term is proportional to the curvature κ of the flame front and accounts for diffusive processes in the flame. It dominates in the thin reaction zone regime.

Accomplishments

In this study we generate a statistically stationary, homogeneous and isotropic turbulent flow field with a pseudo spectral code by Ruetsch [5]. Into that flow field we insert the G-field and let it develop until it reaches the statistically stationary stage. Then we start to gather statistical data to analyse its evolution.

The numerical integration of the scalar G is carried out using a pseudo spectral method with a novel upwind extension to calculate the modulus of the gradient of G, $|\nabla G|$ on the right hand side of equation (1). This extension is necessary to account for the sharp cusps that occur in the G-field with small or vanishing diffusivity. The spatial resolution we used was a 64^3 grid for the statistical analysis and a 128^3 grid for the spectral analysis of the G-field with a Reynolds number based on the Taylor micro scale of 42 and 74 respectively.

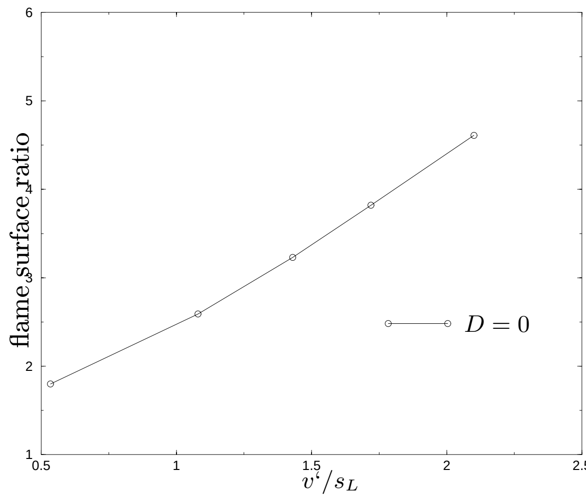

Figure 1: Corrugated flamelet regime.

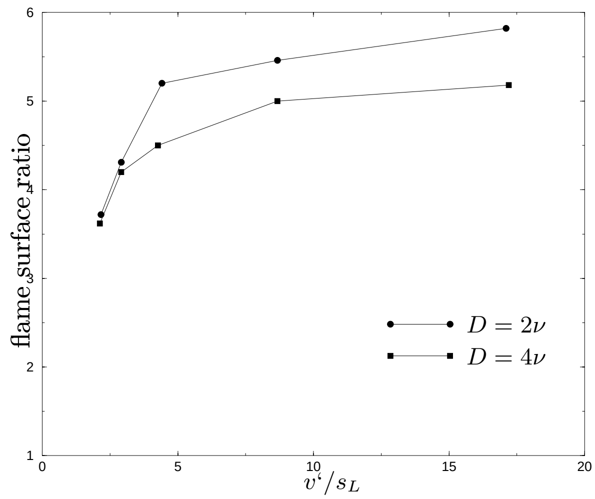

Figure 2: Thin reaction zone regime.

Results

Analysing the turbulent flame surface ratio for different values of the turbulence intensity v'/s_L and the Schmidt-number $\mathrm{S}c = \nu/D$, with ν the kinematical viscosity and D the diffusivity of the scalar, we can verify Damköhler's [6] result of a linear dependence of the turbulent flame speed on the turbulence intensity in the corrugated flamelet regime and a square root dependence in the thin reaction zone regime. Thus we are able to explain the bending effect of the turbulent flame speed by the transition of the burning regime from the corrugated flamelet to the thin reaction zone regime, see figs. 1 and 2.

To check the assumptions that have to be made in modelling the turbulent G-equation by a spectral closure, we examine the auto correlation spectra of G' and the cross correlation spectra of G' with $|\nabla G|'$ for different sets of the parameters v'/s_L and D.

To emphasise the relation between the G-equation and the flame surface density (FSD) approach to modelling premixed turbulent combustion, we present an equation for the time evolution of the surface of an iso-level of G. We show that this is basically the same as the FSD equation given e.g. by Trouve and Poinsot [7]. We illustrate this relationship in figs. 3-5, where we plot the probability density function $P(G')$ of G, the flame surface ratio $\overline{\sigma|_{G=G_0}}$, conditioned on being located on the flame front $G = G_0$, and the FSD Σ, which can be expressed as $\Sigma = \overline{\sigma|_{G=G_0}} P(G')$.

In this context we present the dependence of the different terms in the FSD-equation on their position x' in the turbulent flame brush as well.

The results given above were made by neglecting heat release effects. These are very difficult to include directly into the used numerical framework of pseudo spectral methods. Therefore we model them as a volume source that is located on the flame surface and calculate the induced velocity on every point of the same surface.

We follow a suggestion by Ashurst [8] and replace the laminar burning velocity s_L in eq. (1) by:

$$s_L = s_{L,u}\frac{\epsilon + 1}{2} + (\epsilon - 1)s_{L,u}(\boldsymbol{n} \cdot \boldsymbol{v}_{LR}), \tag{2}$$

where $s_{L,u}$ is the laminar burning velocity with respect to the unburnt mixture, $\epsilon = T_b/T_u$ the expansion coefficient and $\boldsymbol{v}_{LR}$ the velocity induced by the expansion.

The volume source $\boldsymbol{n} \cdot \boldsymbol{v}_{LR}$ can be computed by a Newton potential on the flame surface S_ξ [9]:

$$\boldsymbol{n} \cdot \boldsymbol{v}_{LR} = \frac{1}{4\pi} \int_{S_\xi} \frac{(\boldsymbol{r} - \boldsymbol{\xi}) \cdot \boldsymbol{n}}{|\boldsymbol{r} - \boldsymbol{\xi}|^3} dS_\xi \ . \tag{3}$$

This approach is different to a previous formulation by Dandekar and Collins [10] who implement Sivashinsky's equation [11] directly. It is computationally more expensive but has the advantage that it is consistent with the requirement that every iso-surface of G should depend only on quantities which are defined on the same iso-surface.

Preliminary results calculating the Darrieus-Landau instability show reasonably good agreement with the linear theory over a wide range of expansion factors ϵ and diffusivity D, see fig. 6. We therefore will use this model in analysing the effects of heat release on the turbulent premixed flame.

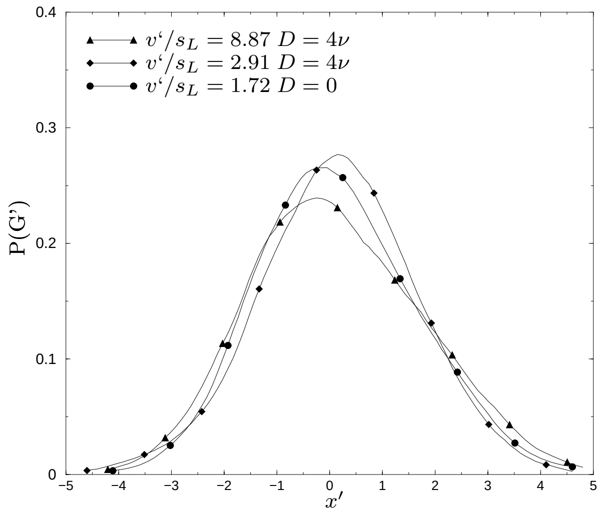

Figure 3: Probability density function of G'.

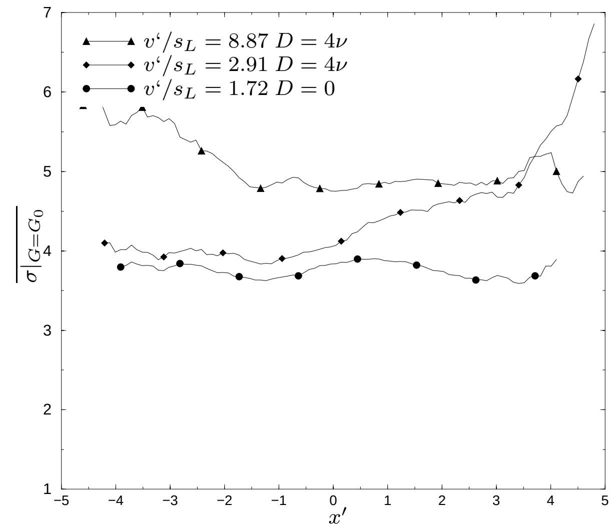

Figure 4: Conditioned flamesurface ratio

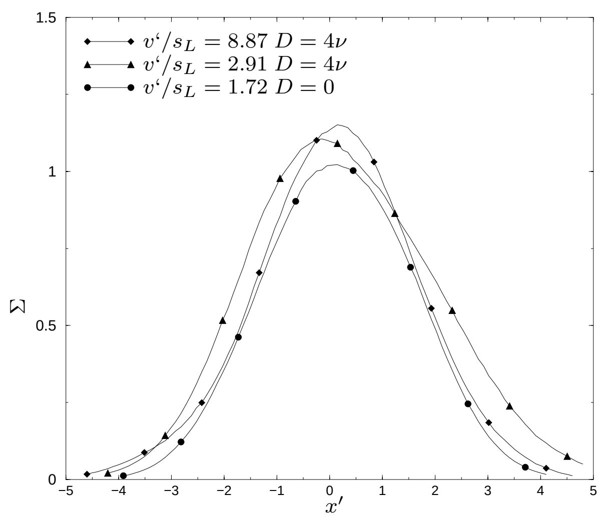

Figure 5: Flame Surface Density.

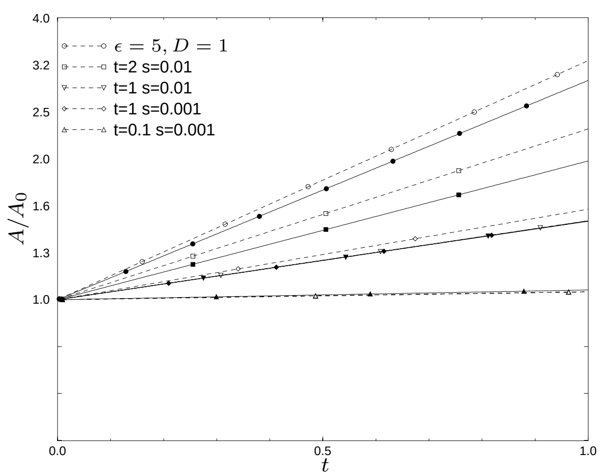

Figure 6: Darrieus-Landau instability. Filled symbols denote the growth from linear theory.

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