

Numerical Investigation of Acoustically Driven Instabilities of Premixed Flames

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Abstract

Acoustically driven instabilities that were experimentally investigated by *Scarby et al.* [5],[6] are simulated using a new numerical method. All cases considered are in the low Mach number regime ($Ma \approx 10^{-3}$). Due to the small Mach number the flow has two characteristic length scales, the acoustic and the hydrodynamic length scale. Therefore, the numerical method consists of two parts. A multi-dimensional finite volume method for zero Mach number variable density flows in combination with a front tracking method based upon a level set formulation resolve the hydrodynamic scale and the flame front geometry. The second component is a one dimensional low Mach number solver that accounts for long wave acoustics. Both solvers are coupled by matching boundary conditions.

Introduction

Unacceptable noise emitted by various combustion devices is caused by acoustic flame instabilities. The interaction between acoustic waves and premixed flame fronts is the subject of our present numerical investigation. *Scarby et al.* [5],[6] has experimentally investigated acoustic instabilities of propagating premixed flame fronts in a tube. He identified four different regimes, which we intend to reproduce with our numerical method. These regimes are a non vibrating cellular flame, a primary acoustic instability of a quasi planar flame and a secondary acoustic unstable regime with high amplitudes, which can break down into an incoherent auto-turbulent fourth regime at sufficiently high acoustic amplitudes.

Governing equations

The following set of conservation equations for zero Mach number combustion was derived by *Majda et al.* [3]. The pressure is decomposed according to $p = P_0 + M^2 p^{(2)}$. For a vanishing Mach number this implies a separation of the two roles the pressure plays: P_0 represents the thermodynamic background pressure whereas $p^{(2)}$ is representing the hydrodynamic pressure. The asymptotic analysis yields spatial homogeneity of P_0 as $M \mapsto 0$ ($\nabla P_0 = 0$) [1],[2]. The integral form for an arbitrary control volume V is:

$$\frac{d}{dt} \int_V \rho dV + \oint_{\partial V} \rho \vec{v} \cdot \vec{n} dA = 0, \quad (1)$$

$$\frac{d}{dt} \int_V \rho \vec{v} dV + \oint_{\partial V} (\rho \vec{v} \circ \vec{v} + I p^{(2)}) \cdot \vec{n} dA = 0, \quad (2)$$

$$\frac{d}{dt} \int_V \rho E dV + \oint_{\partial V} (\rho E + P_0) \vec{v} \cdot \vec{n} dA = 0. \quad (3)$$

$$\frac{d}{dt} \int_V \rho Y_i dV + \oint_{\partial V} \rho Y_i \vec{v} \cdot \vec{n} dA = \int_V \rho \omega_{Y_i} dV, \quad i = 1, n_{species}. \quad (4)$$

We consider an ideal gas with constant specific heat capacities. So that:

$$\rho E = \frac{P_0}{\gamma - 1} + \rho \sum_{i=1}^{n_{species}-1} Y_i (h_i - h_{n_{species}}), \quad P_0 = \rho \mathfrak{R} T, \quad h_i = h_i^0 + \int_{T_0}^T c_{p,i} dT. \quad (5)$$

Numerical method

The numerical method consists of three ingredients: A multi-dimensional finite volume method for unsteady variable density zero Mach number flows which is used to resolve the burnt and unburnt gas flow, a front tracking algorithm that uses a level-set formulation to represent the flame geometry and a one-dimensional low Mach number solver which is used to account for long wave acoustics and to provide boundary conditions for the multi-dimensional zero Mach number solver.

Multi-dimensional zero Mach number method

The unsteady zero Mach solver is based upon the results of asymptotic analyses of the Euler equations by *Klainerman et al.* [1], *Majda et al.* [3] and *Klein* [2]. One key to an extension of a compressible method that survives the incompressible limit is the introduction of a pressure decomposition. By introducing multiple pressure variables the different physical pressure dependent effects can be treated separately. The method basically consists of two parts: a predictor and a corrector step. The predictor is a slightly modified standard TVD scheme which has been successfully used for calculating unsteady compressible flows and yields robust higher order upwind discretisations for the convective fluxes. The corrector part accounts for the change of the mathematical structure of the conservation equations, namely from hyperbolic to hyperbolic/elliptic type as the Mach number vanishes. The elliptic nature implies a divergence constraint to the velocity field. Due to the fact that a non-staggered grid is used two Poisson-type equations have to be solved to ensure both correct advection across the grid cell interfaces and a final cell-centered velocity field that obeys the previously mentioned divergence constraint. A detailed description of this method can be found in [4].

Level set function based flame tracking

The underlying idea of describing a flame front by a scalar level set function $G = G(x, y, z, t)$ is to identify the front with a certain contour level $G = 0$ of the scalar G [7],[8],[9]. The following equation describes the front propagation of the premixed flame :

$$\frac{\partial G}{\partial t} + (\vec{v} + s_l \vec{n}) \cdot \nabla G = 0, \quad \vec{n} = -\frac{\nabla G}{|\nabla G|}. \quad (6)$$

Here \vec{v} is the velocity vector of the flow field and s_l is the scalar laminar burning velocity. As a consequence the set of computational grid cells is divided into three sub sets: unburnt ($G < 0$), burnt ($G > 0$) and mixed ($G \approx 0$) cells, which contain part of the flame surface. In a mixed cell there are at least two states, a burnt one and an unburnt one, which must obey the Rankine-Hugoniot jump conditions. An analysis of these jump conditions in the zero Mach number limit shows that the elliptic pressure field is discontinuous at the front. Thus, a Poisson-type problem with singular source term equivalent to a δ -peak dipole is to be solved. We present a discretisation scheme that guarantees (i) a sharp and non-oscillating transition from unburnt to burnt gas pressures over a single mixed cell and (ii) involves a standard compact stencil for the Laplace type operator.

Accounting for long waves acoustics

Since the flow field is multi-dimensional only in the neighbourhood of the flame front there is no need to resolve the quasi one-dimensional flow a few tube diameters away from the front by a fully multidimensional representation. The given computational resources should be spent on resolving the relevant parts of the flow. On the other hand it is necessary to account for long wave acoustics whose wavelength is much larger than the extensions of the multi-dimensional resolved domain. Acoustic waves are thus resolved by a one-dimensional low Mach number solver and are allowed to interact with the multi-dimensional zero Mach number solver in two ways:

1. The background compression induced by acoustic pressure variations changes the velocity divergence constraint according to:

$$\frac{1}{\gamma P_0} \frac{dP_0}{dt} = -\frac{1}{|V|} \oint_{\partial V} \vec{v} \cdot \vec{n} dA. \quad (7)$$

2. The one-dimensional low Mach number solver yields net mass and momentum fluxes that are matched at the boundaries of the three-dimensional domain by the velocity field in the zero Mach number solver.

The one-dimensional low Mach number solver used has been discussed in [2].

Results

The calculation of the propagating flame front was performed in two space dimensions. A rectangular domain was chosen (20cm \times 5cm) consisting of 200 \times 50 cells. At the inlet an oscillating velocity normal to the boundary of the domain was prescribed :

$$u(x = 0, y, t) = s_l \left(1 + \frac{\Delta u}{s_l} \sin(2\pi f t) \right). \quad (8)$$

The laminar burning velocity s_l was set to $s_l = 20$ cm, the Markstein length is 0.2cm. Since a binary reaction system is considered there is one enthalpy difference - that is equivalent with the heat release - in equation (5) which was set to $h_b - h_u = c_p(T_b - T_u) = 1200 \frac{\text{kJ}}{\text{kgK}}$. The initial flame front had a

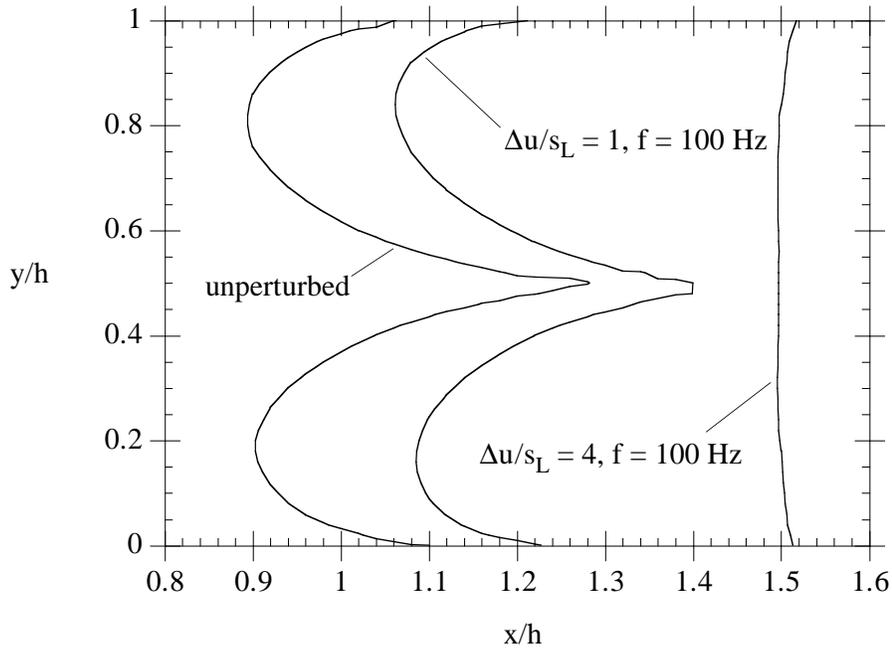


Figure 1: Flame contours in a quasi steady state for three cases: $\Delta u = 0.0$ (unperturbed), $\Delta u = s_l$ (weak perturbation) and $\Delta u = 4 * s_l$ (strong perturbation)

sinusoidal shape with an amplitude of 0.02cm. Three different cases were considered: $\Delta u = 0$ (unperturbed), $\Delta u = s_l$ (weak perturbation) and $\Delta u = 4 * s_l$ (strong perturbation). The different flame fronts corresponding to the three cases are shown in figure no.1. The differences between no perturbation and weak perturbation are slim, both fronts show the characteristics of a Landau-Darrieus instability. In case of strong perturbation however the front is flat. The evolution of the flame front amplitude in time of the is shown in figure no.2.

Even in spite of the strongly simplified assumptions that were introduced for the calculations the first and the second regime *Searby et al.* described were reproduced by the previously described numerical method. These preliminary results were obtained without the one dimensional low Mach solver. Calculations including the accounting for long wave acoustics will performed in the nearby future.

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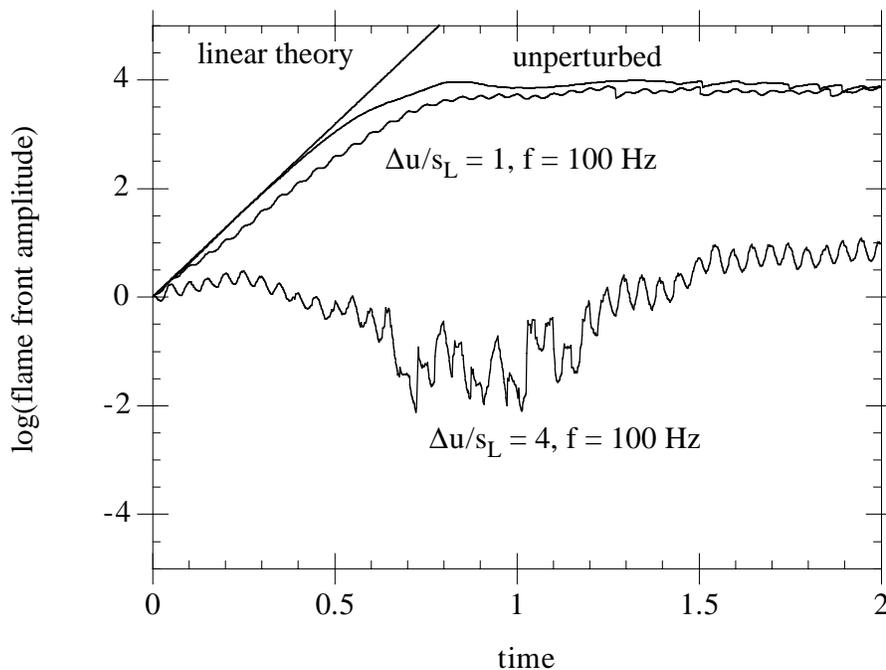


Figure 2: Evolution of flame front amplitude in time for three cases: $\Delta u = 0.0$ (unperturbed), $\Delta u = s_l$ (weak perturbation) and $\Delta u = 4 * s_l$ (strong perturbation)

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