Vibrating gaseous particles model for turbulent flow caused by combustion propagation in a tube.

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Abstract

Turbulent gas combustion propagation in a tube is considered. Averaged equations are obtained on a base of two-component approximation for turbulent flow. To describe pulsation energy dissipation, a model of vibrating gas particles is suggested. Pulsation energy generation is assumed to take place in internal part of a turbulent boundary layer. It is formulated the generation rate hypothesis. Equilibrium conditions of pulsation energy generation and dissipation complete one-dimensional problem of constant velocity propagation of turbulent combustion in a tube.

To investigate a turbulent combustion of methane-air and methane-oxygen mixtures, a flame front model is used.

Introduction

In infinitely long tube with impermeable end-wall flame propagation leads to shock wave generation [1]. If Reynolds number Re is more than 1200, gas flow is turbulent behind it. According to Reynolds scheme any gaseous flow parameter is a sum of averaged and fluctuating values: $A = \overline{A} + A'$, $(A' \ll \overline{A})$. Time averaging gives a density continuity equation in a form differed from equation for summary gas density ρ:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho} \cdot \overline{u}}{\partial x_j} + \frac{\partial \overline{\rho' u'}}{\partial x_j} = 0. \tag{1}$$

Now there is no reliable estimate of contribution of additional item $\overline{\rho' u'}$. Momentum and energy equations also contain such uncertain items.

New approach to a problem of modelling of turbulent combustion propagation is considered in this paper.

Two-components approximation of turbulent flows.

Turbulent flow is suggested to be considered as a flow of virtual (averaged gas flow) and fictitious media. First medium density, velocity, thermal energy and pressure are characterised by magnitudes: $\overline{\rho}, \vec{u}, e, \overline{p} = \overline{\rho} e(\gamma - 1)$. For second medium they are: $\rho', \vec{u}, e, p' = \rho' e(\gamma - 1)$. Mean density of the second component is succeeded from equation (1) to be equal zero: $\overline{\rho'} = 0$. This component increases or decreases summary density with an equal possibility, but it is absent in averaged flow. Therefore it is natural to suppose that fictitious medium contribution in flows of mass, impulse and energy are negligible. This way simplified equations describing averaged motion of turbulent flow in common notations assume the form:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho} \cdot \overline{u_j}}{\partial x_j} = 0,$$

$$\frac{\partial \overline{\rho} \cdot \overline{u_i}}{\partial t} + \frac{\partial \overline{\rho} \cdot \overline{u_i} \cdot \overline{u_j}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial \overline{\tau}_{ij}}{\partial x_j} - \frac{\partial \overline{R}_{ij}}{\partial x_j}, \tag{2}$$

$$\frac{\partial \overline{\rho}(\overline{e} + e_v + |\overline{\vec{u}}|^2/2)}{\partial t} + \frac{\partial \overline{\rho} \cdot \overline{u_j}(\overline{h} + e_v + |\overline{\vec{u}}|^2/2)}{\partial x_j} = \overline{A}_\mu - \overline{A}_\tau - \frac{\partial \overline{Q}_\lambda + \overline{Q}_{\tau j}}{\partial x_j} + \overline{A}_{\tau'},$$

$$\overline{p} = \overline{\rho} \cdot \overline{e}(\overline{\gamma} - 1), \overline{T} = R_o/\mu \cdot \overline{p}/\overline{\rho}.$$

Here $e_v = W^2/2 = \overline{u_i' u_i'} = \overline{|\vec{u}'|}^2$, specific heat ratio γ is assumed to be constant. All essential items, which have evident physical interpretation are taken into account: $\overline{\tau}_{ij}$ and $\overline{R}_{ij}$ are viscosity and Reynolds

stresses, $\overline{A}_\mu = \partial\overline{\rho}\cdot\overline{u_i}\cdot\overline{\tau}_{ij}/\partial x_j$ and $\overline{A}_\tau = \partial\overline{\rho}\cdot\overline{u_i}\overline{R}_{ij}/\partial x_j$ are powers of these stresses in volume unit, $\overline{Q}_\lambda$ and $\overline{Q}_{\tau j} = \overline{\rho}\overline{u'_j e'}$ are molecular and turbulent heat fluxes, $\overline{A}_{\tau'} = \partial\overline{u'_i\tau'_{ij}}/\partial x_j$ is a power of viscosity stresses pulsation in volume unit.

To obtain these equations in the usual manner, it is necessary to assume that $\overline{\rho' u'} \ll \overline{\rho}\cdot\overline{u}$, $\overline{\rho' e'} \ll \overline{\rho}\cdot\overline{e}$ and averaged products of three pulsing values are negligible. First and second assumptions are result of inequalities $A' \ll \overline{A}$. In other words, they mean that difference between time and density averaged velocities and thermal energies [2] is negligible.

Density continuity equation for fictitious medium is:

$$\frac{\partial\rho'}{\partial t} + \frac{\partial\rho'\cdot(\overline{u}_j + u'_j)}{\partial x_j} = 0.$$

Subject to continuty equations for averaged and real flows, it can be obtained a condition:

$$\frac{\partial\overline{\rho}\cdot u'_j}{\partial x_j} = 0. \tag{3}$$

It means that a gaseous particle mass in averaged flow is not changed by pulsation.

A vibrating gas particles model of a turbulence.

Turbulent flow is well known can be considered as a flow of chaotically moving turbulent vortexes [3]. Instantaneous gas dynamics parameters at a fixed point are defined by a vortex, which is located at this point. Averaged parameters are defined by all gas particles, which have enough time to pass this point distance l' during averading period. Subject to equations (2) and (3), substitution of vortexes at a fixed point can be modelling by a one vortex or gas particle vibration near its mean position in averaged motion. This particle averaged density is $\overline{\rho}$, its vibration energy is equal $W^2/2$. Vibration energy dissipation can be estimated by a power of inter-phases force $\vec{f}$ in two-velocity "particles-gas" (vibrating vortexes and averaged flow) suspension: $\vec{u}'\cdot\vec{f} = \vec{u}'\cdot\frac{1}{2}n\overline{\rho}C_*S|\vec{u}'|\vec{u}'$. Here n is vibrating vortexes number in a volume unit, S is vortex cross-section area, C_* is a drag coefficient.

Since vortexes vibrations are chaotic, all directions are supposed to be equiprobable in centre part of flow in a tube. So, averaged product of vibration velocity module and any variable quantity, defined by direction only, is equal a product of averaged module and averaged value of this variable quantity. Under such conditions of locally isotropic turbulence $\overline{u'_i u'_j} = \delta_{ij}W^2/\nu = \delta_{ij}V^2$, turbulent heat flux $\overline{Q}_{\tau j} = \overline{\rho}\overline{u'_j e'} = -\lambda_\tau\partial\overline{T}/\partial x_j$. Here $\nu = 1, 2, 3$ for one, two or three-dimensional cases, respectively, $\lambda_\tau = \overline{\rho}C_v\overline{|\vec{u}'|l'} = \overline{\rho}C_v WL$ is a turbulent heat conductivity coefficient. Magnitude $L = \overline{|\vec{u}'|l'}/\overline{|\vec{u}'|} = \overline{|\vec{u}'|}/W$ can be considered as averaged turbulent length. Equations (2) become essentially simpler. In one-dimensional case these equations are reduced to a form:

$$\frac{\partial\rho}{\partial t} + \frac{\partial\rho u}{\partial x} = 0,$$

$$\frac{\partial\rho u}{\partial t} + \frac{\partial p + \rho u^2 + \rho W^2}{\partial x} = -f_w, \tag{4}$$

$$\frac{\partial\rho(e + W^2/2 + u^2/2)}{\partial t} + \frac{\partial\rho u(h + W^2/2 + u^2/2)}{\partial x} + \frac{\partial\rho u W^2}{\partial x} = \frac{\partial\rho C_v WL\partial T/\partial x}{\partial x} - f_w\cdot u.$$

$$p = \rho e(\gamma - 1), T = R_o/\mu\cdot p/\rho.$$

Here lines above averaged values are turned down, $f_w = 4\tau_w/d$ is a force of friction on walls, molecular heat transfer and viscosity are supposed to be negligible. To complete the problem, it is necessary to formulate relations or equations for averaged turbulent length and vibration velocity.

Pulsation generation hypothesis and equilibrium conditions.

It is known that first pulsation in a tube are mainly generated in internal part of a turbulent boundary layer. Thickness of internal sub-layer δ is about 20% of a boundary layer [2]. In a turbulent boundary layer local isotropy assumptions are not probable because of a wall influence. So, turbulent boundary layer thickness Δ is in order of a turbulent length: $\Delta \approx L$. On the other hand, pulsation generation are

naturally assumed to prevail over dissipation in a turbulent boundary layer. Experiments [4] show, that it takes place at a distance $0.1d$ from a smooth tube wall. Here d is a tube diameter. So, $L \approx \Delta \approx 0.1d$ and $\delta \approx 0.02d$,

A main hypothesis is that pulsation energy is equal "non-compensated heat" of unrealized laminar flow in internal part of a turbulent boundary layer. Generation rate of pulsation energy in volume unit is equal $\rho de_v/dt \approx \rho dq'/dt = \tau_{ij} e_{ij} \approx \tau_w \cdot v_*/l_*$. Here $l_* = \nu_*/v_*$, $v_* = \sqrt{\tau_w/\rho}$, ν_* is a coefficient of viscosity, τ_w is a stress on a pipe wall. So, cross-section averaged generation of pulsation energy is equal $\pi d\delta\tau_w v_*/(l_*\pi d^2/4)$.

In internal sub-layer there is a single represantation length l_*, therefore, it is natural to assume that generated vortex size l is in order of l_*.

Constant velocity propagation of a turbulent combustion in a tube is characterised by equilibrium between generation and dissipation of pulsation in every cross-section of the tube. Stated above assumptions for smooth tubes lead to relation: $W \approx 0.03u(0.0032 + 0.221/Re^{0.237})^{1/2}$, that is $W \simeq 0.015u \cdot Re^{-0.12}$. In experiments, a turbulence caused by a gas combustion is characterised by relations [4]:

$$W = 9 \cdot 10^{-2} u \cdot Re^{-0.16}, L \approx 0.1d. \quad (5)$$

So, experimental relations (5) complete equations (4) and can be named as equilibrium relations.

Flame front model of a turbulent combustion propagation.

Under known conditions, turbulent gas combustion, initiated at impermeable end-wall of a tube can be imagined as a propagation of a infinite thin flame front. Turbulent heat conductivity is assumed to be negligible far from the front. Relative flame velocity is defined by relation: $u_\tau = u_\lambda\sqrt{1 + \chi_\tau/\chi_\lambda}$ [4]. Here χ_λ, χ_τ are temperature conductivity coefficients in laminar and turbulent flows correspondingly, u_λ is a laminar flame velocity.

Well known self-similar solution of a flame propagation problem in combustible gas [1] is used to define initial parameters distributions.

In numerical calculations, S.K.Godunov's method [6] with moving grid is used. One of a grid points is connected with a flame front. In methane-air mixtures relative flame velocity is defined by known relations [5]. Complete flame velocity is defined as a result of solution of an arbitrary break disintegration problem in combustible gas [1].

Results for methane-air and methane-oxygen mixtures.

It is numerically shown that in methane-air mixtures flame velocity and concentration interval of turbulent combustion increase with a pipe diameter. But relative flame velocity does not exceed Chapman-Jouget velocity in averaged flow, formed in front of the flame.

In methane-oxygen mixtures constant flame velocity regimes are absent in limited interval of methane concentrations. This interval is located near stoichiometry composition, its limits expand with a pipe diameter increase. In boundary concentrations mixtures relative flame velocity is equal Chapman-Jouget velocity in averaged flow, formed in front of the flame.

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