A Quasi-Fractal Subgrid Flame Speed Closure for Large Eddy Simulations of Premixed Flames

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Abstract

For the flamelet regime of premixed turbulent combustion, a subgrid flame speed closure is suggested by utilizing the fractal approach and dimensional reasoning. To model the increase in the averaged fractal dimension D with u'/S_L, observed in various measurements, the dependence of the "fractal" dimension on the length of flamelet surface wrinkling is assumed. The processing of available experimental data supports this assumption.

Introduction

Over the past decade, large eddy simulations (LES) have been successfully applied to premixed turbulent flames [1, 2, 3], most often in conjunction with the level set equation [4]. For these purposes, an advanced submodel of a subgrid flame speed S_g is required and the goal of the work is to consider the issue by utilizing and modifying the fractal approach [5].

Within the framework of the approach, the effect of eddies of the size l ($l > \delta_L$, where δ_L is the laminar flame thickness) on flame speed is solely associated with the increase in the flamelet surface area Σ. The fractal approach determines the contribution of these eddies to the area increase, $\Sigma \sim l^{D-2}$, and this relation has been supported by numerous experiments with premixed flames [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Thus, the flame speed can be evaluated as [5]

$$S_t \;\sim\; S_L \left(\frac{e_o}{e_i}\right)^{D-2}, \tag{1}$$

where S_L is the laminar burning velocity. Despite the simplicity, the approach is substantially devaluated by the lack of reliable submodels predicting the outer, e_o, and the inner, e_i, cut-off scales and the fractal dimension D. However, for LES applications, an important merit of the approach consists of the fact that it determines the dependence of the subgrid speed S_g on the grid scale l_g ($S_g \sim l_g^{D-2}$).

A general form of the fractal closure of subgrid flame speed

If the grid scale is inside the inertial range ($\eta \ll l_g \ll L$, where $\eta = (\nu^3/\epsilon)^{1/4}$ and L are the Kolmogorov and integral length scales, respectively, ν is the molecular viscosity, and ϵ is the dissipation rate) of the Kolmogorov turbulence; the subgrid flame speed is, in the general case, controlled by 4 dimensional parameters: ϵ and l_g, characterizing the turbulence, and the laminar flame speed S_L and the thickness $\delta_L = \nu/S_L$, characterizing combustion. Since the dependence of S_g on l_g is determined by the fractal approach, the only dimensionless parameter controlling the flame behavior is the Karlovitz number

$$Ka \;\sim\; \left(\frac{\epsilon\delta_L}{S_L^3}\right)^{1/2} \;=\; \left(\frac{\delta_L}{L_G}\right)^{1/2} \;=\; \left(\frac{\delta_L}{\eta}\right)^{2} = \left(\frac{\eta}{L_G}\right)^{2/3}, \tag{2}$$

where $L_G = S_L^3/\epsilon$ is the so-called Gibson length scale.

Finally, based on (1) the dependence of $S_g \sim l_g^{D-2}$, provided by the fractal approach, and on (2) the above dimensional arguments, we obtain a general closure

$$S_g \;\sim\; S_L \left(\frac{l_g}{L_G}\right)^{D-2} f(Ka) \;=\; S_L \left(\frac{l_g}{g(Ka)L_G}\right)^{D-2}, \tag{3}$$

where $g(Ka)$ or $f(Ka) = g^{2-D}$ are unknown functions. The above consideration emphasizes the key role played by the Karlovitz number and the Gibson length scale in modeling turbulent combustion.

When comparing Eq. 3 with various known models, three points are worth noting. First, the simple closure of $S_g \sim u'_g \sim (\epsilon l_g)^{1/3}$ results from Eq. 3 if $D = 7/3$ and $f = g = 1$. Second, the comparison of Eq. 1, applied to $e_o = l_g$, with Eq. 3 implies the following scaling law for the inner cut-off: $e_i \sim g(Ka)L_G \sim \delta_L Ka^{-2} g(Ka)$. The dependence of the ratio of e_i/δ_L on the Karlovitz number is supported by an analysis of available experimental data, performed by Gülder and Smallwood [15], who have concluded that $e_i \sim \delta_L(c + Ka^{\beta})$ where $\beta = (-1/2 \div -1/3)$ and c is a constant. Third, by varying $g(Ka)$, the aforementioned scaling law can be reduced to various known submodels:

$$g(Ka) \;=\; \left\{ \begin{array}{lll} Ka^2 & \Longrightarrow e_i \sim \delta_L & [7] \\ Ka^{5/3} & \Longrightarrow e_i \sim \delta_L(c + Ka^{-1/3}) & [16] \\ Ka^{3/2} & \Longrightarrow e_i \sim \eta & [17] \\ 1 & \Longrightarrow e_i \sim L_G & [18] \end{array} \right. \tag{4}$$

A quasi-fractal model

Equation 3 does not account for the following important effect. The fractal dimension is known to increase with u'/S_L from $D = 2$ to $D \approx 7/3$ [9, 10]. The upper limit, $D(u'/S_L \to \infty) = 7/3$, is associated with the behavior of material surfaces in non-reacting turbulent flows [19]. The only available, empirical parameterization of $D(u'/S_L)$

$$D \;=\; \frac{2.05}{\frac{u'}{S_L} + 1} \;+\; \frac{2.35}{\frac{S_L}{u'} + 1} \tag{5}$$

has been suggested by North and Santavicca [10]. Despite that Eq. 5 quite satisfactorily approximates the experimental data plotted in Fig. 1, this expression appears to be fundamentally inconsistent. Indeed, since the fractal dimension is relevant to an intermediate range of scales of turbulent eddies, and to the inertial range of the Kolmogorov turbulence, in particular; D must depend on such a turbulence cascade characteristic as the dissipation rate ϵ rather than on a large-scale eddy characteristic such as u'. This inconsistency could be overcome by assuming that D depends on the Karlovitz number determined by Eq. 2. Then, since Ka increases with u'/S_L, an increasing function of $D(Ka)$ could explain the increase in D with u'/S_L, observed in experiments.

However, such a modification does not solve all the fundamental problems. Indeed, in the case of $l \gg \delta_L$, the flamelet thickness cannot affect the increase in the flamelet surface area Σ, caused by the eddies of the size l. The thickness can affect the absolute value of Σ because the flamelet surface production by the smallest eddies can depend on δ_L, for example, the inner cut-off scale depends on δ_L [15]. However, the thickness cannot affect the relative increase in Σ by scales of $l \gg \delta_L$. Thus, since Ka depends directly on δ_L, the use of a function of $D(Ka)$ appears to be fundamentally inconsistent, too.

Moreover, any dependence of D solely on Ka is unacceptable due to the following reasoning. On the one hand, for the limit of $u' \to 0$ and, hence, $Ka \to 0$, D should tend to 2 as in the laminar case. On the other hand, the same limit $D(Ka \to 0) = 2$ is associated with $L \to \infty$ for any ratio of $u'/S_L \gg 1$, as much as desired. On the contrary, when $u'/S_L \gg 1$, the structure of the flamelet surface is expected to tend to the structure of the nonpropagating material surface, characterized with $D \approx 7/3$ [19]. These two limit values, $D(Ka \to 0) = 2$ and $D(u'/S_L \gg 1) \approx 7/3$ appear to be inconsistent.

A possible way of resolving the above problems and modeling the increase in D with u'/S_L is to assume a quasi-fractal structure of flamelet surfaces that is

$$\frac{d\Sigma}{\Sigma} \;=\; \left\{ a + b \ln \frac{l}{L_G} \right\} \frac{dl}{l}. \tag{6}$$

To be completely consistent with the turbulence cascade concept, L_G should be replaced by $L_g = S_g^3/\epsilon$ in Eq. 6. For simplicity, we use the Gibson scale below because the discussed improvement weakly affects the final results but substantially complicates the analysis.

The integration of Eq. 6 from e_i to e_o leads to Eq. 1 with

$$D - 2 \;=\; a \;+\; \frac{b}{2} \left\{ \ln\left(\frac{e_o}{L_G}\right) \;+\; \ln\left(\frac{e_i}{L_G}\right) \right\}. \tag{7}$$

Certainly, Eq. 7 may be used only if the predicted fractal dimension D is between 2 and 7/3.

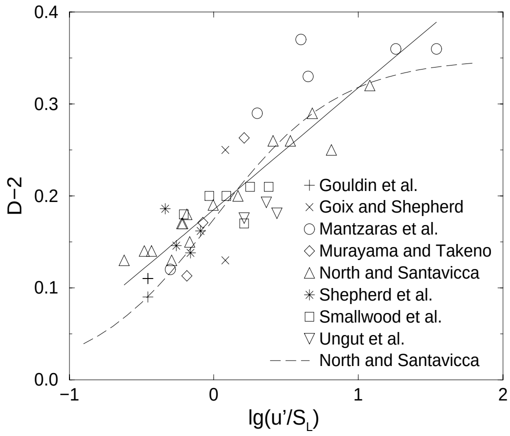

Figure 1: Experimental data (symbols) on the fractal dimension of flamelet surface vs. the ratio of u'/S_L. Solid line shows the linear fit to the data. Dashed curve indicates the approximation of North and Santavicca [10] (see Eq. 5).

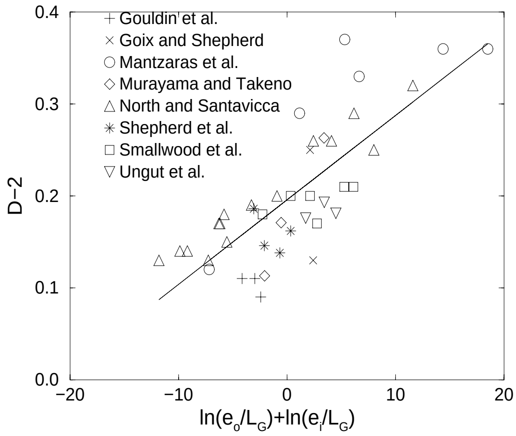

Figure 2: Experimental data (symbols) on the quasi-fractal dimension of flamelet surface for measurements performed in various ranges of scales. Solid line shows the linear fit to the data (see Eq. 7).

It is worth noting that Eq. 7 predicts a dependence of D on L, associated with the last term in the braces (the ratio of e_o/L_G is L-independent as both $e_o \sim L$ and $L_G \sim L$). If $e_i \sim \delta_L(c + Ka^\beta)$ and $\beta \geq -1/2$ [15], the ratio of e_i/L_G decreases with L so that D is a weakly (the first L-independent term in the braces is much larger than the second one) decreasing function of L. However, when $L \to \infty$, D tends to be constant as the assumption that $e_i \sim L_G \sim L$ [18] appears to be more consistent when $\delta_L \ll \eta < L_G$ and the inner cut-off cannot be controlled by δ_L.

Since an increase in u'/S_L is associated with a strong decrease in L_G, Eq. 7 predicts an increase in the averaged fractal dimension with u'/S_L, in accordance with the measurements [9, 10]. The hypothesis that "the apparent u'/S_L dependence of D is a measure of the curvature" of a universal function of l/L_G has been put forward by Niemeyer and Kerstein [20] based on numerical simulations. The above physical and dimensional reasoning supports the hypothesis.

Thus, Eq. 3 with a weak function $2 < D = D(l/L_G, Ka) \leq 7/3$ is a subgrid flame speed closure consistent with (1) a quasi-fractal structure of the flamelet surface area, (2) the theory of homogeneous, locally isotropic turbulence, and (3) dimensional arguments. This result contributes to the problem by substantially narrowing the class of admissible closures of S_g.

For practical applications, the approximation of $e_i \sim \delta_L(c + Ka^\beta)$ with $\beta = (-1/2 \div -1/3)$ [15] can be (1) inserted into Eq. 7 and (2) used to evaluate $g(Ka)$ by comparing Eqs. 1 and 3. Then,

$$S_g \;\sim\; S_L \left(\frac{l_g}{L_G Ka^2 (c + Ka^\beta)} \right)^{D-2}, \tag{8}$$

$$D \;=\; \max\left\{ 2;\; \min\left[a \;+\; \frac{b}{2} \left(\ln \frac{l_g}{L_G} \;+\; 2\ln(Ka) \;+\; \ln\left(c + Ka^\beta\right) \right);\; 7/3 \right] \right\}, \tag{9}$$

The processing of available data on D, performed in the work by employing Eq. 7 (see Fig. 2), has yielded the best fit with $a = 0.196$ and $b = 0.018$.

Conclusions

A general form (Eq. 3 with $2 < D = D(l/L_G, Ka) \leq 7/3$) of subgrid flame speed closure and a semi-empirical version (Eqs. 8 and 9) of it has been suggested for the LES of premixed turbulent combustion. The dependence of the "fractal" dimension on the length of flamelet surface wrinkling has been emphasized.

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