On the Dynamics of Unstable Combustion of Solid Propellants

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Abstract

The unsteady combustion of a homogeneous solid propellant is studied, focusing the problem of intrinsic instability. The governing conservation equations are directly solved numerically, without making the usual assumptions of linearity and quasi-steady gas-phase. It is shown that combustion becomes intrinsically unstable leading to limit-cycle oscillations in burn rate, at critical values of the Denison–Baum stability parameters. Calculations for finite-amplitude pressure driven combustion indicate two distinct resonant modes, (a) the first one near the natural frequency as obtained from intrinsic instability analysis, and (b) a high-frequency second mode. It is found that the first mode is a result of solid phase unsteadiness, while the second mode is a result of gas-phase unsteadiness. Examination of frequency response reveals that the role of gas-phase thermal inertia is, 1) stabilizing near the first mode, but 2) destabilizing near the second mode. Finally, the effects of mean pressure and amplitude of pressure oscillations on pressure frequency response function are studied.

Introduction

Intrinsic, nonacoustic instability of solid propellant combustion is important from the viewpoints of both fundamental combustion theory and practical applications. When subjected to external pressure perturbations of certain frequencies, a resonant mode of highly amplified, periodic burning occurs due to intrinsic instabilities[1–3]. A comprehensive theory explaining its mechanism is not available yet. Despite early work[1, 4], the current knowledge on such instability is largely based on linear stability theories employing activation energy asymptotics[5]. Although such analyses provide trends in combustion behavior, quantitative calculations are needed for *finite* disturbances for applications to real propellants. A modeling restriction that is common to most works is the Quasi Steady gas-phase and Homogeneous solid, One Dimensional Flame (QSHOD) model which neglects the gas-phase thermal inertia. The validity of QSHOD model is limited to pressure perturbations within a low frequency range of below, some 1 *kHz* [1–3] . An analysis considering finite gas-phase thermal inertia, as well as the effects of nonlinear stability becomes formidable, and numerical methods have to be adapted. In spite of some attempts, fully workable solutions are not available.

Two specific issues examined in this computational study are, the role of (1) gas-phase thermal inertia and (2) finite-amplitude pressure disturbance. Briefly, this study is the nonlinear counterpart of [2], and the non-QSHOD, nonlinear, Arrhenius combustion counterpart of [3]. The dynamic combustion model is formulated, and the governing equations are solved for gas-phase and solid phase governing the 1-d, unsteady combustion of a homogeneous solid propellant. In the first part, the external conditions of pressure and ambient temperature are kept constant, and a series of calculations are made for different values of the two stability parameters identified in Ref.[1]. The intrinsic stability boundary is determined by a go/no-go method, where the steady solution changes over to limit-cycle oscillations beyond critical values of these parameters. The computed stability boundary and the natural frequency are compared with the linear stability results of QSHOD [1] and non-QSHOD [5] models. In the second part, combustion of an intrinsically stable propellant is considered. The pressure is externally forced to vary with a finite amplitude and frequency, and the resulting burn rate is computed. The amplification and phase of the response function are computed by a Fourier analysis of the burn rate variations over a range of driving frequencies. These are compared with the linear results of QSHOD [3] and non-QSHOD [2] models. The effect of mean pressure on the response function is also examined. Finally, the calculations are made for different values of the imposed pressure amplitudes at a fixed frequency.

The Physical Problem, Model and Solution Technique

We consider the 1-dimensional, laminar combustion of a solid propellant (see Fig. 1). The assumptions made and the resulting governing equations are standard, and may be found elsewhere[1,2,5]. However, the differences of the problem studied herein are, (1) the gas-phase is truly transient with a finite response time, (2) the pressure perturbations are not small but finite and (3) the gas-phase flame is distributed with an Arrhenius- type reaction. The original partial differential equations for conservation of overall mass, species and energy for transient combustion are taken from [2,6]. The mass-weighted coordinate (von Mises) transformation is applied and the reference velocity of the cold end of the solid is taken as zero. Due to this, the convective terms disappear, and the overall mass conservation is identically satisfied. The variables are non-dimensionalized as in [7]. The gas-phase governing equations are solved employing the *Operator-splitting* technique, which uses an explicit central differencing for the diffusion terms and a Crank-Nicholson differencing for the source terms. For solid phase equation, a fully implicit differencing is used. This technique has been adopted for the propellant burning problem [8] by tracking the instantaneous location of the interface plane. At each time level, the nonlinear interface conditions are satisfied iteratively via a Newton procedure. Thus the instantaneous temperature of the burning surface is computed. The properties of the propellant and practical operating conditions define nine non-dimensional parameters. Their values are obtained from [2,3].

Results and Discussion

Intrinsic Stability Limit

The pressure is kept constant, and the burn rate is calculated for different values of the Denison–Baum[1] parameters (A_{DB}, α_{DB}). Beyond a critical value of (A_{DB}, α_{DB}) the regression exhibits a self-excited limit-cycle oscillations, even when all external conditions are steady. Similar work has been done using asymptotic theory and linear stability analysis [5]. In order to compare the present results with these, the dispersion relations derived in [5] are solved numerically. This provides the critical values of (A_{DB}, α_{DB}) at the pulsating stability limit, and also the frequency of burn rate oscillations at this limit. The results are compared in Fig. 2. It is found that combustion is (1) stabilized by the inclusion of gas-phase thermal inertia and (2) destabilized by distributed gas-phase energy release. Also, at large gas-phase activation energies the stability boundary predicted by all models, both QSHOD and transient gas-phase, are the same. This may be due to small natural frequencies associated with large gas-phase activation energy, where gas-phase has enough time to respond to any change in conditions.

Pressure Driven Combustion

A more useful aspect of propellant combustion is the response of burn rate against forced pressure fluctuations. The properties corresponding to an intrinsically stable regime are chosen. The pressure is forced to vary sinusoidally with a finite amplitude and a constant frequency. The resulting burn rate variations $\dot{r}(t)$ and hence the response function are computed. In Fig.3, the present results are compared with those of [3]. The latter consider linear, QSHOD theory, and obtain closed form expressions for the amplification $R_p(\Omega)$ and phase $\phi(\Omega)$ of the burn rate with respect to the input pressure fluctuations. Ref.[3] considered three different gas-phase heat release models. The present work considers a more realistic, Arrhenius form. It is found that a first *resonant mode* burning occurs when the driving frequency equals the natural frequency of the propellant, and a second resonant mode occurs at a much higher frequency. The first resonant frequency predicted by QSHOD theory is well comparable to the present results. However, the high frequency resonant mode is predicted only by the present non-QSHOD model, and the QSHOD model does not capture this phenomenon. Numerical QSHOD calculations with Arrhenius gas-phase chemistry model are performed to study the effect of gas-phase thermal inertia on the response function. The comparison between QS and transient gas-phase results shows that near the first resonant frequency, at moderately low frequencies, introduction of gas-phase thermal inertia stabilizes burning, where as at higher frequencies it is destabilizing.

Multiple resonant modes have been also predicted in previous linear, non-QSHOD models [2, 9]. In Fig.4 the present nonlinear results (a pressure amplitude of 1% of the mean) are compared with those of [2], which considered infinitesimally small pressure disturbances. Calculations with a transient gas-phase, but quasi-steady solid phase (Fig.4) show that the second resonant frequency is due to the gas-phase thermal inertia only and the inclusion of solid phase thermal inertia stabilizes the burning against high frequency perturbations.

It has been shown by T'ien [2] that the real part of acoustic admittance, and hence burn rate response function, at first resonant frequency decreases with increase in the density ratio (ρ_g/ρ_s) if the mean surface temperature is kept constant. But Fig. 5 shows that increase in mean pressure, which is equivalent to an increase in density ratio, increases the response function at first resonant frequency if the mean surface temperature is allowed to vary with pressure. The first resonant frequency nondimensionalized by solid thermal relaxation time remains almost independent of mean pressure.

The effect of the imposed pressure amplitude on the burn rate response function, at a frequency equal to the first resonant frequency of the previous case, is shown in Fig.6. When the pressure amplitude is large, second (and even third) harmonics appear in the burn rate variations (which was found by the presence of significant values in the higher order Fourier coefficients of $\dot{r}(t)$). Further, a decrease in the mean burn rate, $\bar{r}$, is observed at large amplitudes of pressure. In the present case, mean burn rate is decreased by about 20% when the amplitude is 50% of the mean pressure. These results confirm the importance of nonlinearity in combustion instability. Such nonlinearity could be significant in the behavior of propellant combustion when subjected to large changes in pressures as in L^*- instability or rapid pressurization/depressurization.

Conclusions

The intrinsic, nonlinear stability limit of combustion has been computed considering the finite thermal inertia of the gas phase and compared with the theoretical, linear non-QSHOD model results. The response for pressure driven burning exhibits resonance at two values of the driving frequency. The first resonant mode occurs near the natural frequency at intrinsic stability limit. The second resonant mode occurs at a higher frequency and can be described only by inclusion of finite gas-phase thermal inertia. Inclusion of gas-phase thermal inertia stabilizes the burning near the first resonant frequency and destabilizes at higher frequencies. The mean burn rate decreases with increase in the amplitude of pressure oscillations.

Acknowledgment

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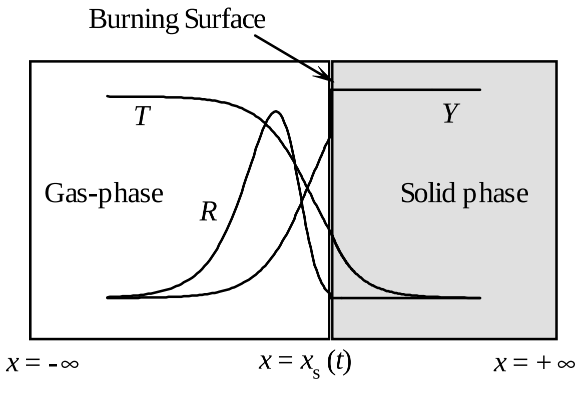

Fig. 1 Problem Domain

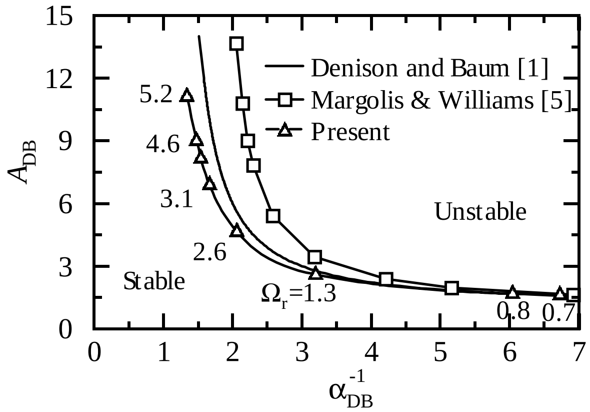

Fig.2 Stable and unstable regions

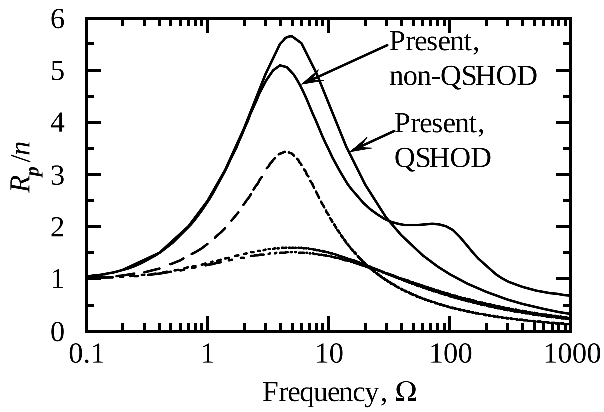

Fig.3 Comparison with linear, QSHOD results of DeLuca [4] and others

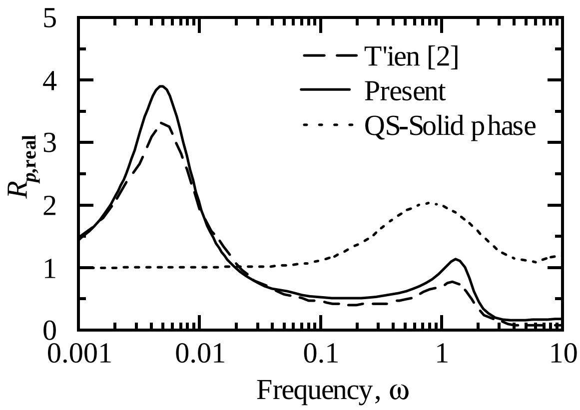

Fig. 4 Comparison with linear, non-QSHOD results of T'ien[2]

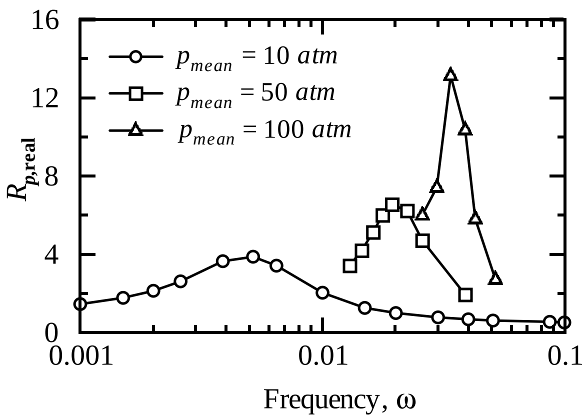

Fig. 5 Effect of mean pressure (p_{mean}) on the real part of response function

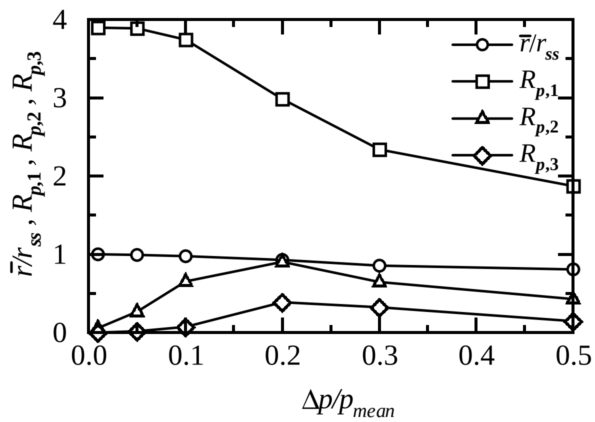

Fig. 6 Effect of amplitude of pressure oscillation (Δp) on response function